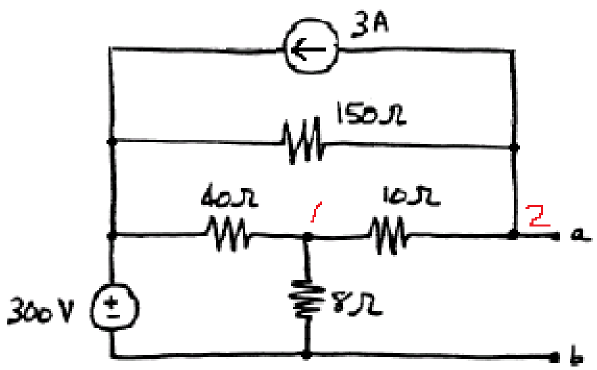
499 HW3 Solution SP22

1



Vth = V2

At node 1:

(V1 - 300)/40 + V1/8 + (V1-V2)/10 = 0 ----(1)

At node 2:

(V2 - V1)/10 + 3 + (V2 - 300)/150 = 0 ---- (2)

From (1) and (2):

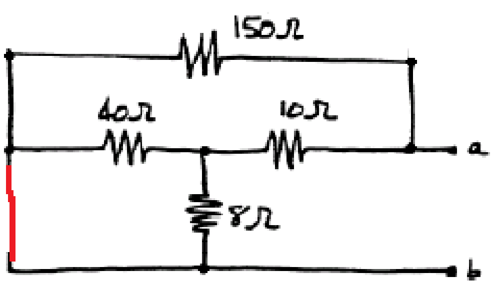
$$V := \begin{bmatrix} V1 \\ V2 \end{bmatrix}$$

$$X := \begin{bmatrix} \frac{1}{40} + \frac{1}{8} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10} + \frac{1}{150} \end{bmatrix} \qquad Y := \begin{bmatrix} \frac{300}{40} \\ \frac{300}{150} - 3 \end{bmatrix}$$

From X*V = Y ---> V = X^*Y

$$V := X^{-1} \cdot Y = \begin{bmatrix} 42 \\ 30 \end{bmatrix} \qquad \boxed{Vth := V_2 = 30}$$

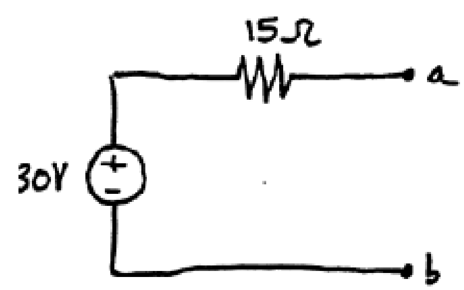
For Rth, we apply input resistance method: (deactivation of all independent source first)



Rth = 150 // (10 + (40//8))

$$R1 := 10 + \frac{40 \cdot 8}{40 + 8} = 16.6666667$$

$$\boxed{Rth := \frac{150 \cdot R1}{150 + R1} = 15}$$



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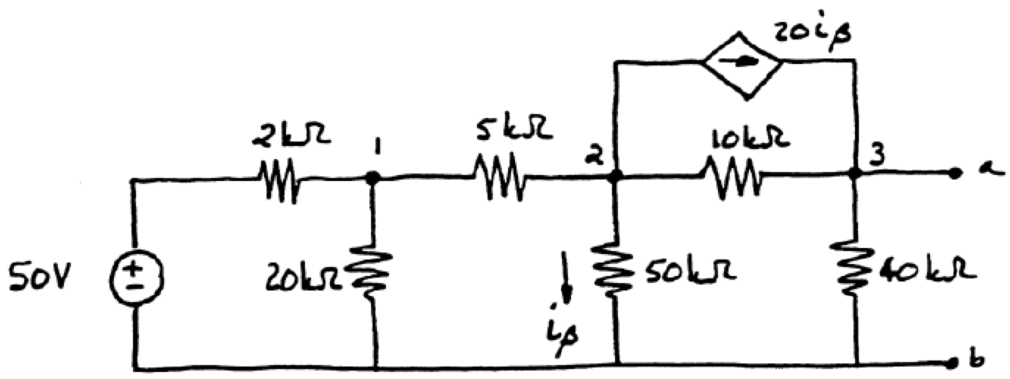


