Surge Impedance and Loading Power

- Remember: Surge Impedance

$$Z_c = \sqrt{\frac{L}{C}} \quad [\Omega]$$

- Lossless Line is assumed
- If a load is purely resistive equal to the surge impedance

$$Z_{load} = \sqrt{\frac{L}{C}}$$

- The, Load Current is given by (where VL is line-to-line voltage)

$$|I_L| = \frac{|V_L|}{\sqrt{3} \cdot \sqrt{\frac{L}{C}}}$$

Surge Impedance and Loading Power

- If a load is purely resistive equal to the surge impedance $Z_{load} = \sqrt{\frac{L}{C}}$
- The, Power (3-phase) delivered to the load:

$$|I_L| = \frac{|V_L|}{\sqrt{3} \cdot \sqrt{\frac{L}{C}}}$$

(in a lossless line)

3ϕ

$$P_L = \sqrt{3}|V_L||I_L|$$

$$= \sqrt{3}|V_L| \frac{|V_L|}{\sqrt{3}\sqrt{\frac{L}{C}}} = \frac{|V_L|^2}{\sqrt{\frac{L}{C}}} \text{ [W]}$$

$= SIL$ (Surge Impedance Loading) [W]

No [Var]

-

Surge Impedance Loading

$$V(x) = \frac{V_R + I_R Z_c}{2} e^{\alpha x} e^{j\beta x} + \underbrace{\frac{V_R - I_R Z_c}{2}}_{0} e^{-\alpha x} e^{-j\beta x}$$

No Reflected Voltage !!

$$e^{\gamma x} = e^{\alpha x} \cdot e^{j\beta x}$$

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{zy}$$

$$V_R = I_R Z_c$$

$$\frac{V_R + I_R Z_c}{2} = V_R \Rightarrow V(x) = V_R$$

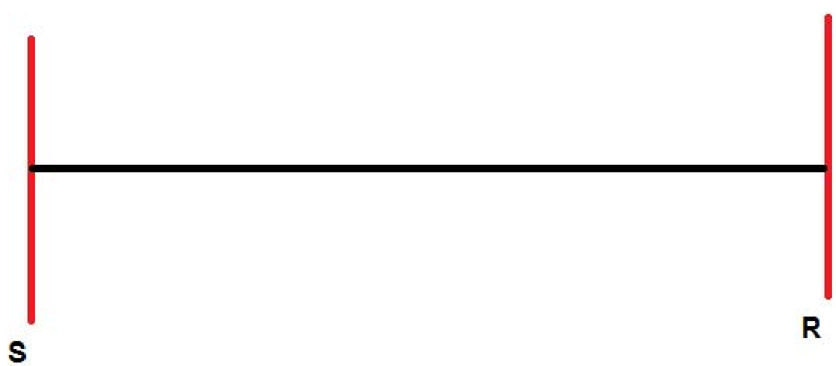
Voltage at any location x is the same as the receiving end voltage **Flat Voltage Profile**

$$j\omega L \; j\omega C$$

$$= j\omega\sqrt{LC}$$

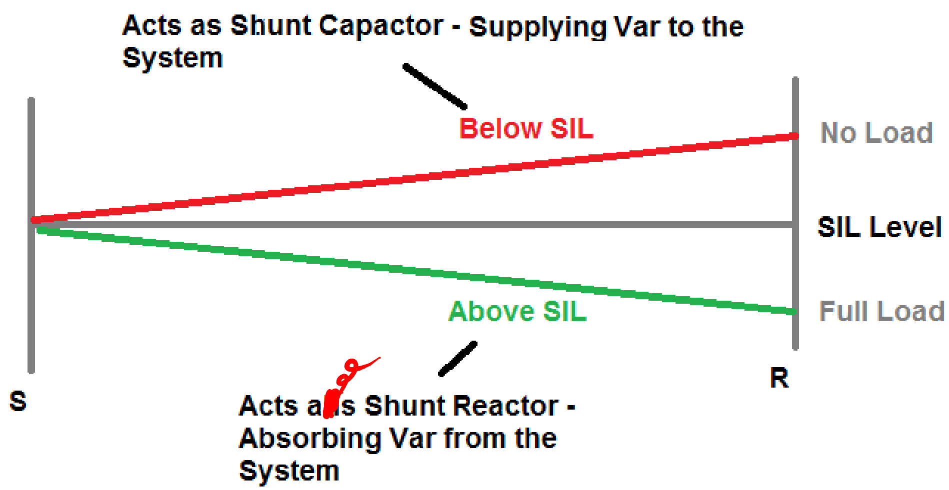
$$= \underbrace{0}_{\alpha = 0} + j\beta$$

No attenuation



Surge Impedance and Loading Power

- Role of Transmission Line



- Practical use of SIL(Surge Impedance Loading)
  - A permissible loading of a transmission may be expressed by a fraction of its SIL
  - SIL may provide a comparison of load carrying capability of lines

Surge Impedance Loading

- Role of Transmission LIne

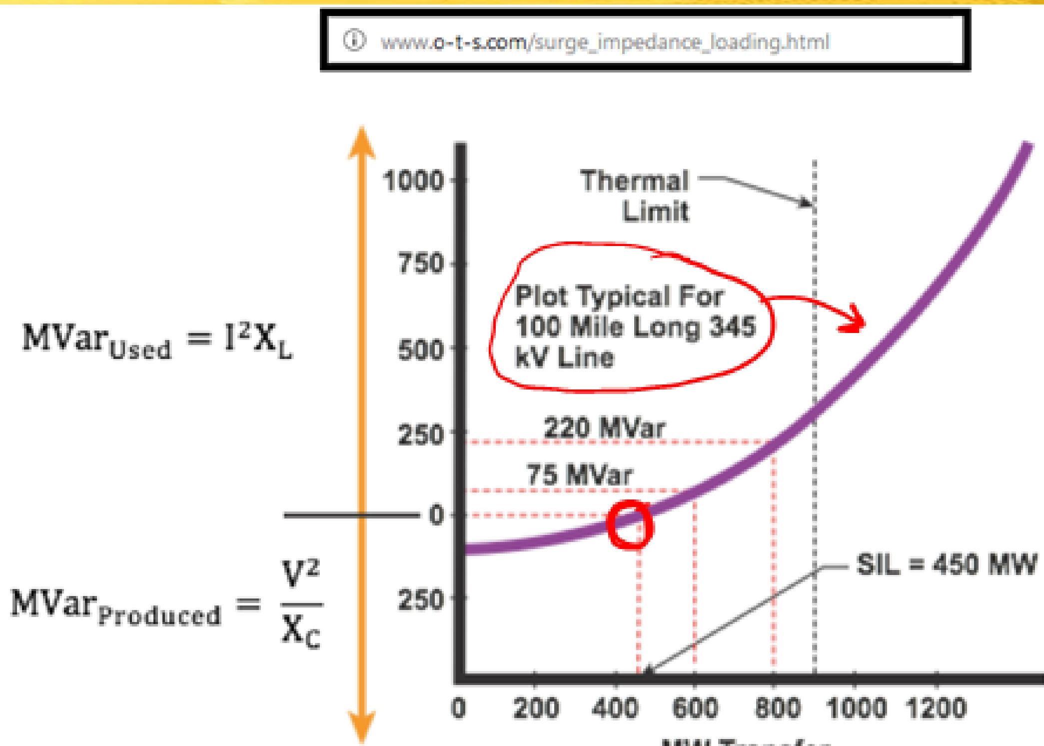


Figure 1
Surge Impedance Loading of a Transmission Line

Hyperbolic Form or the equations

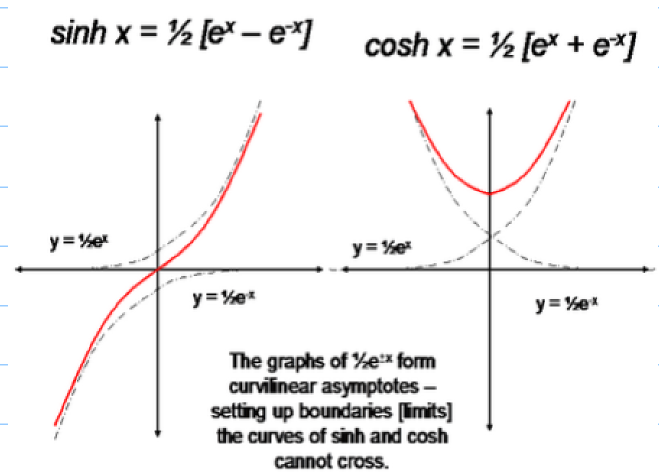
$$V(x) = \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$

$$I(x) = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

more convenient form by hyperbolic functions:

$$\begin{cases} \sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2} \\ \cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2} \end{cases}$$

$$\rightarrow \begin{cases} e^{\theta} = \sinh\theta + \cosh\theta \\ e^{-\theta} = \cosh\theta - \sinh\theta \end{cases}$$



Hyperbolic Form or the equations

$$V(x) = \frac{V_R + I_R \cdot Z_C}{2} e^{\gamma x} + \frac{V_R - I_R Z_C}{2} e^{-\gamma x}$$

$$= \frac{V_R + I_R \cdot Z_C}{2} \left(\sinh \gamma x + \cosh \gamma x \right)$$

$$+ \frac{V_R - I_R \cdot Z_C}{2} \left(\cosh \gamma x - \sinh \gamma x \right)$$

$$= \left[\frac{V_R + \cancel{I_R \cdot Z_C} + V_R - \cancel{I_R \cdot Z_C}}{2} \cosh \gamma x \right] + \left[\left(\frac{\cancel{V_R} + I_R \cdot Z_C - \cancel{V_R} + I_R \cdot Z_C}{2} \right) \sinh \gamma x \right]$$

$$V(x) = V_R \cdot \cosh \gamma x + I_R \cdot Z_C \cdot \sinh \gamma x$$

Similarly,

$$I(x) = I_R \cdot \cosh \gamma x + \frac{V_R}{Z_C} \cdot \sinh \gamma x$$

⌘

Hyperbolic Form or the equations

ABCD Constants? → Need to find V_S.

