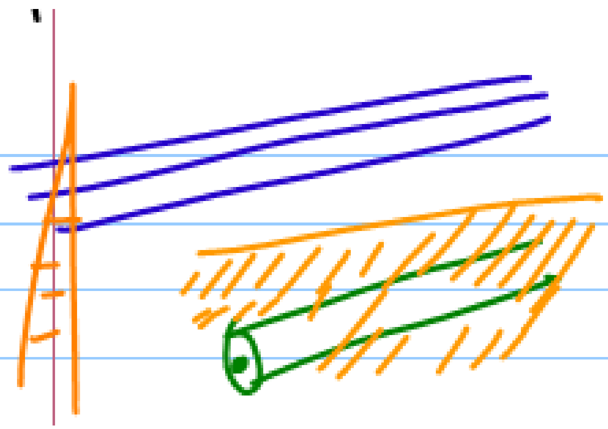
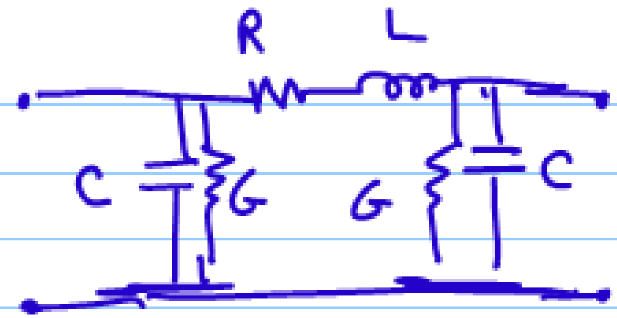
EECE421 Power System Analysis

- Chapter 5:Current and Voltage Relations on a Transmission Line





Focus

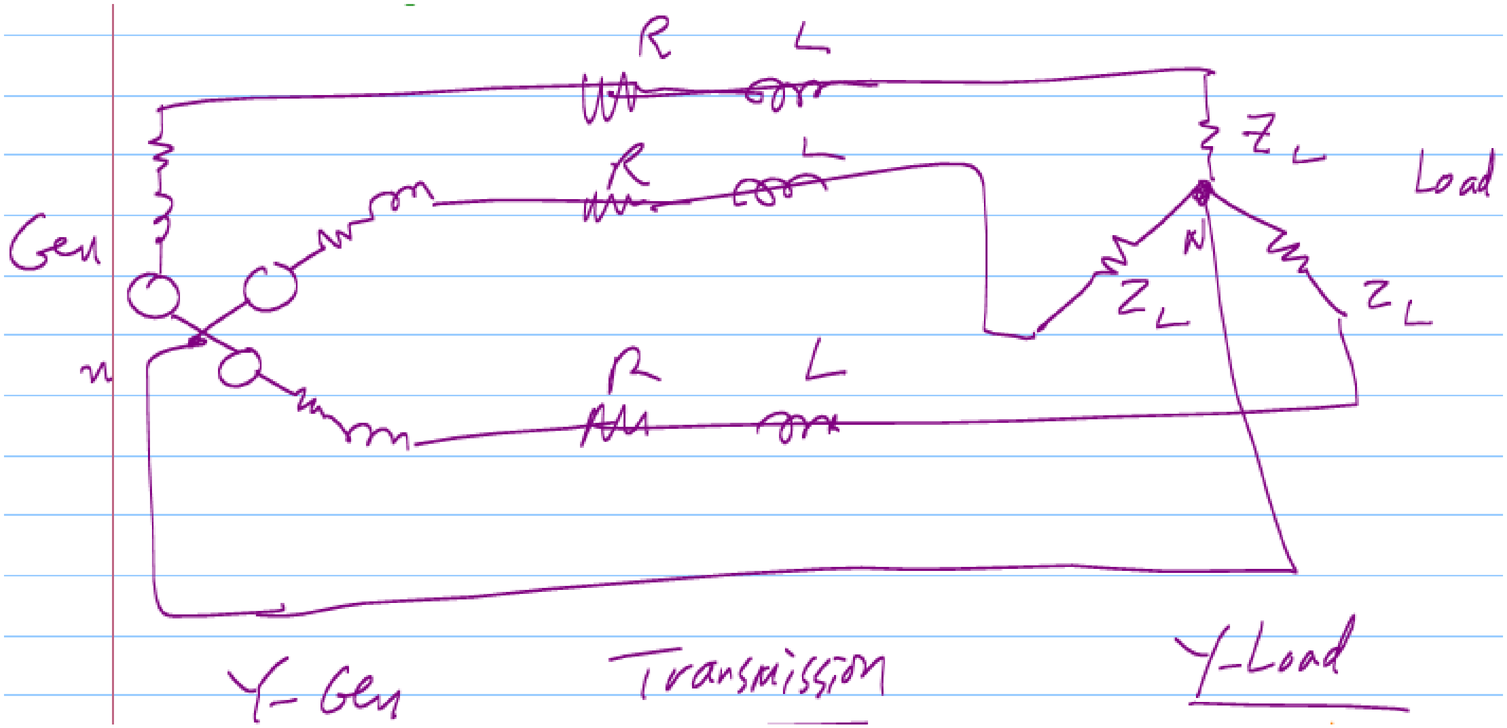
- Maintain voltage at various points within specified limits in the transmission system
- Determination of V, I, and S (P & Q) at any point on a transmission line with assumption that we know these (V, I, and S) at one point (usually at one end of the line)
- Understand the effects of the parameters of the line on V and S flow
- Understand transmission transient (due to surges caused by lightning and switching)