Vth = V3

At node 1:

(V1 - 50)/2 + V1/20 + (V1-V2)/5 = 0 --(1)

At node 2:

(V2 - V1)/5 + V2/50 + (V2-V3)/10 +20*iβ=0

Since iβ = V2/50

(V2 - V1)/5 + V2/50 + (V2-V3)/10 +(20*V2)/50=0 -- (2)

At node 3: V3/40 +(V3-V2)/10 - 20*(V2/50)=0 --(3)

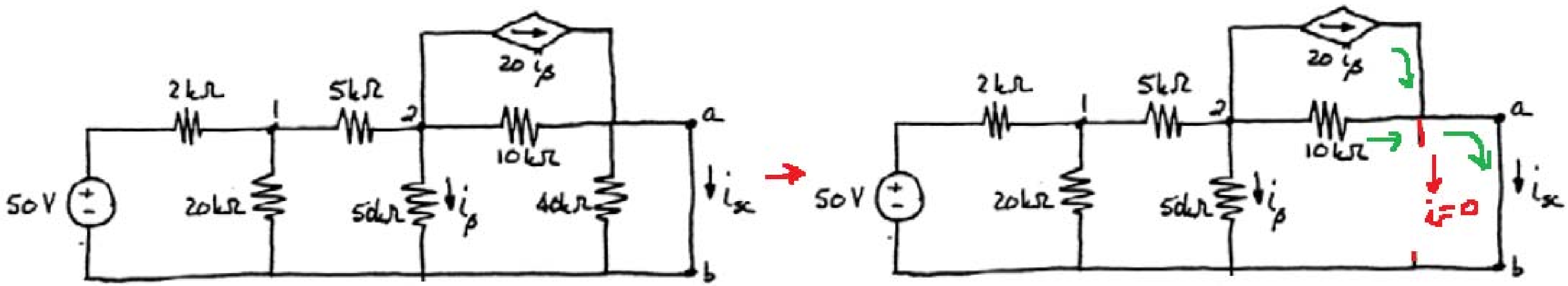
From (1), (2), and (3):

$$X := \begin{bmatrix} \frac{1}{2}+\frac{1}{20}+\frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{50}+\frac{1}{10}+\frac{20}{50} & -\frac{1}{10} \\ 0 & -\frac{1}{10}-\frac{20}{50} & \frac{1}{40}+\frac{1}{10} \end{bmatrix} \qquad Y := \begin{bmatrix} \frac{50}{2} \\ 0 \\ 0 \end{bmatrix}$$

$$V := X^{-1} \cdot Y = \begin{bmatrix} 40 \\ 25 \\ 100 \end{bmatrix}$$

$$Vth := V_3 = 100$$

For Rth, we apply short circuit current approach:



iSC = V2/10 + 20*iβ = V2/10 + 20*(V2/50) [mA]

At node 1:

(V1 - 50)/2 + V1/20 + (V1-V2)/5 = 0 --(1)

At node 2:

(V2 - V1)/5 + V2/50 + V2/10 +(20*V2)/50=0 -- (2)

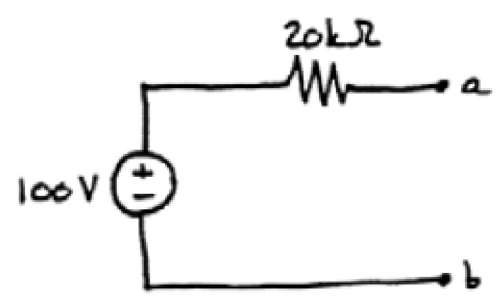
$$X := \begin{bmatrix} \frac{1}{2}+\frac{1}{20}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{50}+\frac{1}{10}+\frac{20}{50} \end{bmatrix} \qquad Y := \begin{bmatrix} \frac{50}{2} \\ 0 \end{bmatrix}$$