$V_S = V(x)\big|_{x=l}$ ← length of the line

S ←— l —→ R

$x=l$ $\qquad$ $x=0$

$$\begin{cases} V_S = V_R \cdot \cosh \gamma l + I_R \cdot Z_c \cdot \sinh \gamma l \\ I_S = \dfrac{V_R}{Z_c} \cdot \sinh \gamma l + I_R \cdot \cosh \gamma l \end{cases}$$

$A = \cosh \gamma l \qquad B = Z_c \cdot \sinh \gamma l$

$C = \dfrac{\sinh \gamma l}{Z_c} \qquad D = \cosh \gamma l$

S $\boxed{ABCD}$ R

NOTE:

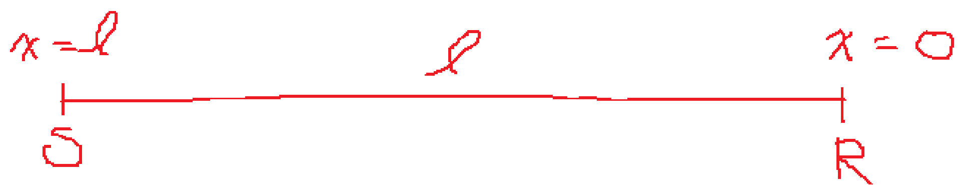
V_S, V_R : Line-to-neutral Voltage (V_P)

I_S, I_R : Line current.

$$\begin{cases} V_R = V_S \cdot \cosh \gamma l - I_S \cdot Z_c \cdot \sinh \gamma l \\ I_R = I_S \cdot \cosh \gamma l - \dfrac{V_S}{Z_c} \cdot \sinh \gamma l \end{cases}$$

What if your calculator does not do Hyperbolic?

- $$V(x) = \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$
  $$I(x) = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$



- $$\boxed{\begin{aligned} V_S &= \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma \ell} + \frac{V_R - I_R Z_c}{2} e^{-\gamma \ell} \\ I_S &= \frac{V_R/Z_c + I_R}{2} e^{\gamma \ell} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma \ell} \end{aligned}}$$

Hyperbolic Form or the equations

More on hyperbolic cos & sin

1
$$\cosh(\alpha l + j\beta l) = \cosh \alpha l \cos\beta l + j \sinh \alpha l \sin\beta l$$
$$\sinh(\alpha l + j\beta l) = \sinh \alpha l \cos\beta l + j \cosh \alpha l \sin\beta l$$

Maclaurin's Series

2
$$\cosh\theta = 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \frac{\theta^6}{6!} + \cdots$$
$$\sinh\theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \frac{\theta^7}{7!} + \cdots$$

Phasor

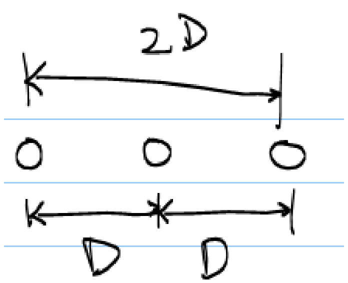
3
$$\cosh(\alpha + j\beta) = \frac{e^{\alpha}e^{j\beta} + e^{-\alpha}e^{-j\beta}}{2} = \frac{1}{2}\left(e^{\alpha}\angle\beta + e^{-\alpha}\angle{-\beta}\right)$$
$$\sinh(\alpha + j\beta) = \frac{e^{\alpha}e^{j\beta} - e^{-\alpha}e^{-j\beta}}{2} = \frac{1}{2}\left(e^{\alpha}\angle\beta - e^{-\alpha}\angle{-\beta}\right)$$

Example 5.1

- A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor's outside diameter is 0.977in. And the flat horizontal spacing is 7.25 m (or 23.8 ft)between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b)Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.

Example 5.1

⌘ A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor's outside diameter is 0.977in. And the flat horizontal spacing is 7.25 m (or 23.8 ft)between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b)Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.



D= 23.8 ft

d = 0.977 in

R = 0.1603/mile

$$Deq := \sqrt[3]{D12 \cdot D23 \cdot D31} = 29.9861 \quad ft$$

$$d := 0.977 \quad in \qquad len := 230 \quad mile$$

$$R0 := 0.1603 \quad \Omega \,/\, mile$$

$$r := \frac{d}{2 \cdot 12} = 0.0407 \quad ft$$

$$r' := r \cdot 0.7788 \quad ft$$

$$L := 2 \cdot 10^{-7} \cdot \ln\left(\frac{Deq}{r'}\right) = 1.3704 \cdot 10^{-6} \quad H \,/\, m$$

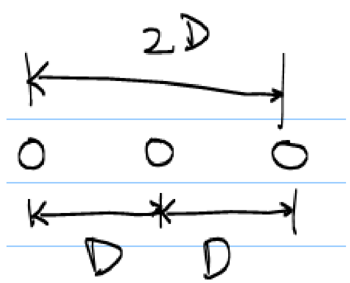
$$XL := 2 \cdot \pi \cdot 60 \cdot L \cdot 1609 = 0.8313 \quad \Omega \,/\, mile$$

$$z := R0 + i \cdot XL = 0.1603 + 0.8313 \cdot i \quad \Omega \,/\, mile$$

⌘

Example 5.1

⌘ A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor's outside diameter is 0.977in. And the flat horizontal spacing is 7.25 m (or 23.8 ft)between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b)Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.



$D = 23.8$ ft

$d = 0.977$ in

$R = 0.1603/\text{mile}$

$$k := 8.85\cdot 10^{-12}$$

$$Cn := \frac{2\cdot\pi\cdot k}{\ln\left(\frac{Deq}{r}\right)} = 8.4226\cdot 10^{-12} \quad F/m$$

$$Xc := \frac{1}{2\cdot\pi\cdot 60\cdot Cn\cdot 1609\cdot i} = -1.9574\cdot 10^{5}\cdot i \quad \Omega\ \text{mile}$$

$$y := \frac{1}{Xc} = 5.1089426\cdot 10^{-6}\cdot i \quad S/\text{mile}$$

$$\gamma l := \sqrt{y\cdot z}\cdot len = 0.0454922 + 0.4761608\cdot i$$

⌘

Example 5.1

⌘ $V_S = V_R \cdot \cosh \gamma l + I_R \cdot Z_C \cdot \sinh \gamma l$

$I_S = \frac{V_R}{Z_C} \cdot \sinh \gamma l + I_R \cdot \cosh \gamma l$

km (or 230 mi) long. The conductor's outside
pacing is 7.25 m (or 23.8 ft)between
0.1603 ohm/mile. The load on the line is 125
d the voltage, current, and power at the
the line. (c) Determine the wavelength and

$\gamma l := \sqrt{y \cdot z} \cdot len = 0.0454922 + 0.4761608 \cdot i$

$zc := \sqrt{\frac{z}{y}} = 405.2236295 - 38.7148641 \cdot i$

$PR := 125 \cdot 10^{6}$

$VR := \frac{215000}{\sqrt{3}} = 1.2413031 \cdot 10^{5}$

$IR := \frac{PR}{3 \cdot VR} = 335.6687612$

$\cosh(\gamma l) = 0.8896811 + 0.0208595 \cdot i$

$\sinh(\gamma l) = 0.0404456 + 0.4588448 \cdot i$

⌘

Example 5.1

⌘ $V_S = V_R \cdot \cosh \gamma l + I_R \cdot Z_C \cdot \sinh \gamma l$

$I_S = \dfrac{V_R}{Z_C} \cdot \sinh \gamma l + I_R \cdot \cosh \gamma l$

$\cosh(\gamma l) = 0.8896811 + 0.0208595 \cdot i$

$\sinh(\gamma l) = 0.0404456 + 0.4588448 \cdot i$

$VS := VR \cdot \cosh(\gamma l) + IR \cdot Zc \cdot \sinh(\gamma l) = 1.2190069 \cdot 10^{5} + 64476.1743055 \cdot i$

$|VS| = 1.3790198 \cdot 10^{5}$ $\quad \arg(VS) \cdot \dfrac{180}{\pi} = 27.8754275$

$IS := \dfrac{VR}{Zc} \cdot \sinh(\gamma l) + IR \cdot \cosh(\gamma l) = 297.6084302 + 147.4593272 \cdot i$

$|IS| = 332.1371$ $\quad \arg(IS) \cdot \dfrac{180}{\pi} = 26.3575$

$pf := \cos(\arg(VS) - \arg(IS)) = 0.9996491$

$PS := 3 \cdot |VS| \cdot |IS| = 1.3740708 \cdot 10^{8}$

⌘

Example 5.1

- A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor's outside diameter is 0.977in. And the flat horizontal spacing is 7.25 m (or 23.8 ft)between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b)Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.

