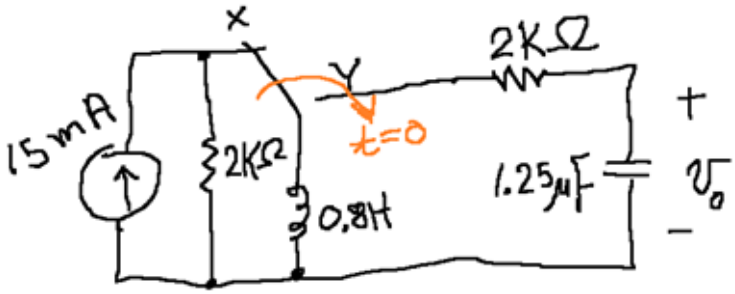
499 HW4 SP22

1

$L := 0.8 \qquad C := 1.25 \cdot 10^{-6} \qquad R := 2000$



Before t=0:

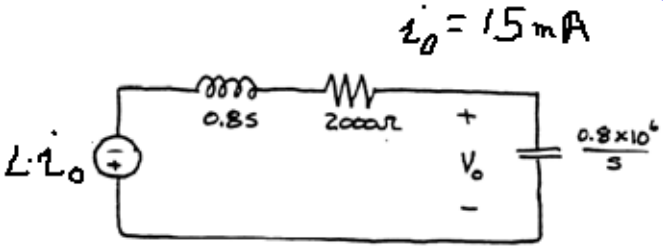
$v0 := 0$ $\qquad$ $i0 := 15 \cdot 10^{-3}$

L becomes short circuit

$L \cdot i0 = 0.012$

After t=0: s - domain circuit

Voltage across the C by voltage division



$$V := -\frac{L \cdot i0 \cdot \left(\frac{1}{sC}\right)}{R + s \cdot L + \frac{1}{s \cdot C}}$$

$$V := -\frac{\frac{L \cdot i0}{L \cdot C}}{s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C}}$$

$$V := -\frac{L \cdot i0}{R \cdot s \cdot C + s^2 \cdot L \cdot C + 1}$$

$\frac{R}{L} = 2500$ $\qquad$ $\frac{1}{L \cdot C} = 1 \cdot 10^6$ $\qquad$ $\frac{i0}{C} = 12000$

$$V := -\frac{\frac{i0}{C}}{s^2 + \frac{R \cdot s}{L} + \frac{1}{L \cdot C}}$$

$$V := -\frac{12000}{s^2 + 2500 \cdot s + 1 \cdot 10^6}$$

$\frac{30}{2000} = 0.015$

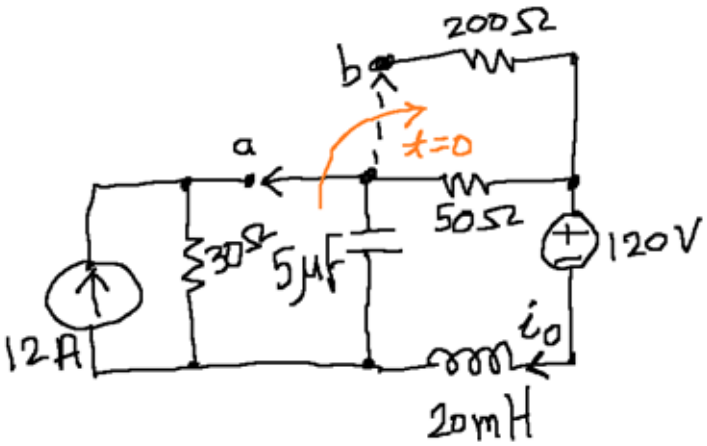
$$V := \frac{-12000}{(s + 500) \cdot (s + 2000)}$$

$$V := \frac{K1}{s + 500} + \frac{K2}{s + 2000}$$

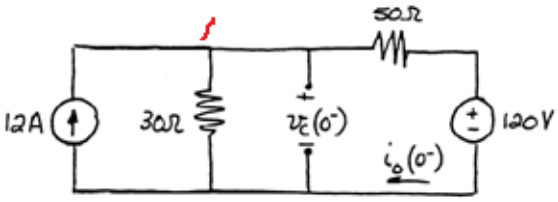
$K1 := -8 \qquad K2 := 8$

v(t)=(-8*exp(-500t) + 8*exp(-2000t)) u(t)