5.1 Representation of Lines

- 3 classes by the length of the line
  - Short Line (L < 50 miles):
    - lumped parameters with C ignored
  - Medium-length Line (50 <L < 150 miles):
    - Lumped Parameters with C placed at both ends with amount of C/2 each
  - Long Line (L > 150 miles)
    - Distributed parameter approach

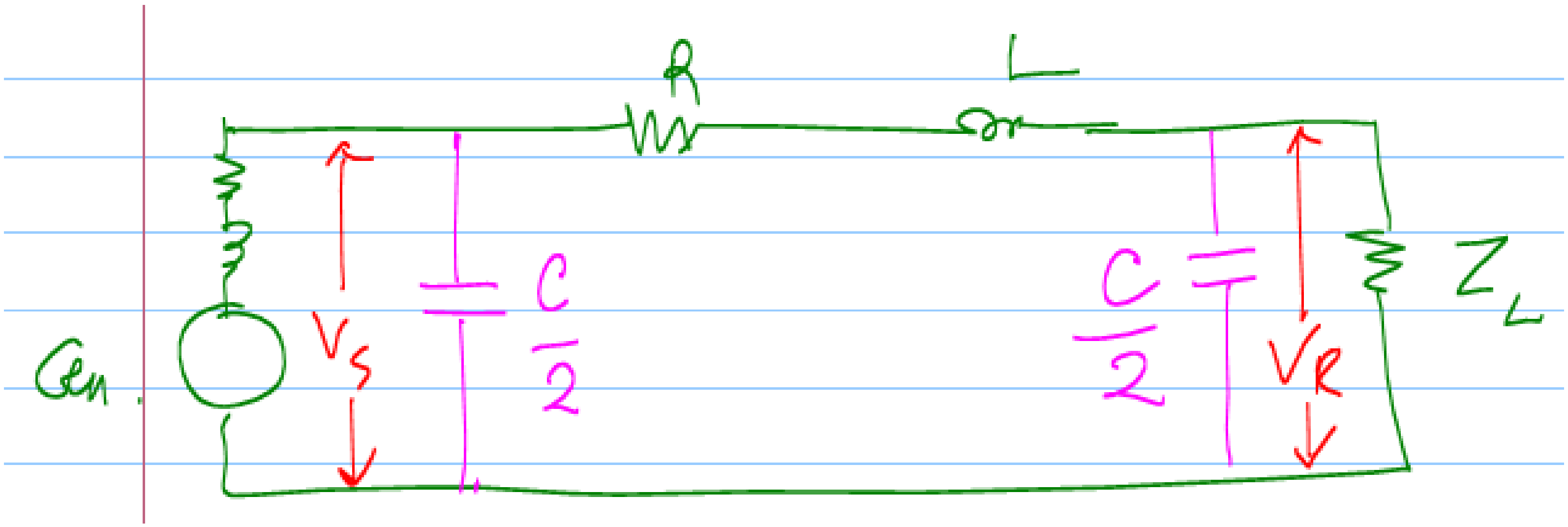
Short Line - Lumped Parameter

- R and L for the entire line (of < 50 miles)
- C ignored



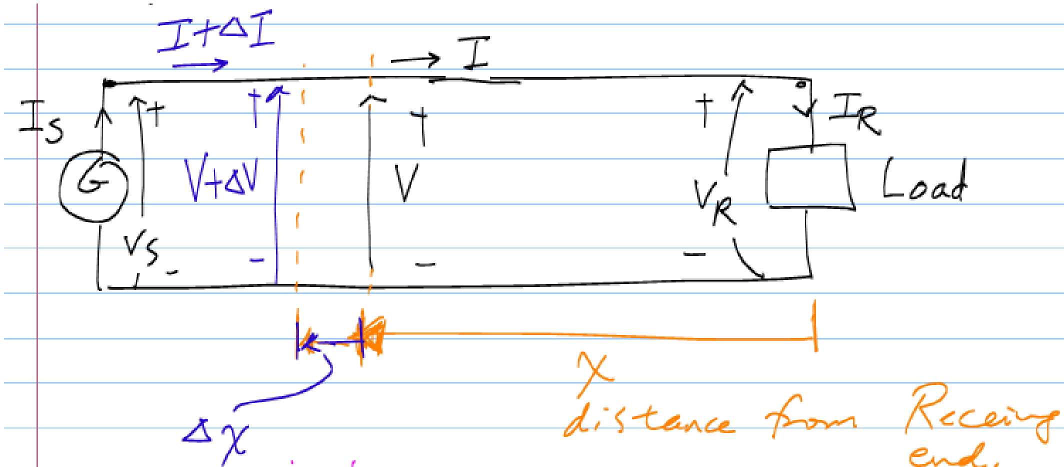
Medium Line – Lumped Parameter

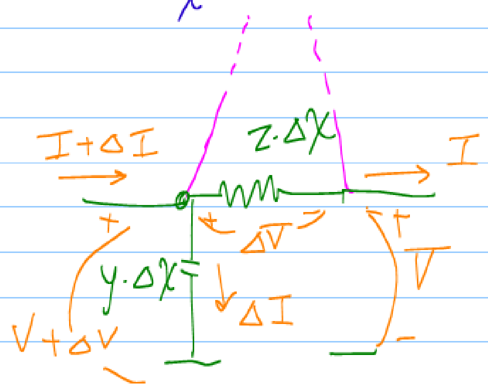
- R and L for the entire line of [50, 150] miles