No Load Voltage: VR at IR=0

$$VRnl := \frac{VS}{\cosh(\gamma l)} = 1.3863912 \cdot 10^{5} + 69220.5777301 \cdot i$$

$$Vreg := \frac{|VRnl| - |VR|}{|VR|} \cdot 100 = 24.8357524 \text{ percent}$$

Example 5.1

- A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor's outside diameter is 0.977in. And the flat horizontal spacing is 7.25 m (or 23.8 ft)between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b)Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.

phase shift per mile

Since γl = αl+j*βl

$$\beta l := \mathrm{Im}(\gamma l) = 0.4761608$$

$$\beta := \frac{\beta l}{len} = 0.0020703$$

wavelength

$$\lambda := \frac{2\cdot\pi}{\beta} = 3034.9677489 \quad \text{mile}$$

Velocity

$$f := 60 \quad \text{Hz}$$

$$v := f\cdot\lambda = 1.8209806\cdot 10^{5} \quad \text{mile / sec}$$

Class Activity

⌘ Class Activity on Transmission Parameters

A 3-phase 60-Hz transmission line is 250 miles long. The voltage at the sending end is 220 kV. The parameters of the line are R = 0.2 Ω/mile, X= 0.8 Ω/mile, and Y = 5.3 μS/mile. Find the sending-end current when there is no load on the line.

$$V_S = V_R \cdot \cosh \gamma l + I_R \cdot Z_C \cdot \sinh \gamma l$$

$$I_S = \frac{V_R}{Z_C} \cdot \sinh \gamma l + I_R \cdot \cosh \gamma l$$

$$V_S = \frac{V_R + I_R \cdot Z_C}{2} e^{\gamma l} + \frac{V_R - I_R Z_C}{2} e^{-\gamma l}$$

$$I_S = \frac{V_R/Z_C + I_R}{2} e^{\gamma l} - \frac{V_R/Z_C - I_R}{2} e^{-\gamma l}$$

⌘ $\gamma l := \sqrt{y \cdot z} \cdot len$ $\quad Zc := \sqrt{\frac{z}{y}}$ $\quad VS := \frac{220}{\sqrt{3}} \cdot 10^{3}$

For IR=0: $\quad VR := \frac{VS}{\cosh(\gamma l)}$ $\quad IS := \left(\frac{VR}{Zc}\right) \cdot \sinh(\gamma l)$

What if your calculator does not do Hyperbolic?

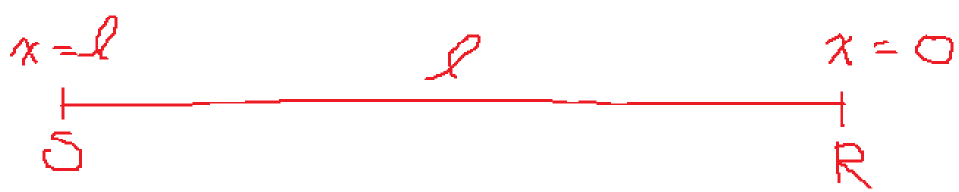
$$V(x) = \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$

$$I(x) = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

$\gamma = \alpha + j\beta$

$e^{\gamma x}$

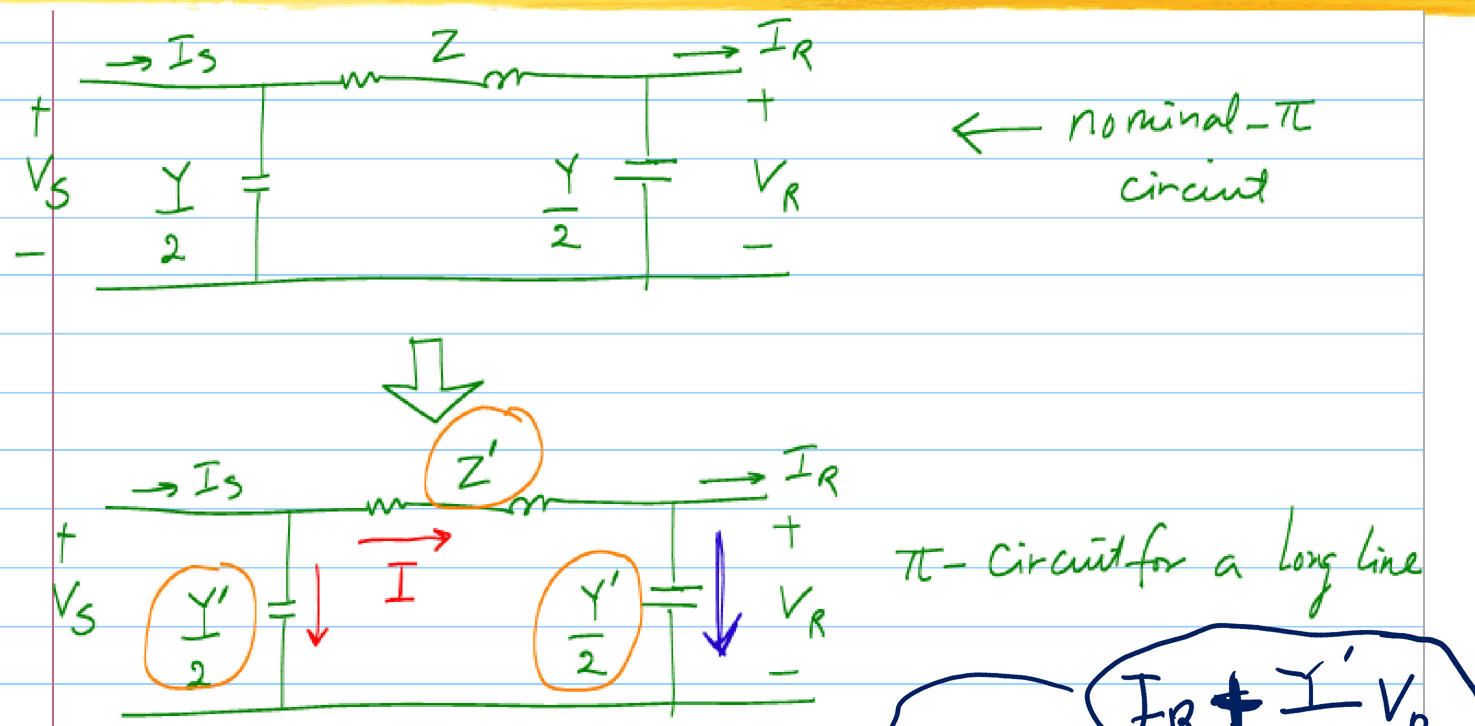
$= e^{\alpha x} \cdot \underbrace{e^{j\beta x}}_{(\cos \beta x + j \sin \beta x)\ [rad]}$



$$\boxed{\begin{aligned} V_S &= \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma \ell} + \frac{V_R - I_R Z_c}{2} e^{-\gamma \ell} \\ I_S &= \frac{V_R/Z_c + I_R}{2} e^{\gamma \ell} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma \ell} \end{aligned}}$$

Equivalent Circuit of a Long Line

- Can we use Pi-circuit even for a long line?