$$V := X^{-1} \cdot Y = \begin{bmatrix} 36 \\ 10 \end{bmatrix}$$

$$V2 := V_2 = 10$$

$$Isc := \frac{V2}{10} + 20 \cdot \left(\frac{V2}{50}\right) = 5$$ mA

$$\boxed{Rth := \frac{Vth}{Isc} = 20}$$ kΩ



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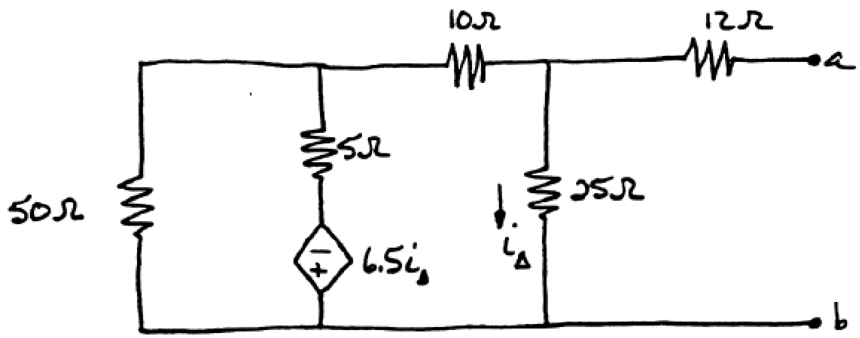
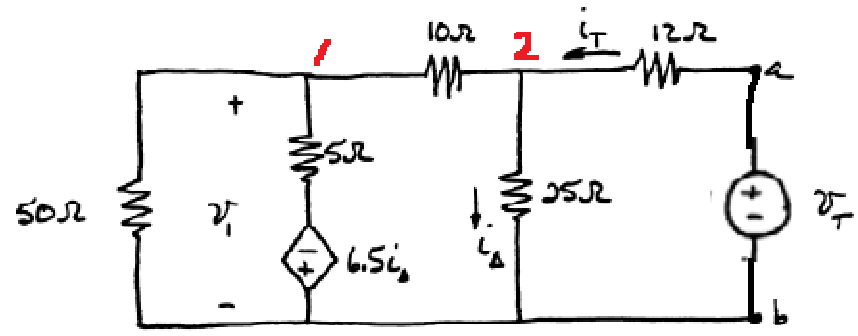


Vth = 0 since there is no independent source

And the only approach to get Rth

is the test source(voltage) method and

Rth = VT/IT



At node 1:

V1 /50 + (V1+6.5*iΔ)/5 + (V1-V2)/10 =0

But since iΔ = V2/25

V1 /50 + (V1+6.5*(V2/25))/5 + (V1-V2)/10 =0 ---- (1)

At node 2:

(V2 - V1)/10 + V2/25 + (V2-VT)/12 = 0

And IT= (VT-V2)/12

For calculation purpose (Especially with Smath), let us assume VT = 1,

Then IT= (1-V2)/12 and Rth = VT/IT = 1/IT

(V2 - V1)/10 + V2/25 + (V2-1)/12 = 0 ----- (2)

From (1) and (2):

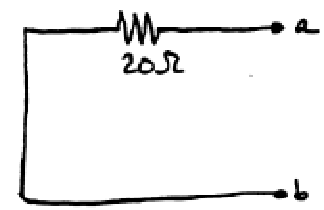
$$X := \begin{bmatrix} \frac{1}{50}+\frac{1}{5}+\frac{1}{10} & \frac{6.5}{25\cdot 5}-\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{25}+\frac{1}{12} \end{bmatrix} \qquad Y := \begin{bmatrix} 0 \\ \frac{1}{12} \end{bmatrix}$$