2



Before t<0, C becomes open and L becomes short



V1 = v0 and i0 = current through 50 ohm resistor

-12 + V1/30+ (V1-120)/50 = 0

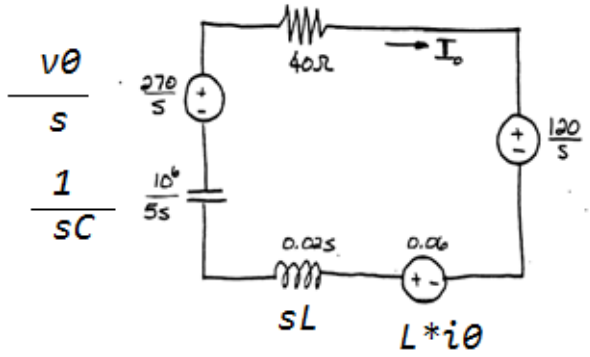
V1(1/30+ 1/50) = 12 + 120/50

$$V1 := \frac{12 + \frac{12}{5}}{\frac{1}{30} + \frac{1}{50}} = 270$$

v0 := V1 = 270

$$i0 := \frac{V1 - 120}{50} = 3$$

After t>0:



$$I := \frac{\frac{270}{s} + L \cdot i0 - \frac{120}{s}}{s \cdot L + 40 + \frac{1}{s \cdot C}}$$

$$I := \frac{270 \cdot C + L \cdot i0 \cdot C \cdot s - 120 \cdot C}{s^2 \cdot L \cdot C + 40 \cdot s \cdot C + 1}$$

$$I := \frac{\frac{270}{L} + s \cdot i0 - \frac{120}{L}}{s^2 + \frac{40}{L} \cdot s + \frac{1}{L \cdot C}}$$

L := 0.02 $\quad$ $C := 5 \cdot 10^{-6}$

$$\frac{270}{L} - \frac{120}{L} = 7500 \qquad \frac{40}{L} = 2000 \qquad \frac{1}{L \cdot C} = 1 \cdot 10^7$$

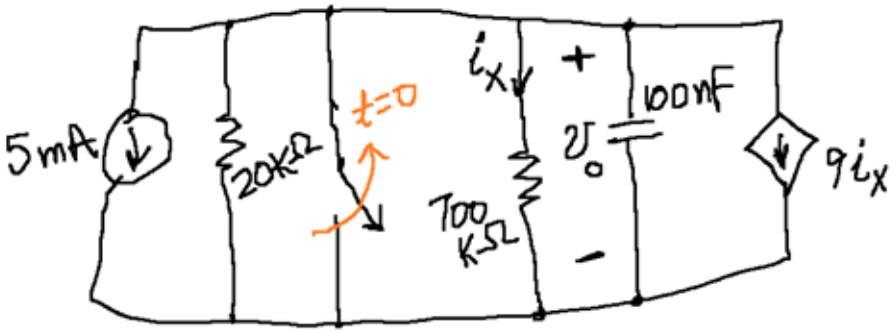
$$I := \frac{3 \cdot s + 7500}{s^2 + 2000 \cdot s + 10^7}$$

$$I := \frac{3 \cdot s + 7500}{(s + 1000)^2 + 3000^2}$$

$$I := \frac{3 \cdot (s + 1000)}{(s + 1000)^2 + 3000^2} + \frac{\frac{4500}{3000} \cdot 3000}{(s + 1000)^2 + 3000^2}$$

i(t) = exp(-1000t)*(3*cos(3000t) + 1.5*sin(3000t)) u(t)

3



Before t<0, all current flows through the shorted line, voltage is zero. So the initial voltage in the capacitor is zero.

After t>0, s=domain circuit

Apply node voltage method, the node voltage is voltage across the capacitor, v

$$\frac{I}{s}+\frac{V(s)}{200000}+\frac{V(s)}{700000}+9\cdot\frac{V(s)}{700000}+SCV(s)=0 \quad \text{because} \quad ix=V/700K$$

$I := 5\cdot 10^{-3}$ $C := 100\cdot 10^{-9}$

$$V := \frac{\frac{-I}{s}}{s\cdot C+\frac{1}{20000}+\frac{10}{700000}}$$

$$\frac{I}{C}=50000$$

$$V := \frac{-I}{\left(s^{2}\cdot C+\left(\frac{1}{20000}+\frac{10}{700000}\right)\cdot s\right)}$$

$$\frac{\frac{1}{20000}+\frac{10}{700000}}{C}=642.8571429$$

$$V := \frac{-\frac{I}{C}}{\left(s^{2}+\left(\frac{\frac{1}{20000}+\frac{10}{700000}}{C}\right)\cdot s\right)}$$

$$K2 := -\frac{50000}{642.86}=-77.7774321$$

$$V := \frac{-50000}{s\cdot(s+642.86)}$$

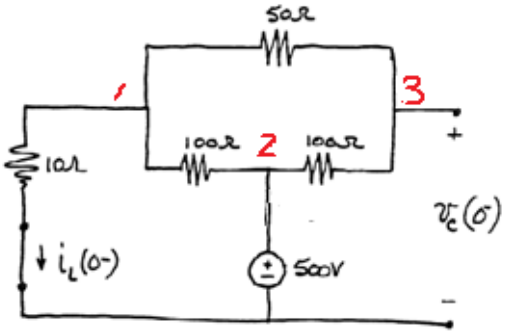
$K1 := -K2 = 77.7774321$

$$V := \frac{K1}{s}+\frac{K2}{s+642.86}$$

v (t) = 77.78*(1-exp(-642.86t)) u(t)

4

For t<0:



V2 = 500 and V3 = v0, i0 = V1/10

Node 1: V1/10 + (V1-500)/100 + (V1-V3)/50 = 0

Node 3: (V3-500)/100 + (V3-V1)/50 = 0

$V2 := 500$

$$X := \begin{bmatrix} \frac{1}{10}+\frac{1}{100}+\frac{1}{50} & -\frac{1}{50} \\ -\frac{1}{50} & \frac{1}{100}+\frac{1}{50} \end{bmatrix} \quad Y := \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$V := X^{-1} \cdot Y = \begin{bmatrix} 71.4285714 \\ 214.2857143 \end{bmatrix}$$

$V1 := V_1 = 71.4285714$

$V3 := V_2 = 214.2857143$

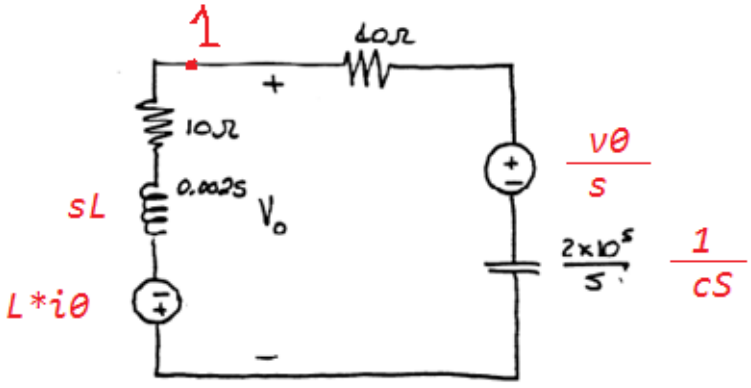
Therefore

$$i0 := \frac{V1}{10} = 7.1428571$$

$v0 := V3 = 214.2857143$

After t>0 s-domain circuit

$L := 0.002$ $C := 5 \cdot 10^{-6}$ $L \cdot i0 = 0.0142857$



By node voltage method

V(s) = V1

$$\frac{V1 + L \cdot i0}{10 + s \cdot L} + \frac{V1 - \frac{v0}{s}}{40 + \frac{1}{C \cdot s}} = 0$$

$$V1 \cdot \left(\frac{1}{10 + s \cdot L} + \frac{1}{40 + \frac{1}{s \cdot C}} \right) + \left(\frac{L \cdot i0}{10 + s \cdot L} + \frac{-\frac{v0}{s}}{40 + \frac{1}{s \cdot C}} \right) = 0$$

$$V1 \cdot \left(\frac{1}{10 + s \cdot L} + \frac{1 \cdot C \cdot s}{40 \cdot C \cdot s + 1} \right) + \left(\frac{L \cdot i0}{10 + s \cdot L} + \frac{-v0 \cdot C}{40 \cdot C \cdot s + 1} \right) = 0$$

$$V1 \cdot \left(\frac{1 + 40 \cdot C \cdot s + C \cdot s \cdot (10 + s \cdot L)}{(10 + s \cdot L) \cdot (40 \cdot C \cdot s + 1)} \right) + \frac{L \cdot i0 \cdot (40 \cdot C \cdot s + 1) - v0 \cdot C \cdot (10 + s \cdot L)}{(10 + s \cdot L) \cdot (40 \cdot C \cdot s + 1)} = 0$$

$$V1 := \frac{L \cdot i0 \cdot (40 \cdot C \cdot s + 1) - v0 \cdot C \cdot (10 + s \cdot L)}{C \cdot L \cdot s^2 + 50 \cdot C \cdot s + 1}$$

$$V1 := \frac{\frac{i0}{C} \cdot (40 \cdot C \cdot s + 1) - \frac{v0}{L} \cdot (10 + s \cdot L)}{s^2 + \frac{50}{L} \cdot s + \frac{1}{L \cdot C}}$$

$$V1 := \frac{(40 \cdot i0 - v0) \cdot s + \frac{i0}{C} - \frac{10 \cdot v0}{L}}{s^2 + \frac{50}{L} \cdot s + \frac{1}{L \cdot C}}$$

$$\frac{50}{L} = 25000 \qquad \frac{1}{L \cdot C} = 1 \cdot 10^8$$

$$40 \cdot i0 - v0 = 71.4285714$$

$$\frac{i0}{C} - \frac{10 \cdot v0}{L} = 3.5714286 \cdot 10^5$$

$$V1 := \frac{71.43 \cdot s + 357143}{s^2 + 25000 \cdot s + 10^8}$$

$$V1 := \frac{71.43 \cdot s + 357143}{(s + 5000) \cdot (s + 20000)}$$

$$V1 := \frac{K1}{s + 5000} + \frac{K2}{s + 20000}$$

(K1 + K2)*s + K1*20000+K2*5000 = 71.43s + 357143

K 1 + K1 = 71.43 --> K1=71.43-K2

$$K2 := \frac{71.43 \cdot 20000 - 357143}{15000} = 71.4304667$$

$$K1 := 71.43 - K2 = -0.0004667$$

$$K1 + K2 = 71.43$$

$$K1 \cdot 20000 + K2 \cdot 5000 = 3.57143 \cdot 10^5$$

v (t) = (-0.00047*exp(-5000t)+71.4*exp(20000t))u(t)