$$I = I_R + \frac{Y'}{2} V_R \rightarrow V_S = I Z' + V_R$$

Equi

⌘ Can we

$$V_S = \left(I_R + \frac{Y'}{2}V_R\right)Z' + V_R$$

$$V_S = \left(\frac{Z'Y'}{2} + 1\right)V_R + Z' I_R$$

$$V_S = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l$$

$$I_S = I_R \cosh \gamma l + \frac{V_R}{Z_c} \sinh \gamma l$$

$$Z_c = \sqrt{\frac{z}{y}} \qquad \gamma = \sqrt{zy}$$

$$V_S = \left(\frac{ZY}{2}+1\right)V_R + Z \cdot I_R$$

$$I_S = Y\left(\frac{ZY}{4}+1\right)V_R + \left(\frac{ZY}{2}+1\right)I_R$$

Nominal π-ckt

$$Z' = Z_c \cdot \sinh \gamma l$$

$$= \sqrt{\frac{z}{y}} \cdot \sinh \gamma l = \frac{\sqrt{z}\cdot\sqrt{z}\cdot l}{\sqrt{y}\cdot\sqrt{z}\cdot l} \cdot \sinh \gamma l$$

$$\sqrt{z}\cdot\sqrt{z}\cdot l = zl = Z \text{ (total series Impedance)}$$

$$Z' = Z \frac{\sinh \gamma l}{\sqrt{zy}\, l} = Z \cdot \frac{\sinh \gamma l}{\gamma l}$$

multiplying factor to convert nominal-π ckt to long-line equivalent-π ckt

If γl is small, $\frac{\sinh \gamma l}{\gamma l} \sim 1$

medium/short Transmission line

E ne

⌘ Can

For Y'!

$$V_S = \boxed{\left(\frac{Z'Y'}{2}+1\right)} V_R + Z' \cdot I_R$$

Equate

$$V_S = V_R \boxed{\cosh \gamma l} + I_R Z_c \sinh \gamma l$$

$\left(Z = zl, \; \gamma l = \sqrt{yz} \cdot l \right)$

From above

$$Z' = Z \cdot \frac{\sinh \gamma l}{\gamma l} = Z_c \cdot \sinh \gamma l$$

$$\frac{Z'Y'}{2} + 1 = \cosh \gamma l$$

$$\frac{Y'(Z_c \cdot \sinh \gamma l)}{2} + 1 = \cosh \gamma l$$

$$\frac{Y'}{2} = \frac{\cosh \gamma l - 1}{Z_c \cdot \sinh \gamma l} = \frac{1}{Z_c} \cdot \frac{\cosh \gamma l - 1}{\sinh \gamma l}$$

Ec ... ne

⌘ Can

$$\tanh\left(\frac{\gamma l}{2}\right) = \frac{\cosh \gamma l - 1}{\sinh \gamma l}$$

$$\frac{Y'}{2} = \left(\frac{1}{Z_c}\right) \cdot \tanh\left(\frac{\gamma l}{2}\right)$$

$$\begin{cases} Z_c = \sqrt{\frac{z}{y}} \\ Y = y\,l \end{cases} \qquad \gamma = \sqrt{yz}$$

$$\frac{1}{Z_c} = \sqrt{\frac{y}{z}} = \sqrt{\frac{y \cdot y}{zy}} = \frac{y}{\sqrt{zy}} = \frac{y\,l}{\sqrt{zy}\;l} = \frac{Y/2}{\gamma l/2}$$

$$= \frac{Y}{2} \cdot \frac{1}{\gamma l/2}$$

$$\tanh x = \frac{e^{2x}-1}{e^{2x}+1}$$

$$\rightarrow \frac{Y'}{2} = \frac{Y}{2} \cdot \frac{\tanh\left(\frac{\gamma l}{2}\right)}{\left(\frac{\gamma l}{2}\right)}$$

Correction factor from nominal-π ckt to long line equivalent-π circuit.

Nominal π to long-line π

- From Nominal Pi-ckt equation (50 < l < 150)

$$V_S = \left(\frac{ZY}{2} + 1\right) V_R + Z \cdot I_R$$

$$I_S = Y\left(\frac{ZY}{4} + 1\right) V_R + \left(\frac{ZY}{2} + 1\right) \cdot I_R$$

- Lon-line equivalent pi-ckt equation (l > 150 miles)

$$Z \leftarrow Z \cdot \frac{\sinh \gamma l}{\gamma l} \qquad (Z')$$

$$Y \leftarrow Y \cdot \frac{\tanh\left(\frac{\gamma l}{2}\right)}{\tanh\left(\frac{\gamma l}{2}\right)} \qquad (Y')$$

Example 5.3

⌘ Find the equivalent pi-circuit for the line described in Example 5.1 and compare it with the nominal pi.

$z := R0 + i \cdot XL = 0.1603 + 0.8313 \cdot i \quad \Omega \,/\, \text{mile}$

$|z| = 0.8465775 \;\Omega$

$Z := z \cdot len = 36.869 + 191.1903779 \cdot i$

$\arg(z) \cdot \frac{180}{\pi} = 79.0851086$

$|Z| = 194.7128238 \qquad \arg(Z) \cdot \frac{180}{\pi} = 79.0851086$

deg

$k := 8.85 \cdot 10^{-12}$

$Cn := \frac{2 \cdot \pi \cdot k}{\ln\left(\frac{Deq}{r}\right)} = 8.4226 \cdot 10^{-12} \quad F \,/\, m$

$Xc := \frac{1}{2 \cdot \pi \cdot 60 \cdot Cn \cdot 1609 \cdot i} = -1.9574 \cdot 10^{5} \cdot i \quad \Omega \;\text{mile}$

$y := \frac{1}{Xc} = 5.1089426 \cdot 10^{-6} \cdot i \quad S \,/\, \text{mile} \qquad |y| = 5.1089426 \cdot 10^{-6}$

$\arg(y) \cdot \frac{180}{\pi} = 90 \quad \text{deg}$

$Y := y \cdot len = 0.0011751 \cdot i$

$Y2 := \frac{Y}{2} = 0.0005875 \cdot i \qquad |Y2| = 0.0005875 \qquad \arg(Y2) \cdot \frac{180}{\pi} = 90$

$\gamma l := \sqrt{y \cdot z} \cdot len = 0.0454922 + 0.4761608 \cdot i$

$Zc := \sqrt{\frac{z}{y}} = 405.2236295 - 38.7148641 \cdot i$

Example 5.3

- Find the equivalent pi-circuit for the line described in Example 5.1 and compare it with the nominal pi.

$$Z' := Zc \cdot \sinh(\gamma l) = 34.1536353 + 184.3689102 \cdot i$$

$$|Z'| = 187.5056422 \qquad \arg(Z') \cdot \frac{180}{\pi} = 79.5051445$$

$$|Z| = 194.7128238$$

$$\frac{|Z'| - |Z|}{\|Z\|} \cdot 100 = -3.7014417 \quad \text{percent}$$

$$Y2' := Y2 \cdot \frac{\tanh\left(\frac{\gamma l}{2}\right)}{\frac{\gamma l}{2}} = 2.2197466 \cdot 10^{-6} + 0.0005988 \cdot i$$

$$|Y2'| = 0.0005988 \qquad |Y2| = 0.0005875$$

$$\frac{|Y2'| - |Y2|}{\|Y2\|} \cdot 100 = 1.9142259 \quad \text{percent}$$

$$\left\{ \begin{array}{l} Z' : 3.7\% \text{ lower} \\ Y' : 2\% \text{ higher} \end{array} \right\} \rightarrow \underline{\underline{\text{Ignorable}}}.$$

Nominal-π circuit can represent Even long lines!!

Power Flow

- Power Flow (P & Q)
  - Expressed by V, I, and pf
  - Expressed by ABCD circuit constants
    - Focus: How V_R and I_R affects V_S
    - ABCD Constants: All Complex Numbers

$$\bar{V}_S = \bar{A} \cdot \bar{V}_R + \bar{B} \cdot \bar{I}_R \quad (1)$$

$$\bar{I}_S = \bar{C} \cdot \bar{V}_R + \bar{D} \cdot \bar{I}_R \quad (2)$$

Let
$$\begin{cases} \bar{A} = A\angle\alpha \\ \bar{B} = B\angle\beta \\ \bar{V}_R = V_R\angle 0^\circ \\ \bar{V}_S = V_S\angle\delta \end{cases}$$

Power Flow

$$\bar{V}_S = \bar{A}\cdot\bar{V}_R + \bar{B}\cdot\bar{I}_R \quad (1)$$

Let

$$\begin{cases} \bar{A} = A\angle\alpha \\ \bar{B} = B\angle\beta \\ \bar{V}_R = V_R\angle 0^\circ \\ \bar{V}_S = V_S\angle\delta \end{cases}$$

$(1) \rightarrow$:

$$\bar{I}_R = \frac{\bar{V}_S - \bar{A}\cdot\bar{V}_R}{\bar{B}} = \frac{\bar{V}_S}{\bar{B}} - \frac{\bar{A}}{\bar{B}}\bar{V}_R$$

$$= \frac{V_S}{B}\angle\delta-\beta - \frac{A\cdot V_R}{B}\angle\alpha-\beta$$

From

$$\bar{S}_R = \bar{V}_R\cdot\bar{I}_R^{*} = \underbrace{\frac{V_S\cdot V_R}{B}\angle\beta-\delta}_{} - \underbrace{\frac{A\cdot V_R^2}{B}\angle\beta-\alpha}_{}$$