$$V := X^{-1} \cdot Y = \begin{bmatrix} 0.06 \\ 0.4 \end{bmatrix}$$

$$V2 := V_2 = 0.4$$

$$IT := \frac{(1 - V2)}{12} = 0.05$$

$$\boxed{Rth := \frac{1}{IT} = 20}$$



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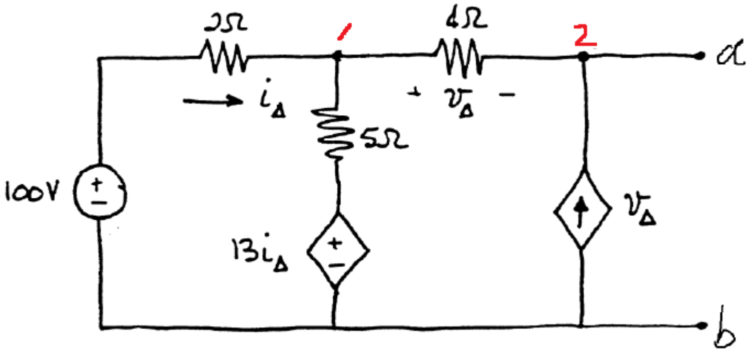


Vth = V2

At node 1:

(V1 - 100)/2 + (V1 - 13*iΔ)/5 + (V1 -V2)/4 = 0

Since iΔ = (100-V1)/2

(V1 - 100)/2 + (V1 - 13*((100-V1)/2)/5 + (V1 -V2)/4 = 0 -(1)

At node 2: (V2 - V1)/4 - vΔ = 0

But vΔ = (V1-V2)

(V2 - V1)/4 - (V1-V2)=0 -- (2)

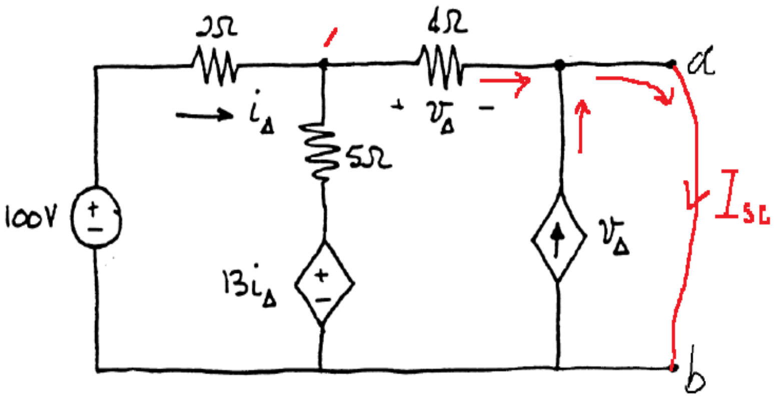
From (1) and (2):

$$X := \begin{bmatrix} \frac{1}{2} + \frac{1}{5} + \frac{\frac{13}{2}}{5} + \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} - 1 & \frac{1}{4} + 1 \end{bmatrix} \qquad Y := \begin{bmatrix} \frac{100}{2} + \frac{\frac{13 \cdot 100}{2}}{5} \\ 0 \end{bmatrix}$$

$$V := X^{-1} \cdot Y = \begin{bmatrix} 90 \\ 90 \end{bmatrix}$$

$$\boxed{Vth := V_2 = 90}$$

For Rth, let us apply short circuit approach:



Isc = vΔ + V1/4 but vΔ=V1

Isc = V1+V1/4 = (5/4)V1

At node 1:

(V1 - 100)/2 + (V1 - 13*(100-V1)/2)/5 + V1/4 = 0 -- (1)

V1 (1/2 + 1/5 + (13/2)/5 + 1/4) = 100/2 +13*(100/2)/5

$$V1 := \frac{\frac{100}{2} + \frac{\frac{13 \cdot 100}{2}}{5}}{\frac{1}{2} + \frac{1}{5} + \frac{\frac{13}{2}}{5} + \frac{1}{4}} = 80$$

$$Isc := \frac{5}{4} \cdot V1 = 100$$

$$Rth := \frac{Vth}{Isc} = 0.9$$