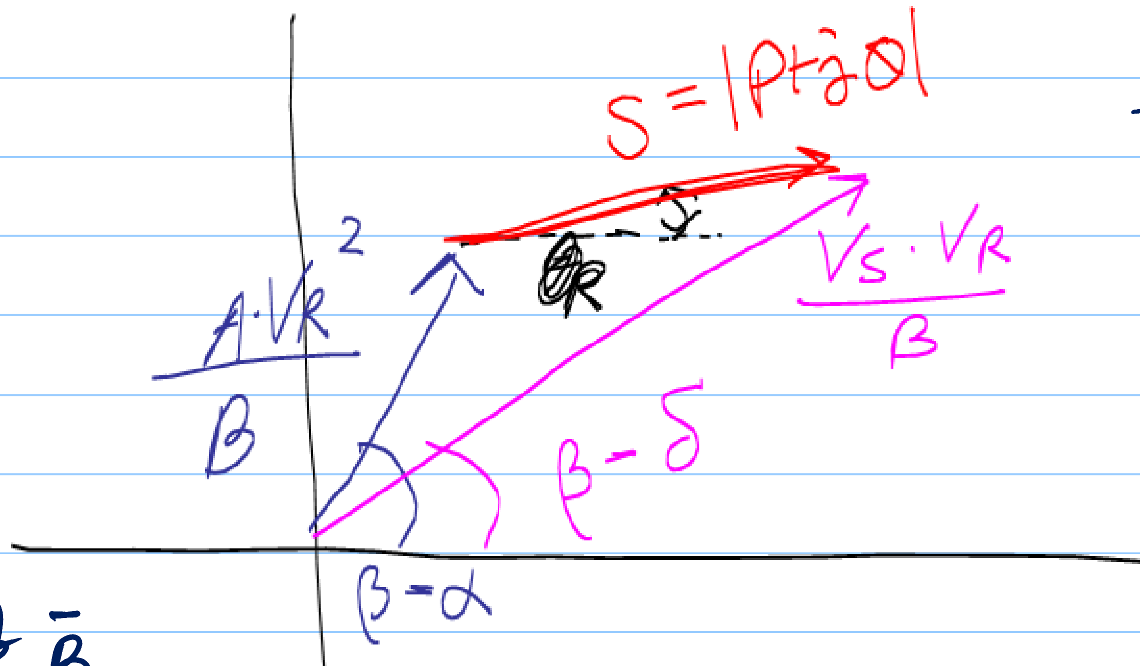
Power Flow

$$\bar{S}_R = \bar{V}_R \cdot \bar{I}_R^{*} = \frac{V_S \cdot V_R}{B} \angle \beta - \delta - \frac{A \cdot V_R^{2}}{B} \angle \beta - \alpha$$

δ angle of $\bar{V}_S$

α angle of $\bar{A}$

β angle of $\bar{B}$

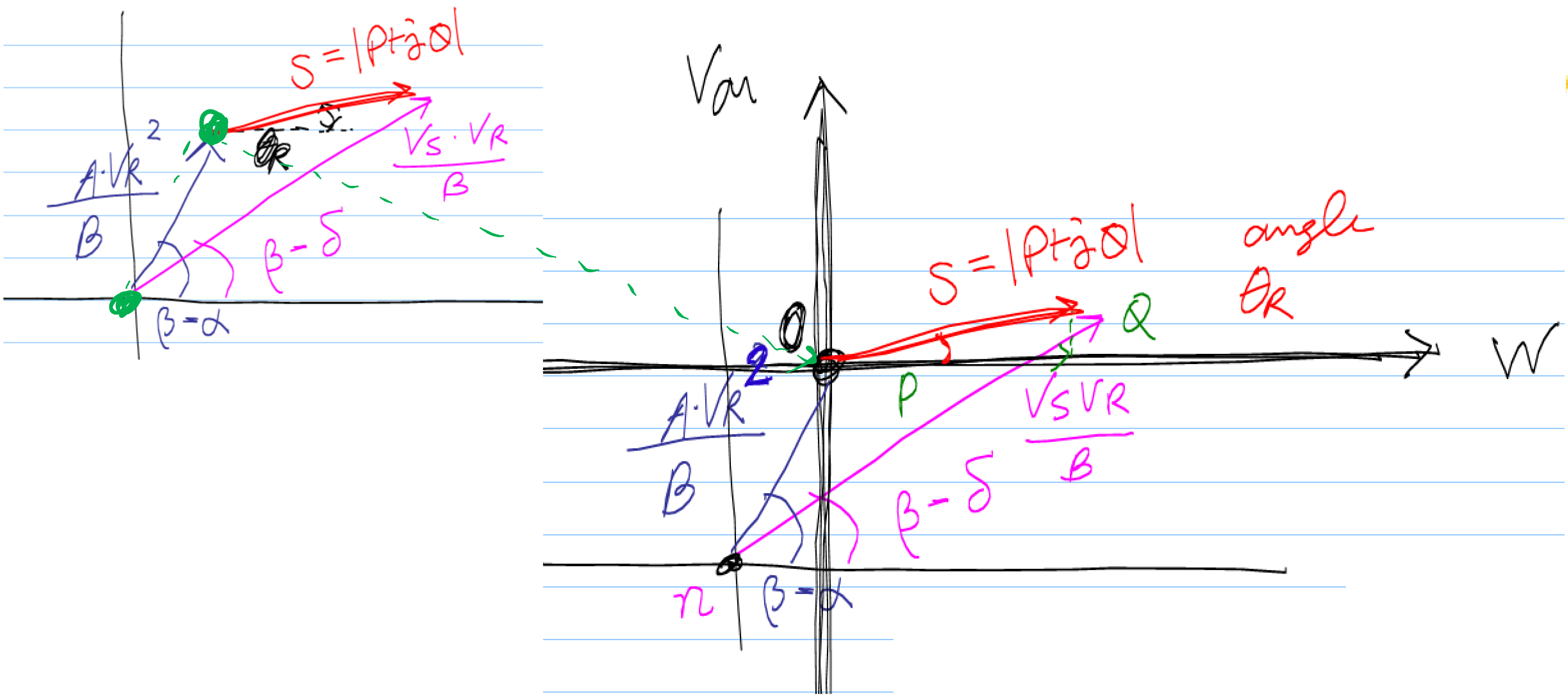


$A = \frac{ZY}{2} + 1$

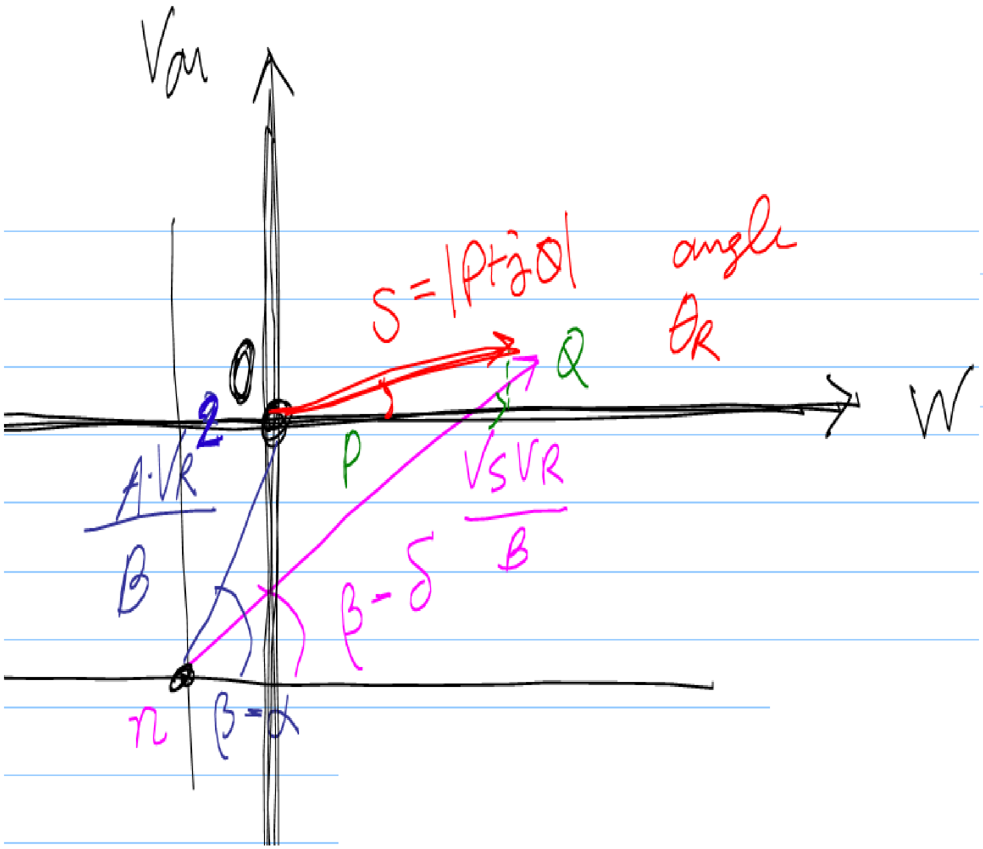
$B = Z$

nominal π − ckt

Power Flow: Origin Shift



Power Flow: Origin Shift



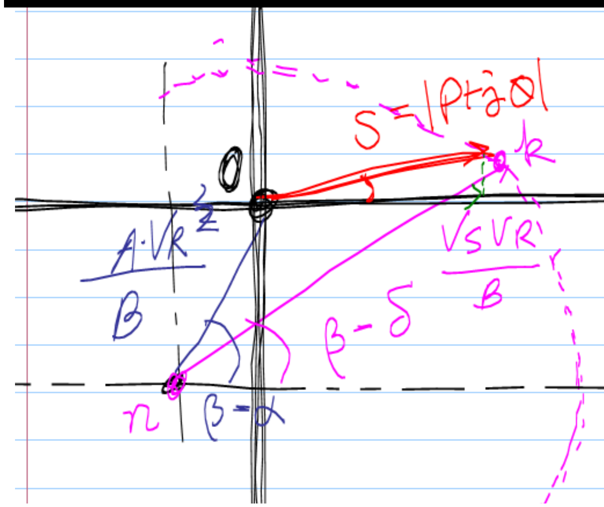
$$P_R = \frac{V_S \cdot V_R}{B} \cos(\beta - \delta) - \frac{A V_R^2}{B} \cos(\beta - \alpha)$$

$$\rightarrow \quad P_R = V_R \cdot I_R \cdot \cos \theta_R$$

$$Q_R = \frac{V_S \cdot V_R}{B} \sin(\beta - \delta) - \frac{A V_R^2}{B} \sin(\beta - \alpha)$$

$$\rightarrow \quad Q_R = V_R I_R \sin \theta_R$$

Power Flow



$$P_R = \frac{V_S \cdot V_R}{B}\cos(\beta-\delta) - \frac{A V_R^2}{B}\cos(\beta-\alpha)$$

$$\rightarrow P_R = V_R \cdot I_R \cdot \cos\theta_R$$

$$Q_R = \frac{V_S \cdot V_R}{B}\sin(\beta-\delta) - \frac{A V_R^2}{B}\sin(\beta-\alpha)$$

$$\rightarrow Q_R = V_R\, I_R \sin\theta_R$$

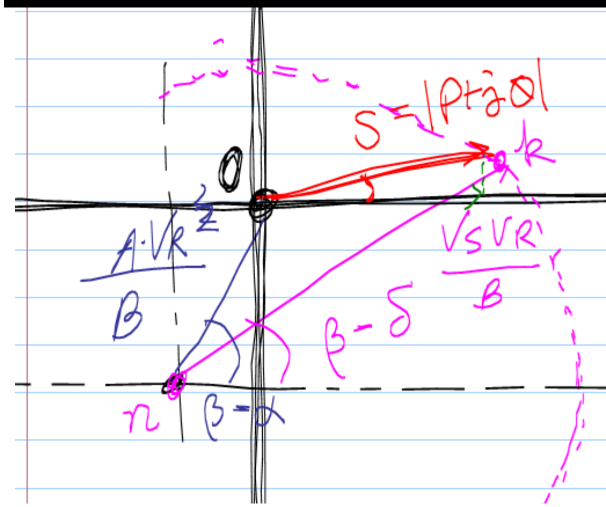
⌘Observations

- Point *n* is not dependent on IR
- Point n will not change if VR is constant: (A*VR^2)/B
- Distance between *n* and *k* is constant for fixed value of VS and VR: (VS*VR)/B
- Distance between *o* and *k* changes with changing load (IR)

$$\bar{S} = \bar{V}_R \cdot \bar{I}_R^{\,*}$$

Power Flow

- Observations
  - Because the distance between *n* and *k* is constant, the distance between *o* and *k* is constrained to move in a circle whose center is at *n*. for fixed V_R with Varying I_R
  - If VR is held constant, a different circle can be drawn for different values of VS's



$$P_R = \frac{V_S \cdot V_R}{B}\cos(\beta-\delta) - \frac{AV_R^2}{B}\cos(\beta-\alpha)$$

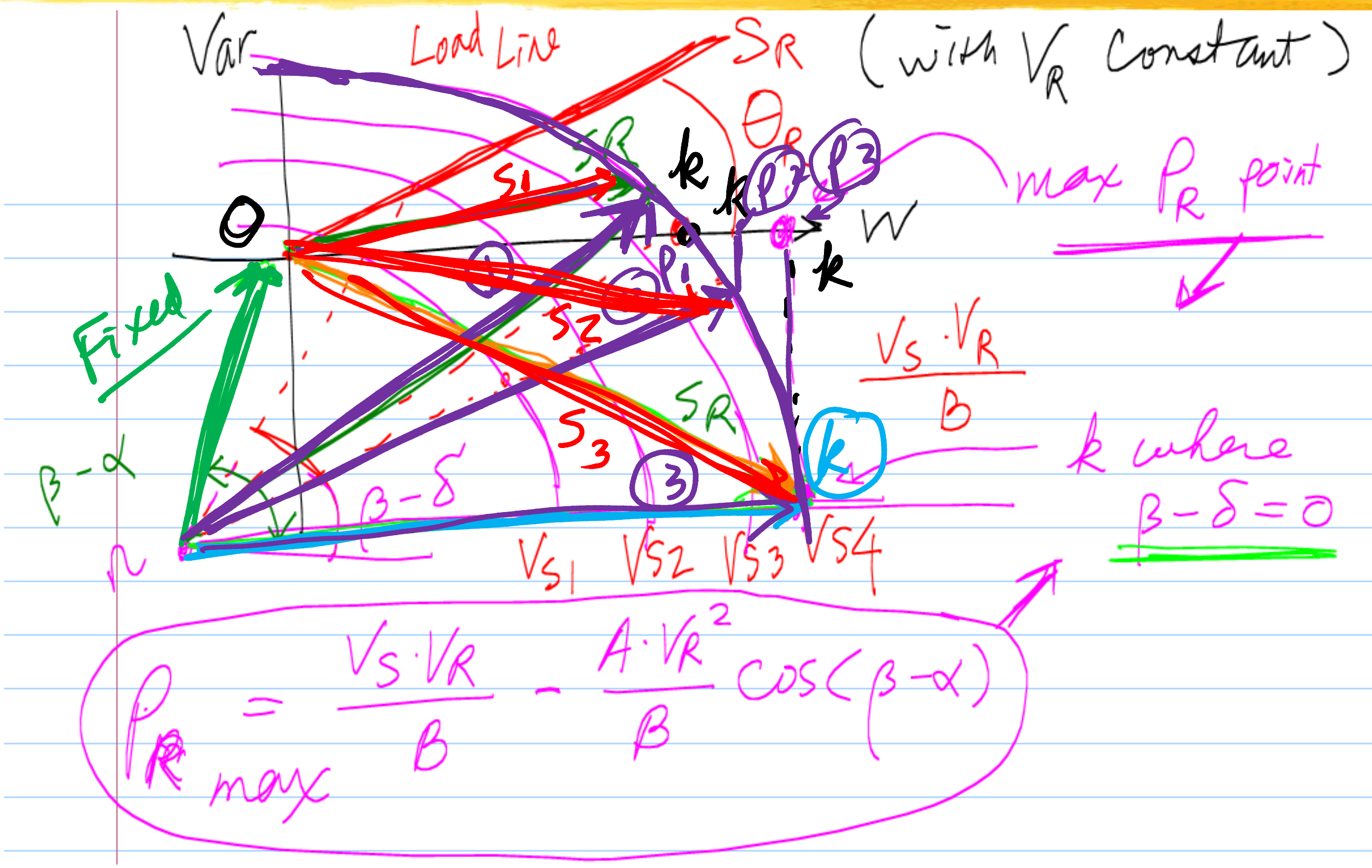
$$\rightarrow P_R = V_R \cdot I_R \cdot \cos\theta_R$$

$$Q_R = \frac{V_S \cdot V_R}{B}\sin(\beta-\delta) - \frac{AV_R^2}{B}\sin(\beta-\alpha)$$

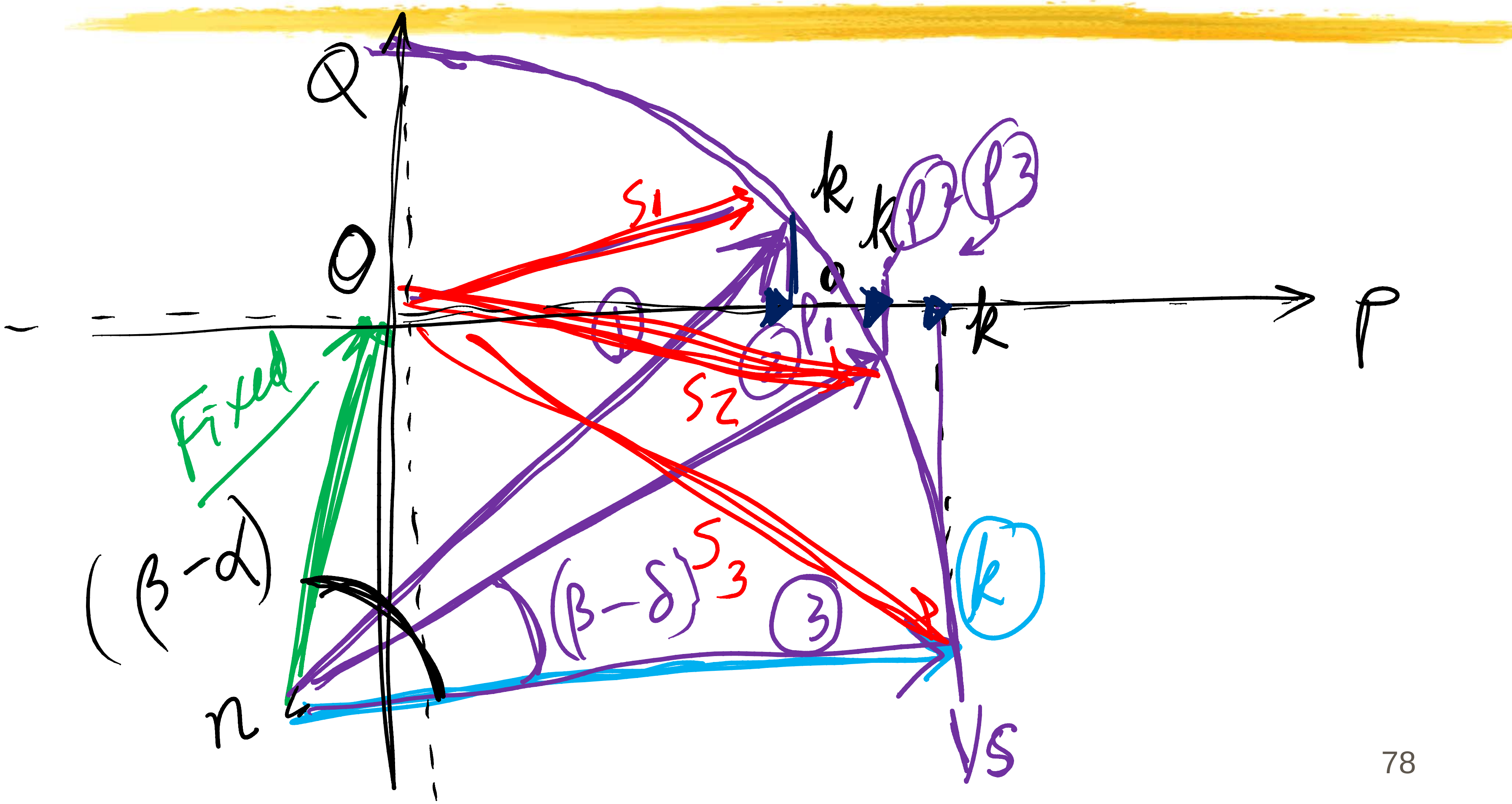
$$\rightarrow Q_R = V_R I_R \sin\theta_R$$

Power Flow for Max Power

- If VR is held constant, a different circle can be drawn for different values of VS's

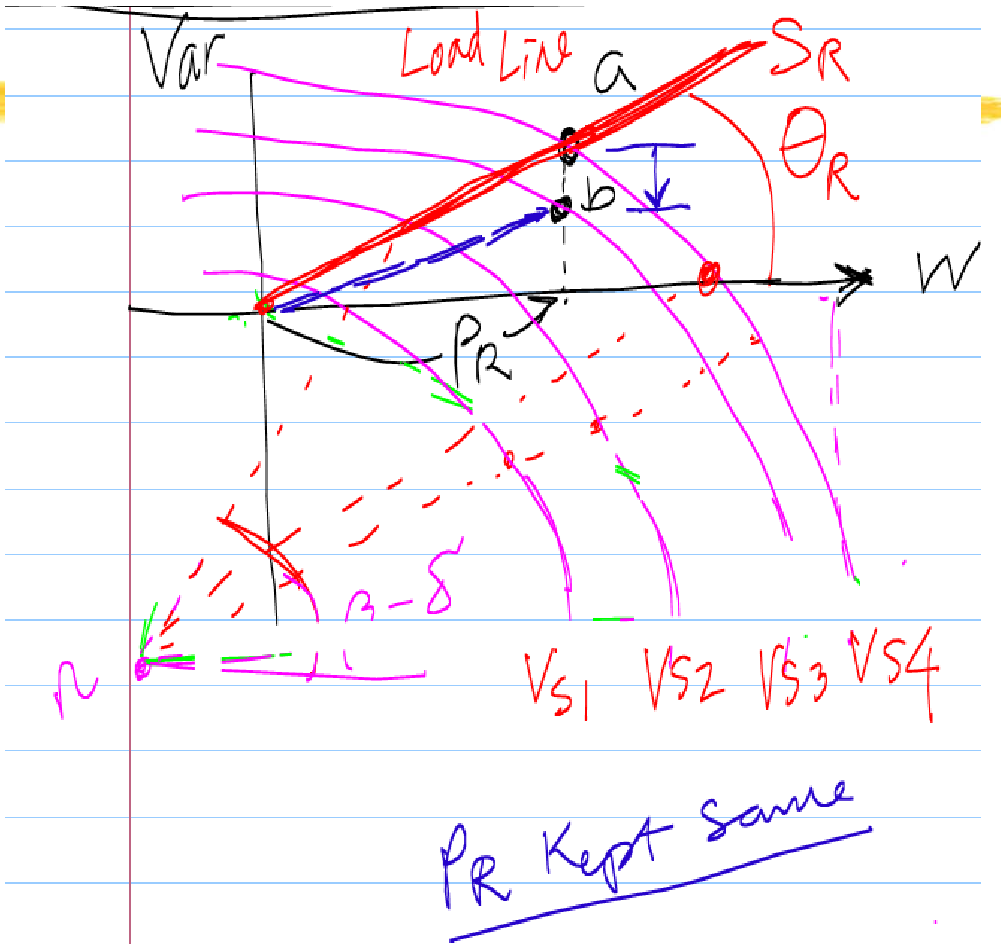


Power Flow for Max Power



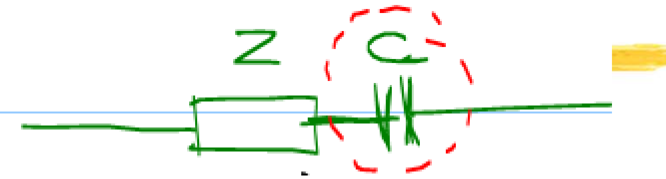
Power Flow: Impact of Vs and Var

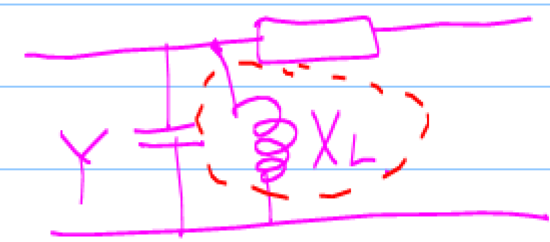
- When PR has to be maintained, moving from *a* to *b* involves:
  - Sending end voltage reduces from VS4 to VS3
  - (To keep the VR intact), due to the VS decrease, the Var has to be decreased, which means negative reactive power must be supplied by parallel capacitors



Reactive Compensation of Transmission Line

- Reactive Compensation by:
  - Series Compensation: Capacitor placed in each conductor reduces the series impedance
  - Shunt Compensation: Placement of inductors between conductor and neutral reduces the susceptance (admittance)





- Pi-Circuit Case

$$V_S = \overbrace{\left(\frac{ZY}{2}+1\right)}^{A} V_R + \overbrace{Z}^{B} \cdot I_R$$

$$I_S = \underbrace{Y\left(\frac{ZY}{4}+1\right)}_{C} V_R + \underbrace{\left(\frac{ZY}{2}+1\right)}_{D} \cdot I_R$$

Nominal-π (medium)

OR $Z \leftarrow Z'$, $Y \leftarrow Y'$

$$V_S = V_R \cdot \underbrace{\cosh \gamma l}_{A} + I_R \cdot \underbrace{Z_c \cdot \sinh \gamma l}_{B}$$

$$I_S = \underbrace{\frac{V_R}{Z_c} \cdot \sinh \gamma l}_{C} + I_R \cdot \underbrace{\cosh \gamma l}_{D}$$

Equivalent-π (long)

Reactive Compensation of Transmission Line

- Pi-Circuit Case
- Maximum Power Equation

$$P_{R_{max}} = \frac{V_S \cdot V_R}{B} - \frac{A \cdot V_R^2}{B} \cos(\beta - \alpha)$$

dominant factor

$$B = \begin{cases} Z \\ Z_c \cdot \sinh \gamma l = \sqrt{\frac{z}{y}} \cdot \sqrt{\frac{z}{z}} \cdot \frac{l}{l} \cdot \sinh \gamma l \end{cases}$$

$$= \frac{zl}{\sqrt{yz} \cdot l} \sinh \gamma l = Z \cdot \frac{\sinh \gamma l}{\gamma l}$$

$$\text{"Compensation Factor"} = \frac{X_C}{X_L}$$

capacitive reactance of series capacitance

Total inductive reactance the line

$$V_S = \underbrace{\left(\frac{ZY}{2} + 1\right)}_{A} V_R + \underbrace{Z}_{B} \cdot I_R$$

$$I_S = \underbrace{Y\left(\frac{ZY}{4} + 1\right)}_{C} V_R + \underbrace{\left(\frac{ZY}{2} + 1\right)}_{D} \cdot I_R$$

Nominal-π (medium)

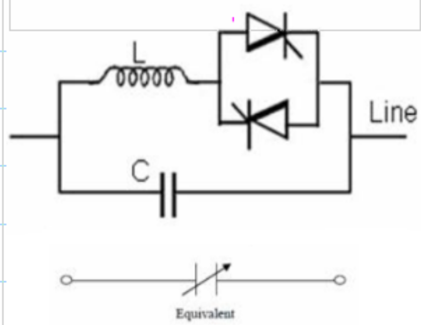
$$V_S = V_R \cdot \underbrace{\cosh \gamma l}_{A} + I_R \cdot \underbrace{Z_c \cdot \sinh \gamma l}_{B}$$

$$I_S = \underbrace{\frac{V_R}{Z_c} \cdot \sinh \gamma l}_{C} + I_R \cdot \underbrace{\cosh \gamma l}_{D}$$

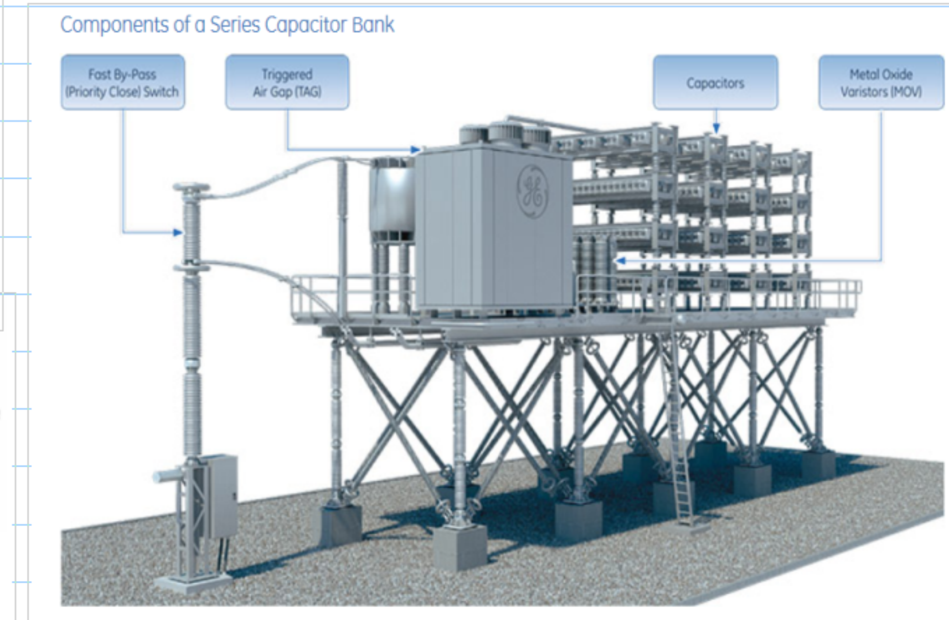
Equivalent-π (long)

Reactive Compensation of Transmission Line

In the southwestern part of the United States series compensation is especially important because large generating plants are located hundreds of miles from load centers and large amounts of power must be transmitted over long distances. The lower voltage drop in the line with series compensation is an additional advantage. Series capacitors are also useful in balancing the voltage drop of two parallel lines.



Thyristor Controlled Series Capacitor



EXAMPLE 5.4: Reactive Compensation of Transmission Line

- In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%

EXAMPLE 5.4: Reactive Compensation of Transmission Line

- In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%

From 5.1

$$len = 230$$

$$z = 0.1603 + 0.831263 \cdot i \qquad y = 5.108943 \cdot 10^{-6} \cdot i$$

$$Zc = 405.223629 - 38.714864 \cdot i \qquad \gamma l = 0.045492 + 0.476161 \cdot i$$

For the Uncompendated Line:

$$A := \cosh(\gamma l) = 0.889681 + 0.020859 \cdot i \qquad D := A$$

$$B := Zc \cdot \sinh(\gamma l) = 34.153635 + 184.36891 \cdot i \qquad |B| = 187.505642$$

$$C := \frac{\sinh(\gamma l)}{Zc} = -8.295422 \cdot 10^{-6} + 0.001132 \cdot i$$

EXAMPLE 5.4: Reactive Compensation of Transmission Line

⌘ In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%

Now the compensation with COmpensation Factor of 70%: Xc/XL = 0.7

$$\text{Comp} := 0.7$$

$$\text{Xc} := \text{Comp}\cdot(-i)\cdot\text{XL}\cdot\text{len} = -133.833265\cdot i \qquad \text{XL} = 0.831263$$

Now new B constant:

$$\text{Bnew} := \text{B} + \text{Xc} = 34.153635 + 50.535646\cdot i \qquad |\text{Bnew}| = 60.994445$$

$$\frac{|\text{Bnew}|}{|\text{B}|} = 0.325294$$

EXAMPLE 5.4: Reactive Compensation of Transmission Line

⌘ In o … change of the A, C, a … constants for the line … 0%

Other Constants

$$z = 0.1603 + 0.831263 \cdot i$$

$$Z := z \cdot len = 36.869 + 191.190378 \cdot i$$

$$Znew := Re(z \cdot len) + i \cdot Im(z \cdot len) \cdot (1 - Comp) = 36.869 + 57.357113 \cdot i$$

$$znew := \frac{Znew}{len} = 0.1603 + 0.249379 \cdot i$$

$$\gamma lnew := \sqrt{znew \cdot y} \cdot len = 0.079759 + 0.271587 \cdot i$$

$$Anew := \cosh(\gamma lnew) = 0.966412 + 0.021419 \cdot i \qquad |Anew| = 0.96665$$

$$|A| = 0.889926$$

$$Zcnew := \sqrt{\frac{znew}{y}} = 231.126571 - 67.876998 \cdot i \qquad \arg(Anew) \cdot \frac{180}{\pi} = 1.269661$$

$$Cnew := \frac{\sinh(\gamma lnew)}{Zcnew} = -8.427465 \cdot 10^{-6} + 0.001162 \cdot i \qquad |Cnew| = 0.001162$$

$$|C| = 0.001132$$

$$Bnew2 := Zcnew \cdot \sinh(\gamma lnew) = 36.044308 + 56.97852 \cdot i$$

EXAMPLE 5.4: Reactive Compensation of Transmission Line

- In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%

$|Bnew| = 60.994445$

$|Bnew2| = 67.422132$

$|B| = 187.505642$

Consider The maximum Power Equation:

$$P_{R_{max}} = \frac{V_S \cdot V_R}{B} - \frac{A \cdot V_R^2}{B} \cos(\beta - \alpha)$$

Shunt Compensation

⌘ Charging Current Reduction with Shunt Compensation

Charging Current reduction with Shunt Compensation (p 112)

"capacitive susceptance"

Remember : $I_{chg} = j\omega C_n V_{an} = \dfrac{V_{an}}{jX_C}$

$\rightarrow I_{chg} = \dfrac{V_{an}}{|X_C|} = B_C \cdot V_{an}$

Connection of Inductors from line to Neutral with amount of B_L ← "inductive susceptance" $= \dfrac{1}{|jX_L|}$

After Shunt Compensation:

$\rightarrow I_{chg} = (B_C - B_L) \cdot V_{an}$

$= (B_C - B_L)\dfrac{B_C}{B_C} \cdot V_{an} = B_C \cdot V_{an}\left(1 - \dfrac{B_L}{B_C}\right)$

"Shunt Compensation Factor"