- C/2 at both ends: C is the capacitance of the entire line



Distributed Parameter – Long Line

(L > 150 miles)





z: series impedance per unit length $= (r + j\omega L)$ per Δx

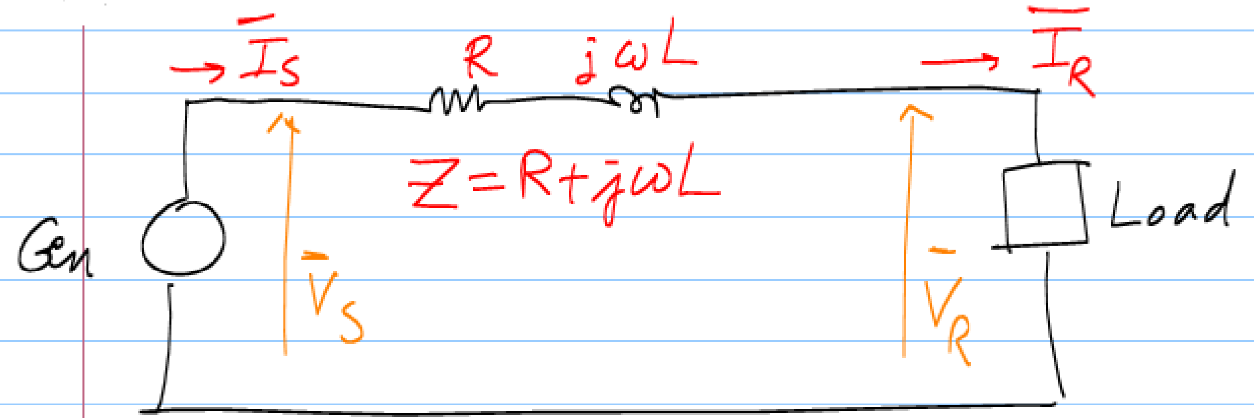
y: shunt admittance per unit length =

l : length of line

$Z = zl$: total series impedance per phase

$Y = yl$: Total shunt admittance per phase to neutral

Short Transmission Line



$$\bar{I}_R = \bar{I}_S$$

$$\bar{V}_S = \bar{V}_R + \bar{I}_R \bar{Z}$$

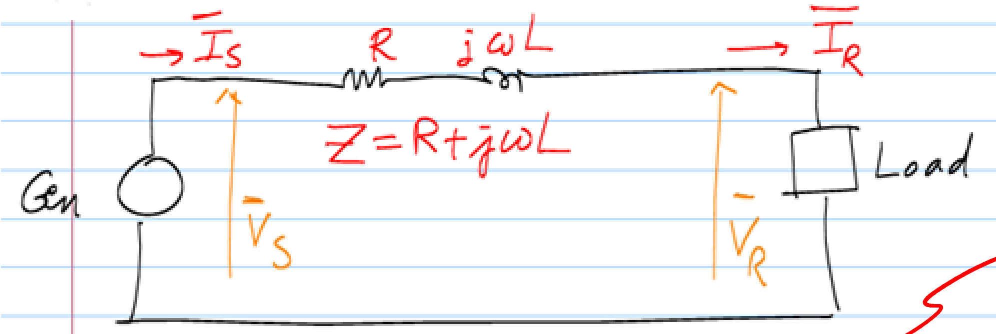
* "Voltage Regulation"

→ Rise in Voltage at the receiving end

Leading pf case!

⇒ drop at R (lagging)

Voltage Regulation



$|V_s|$

$$\%\text{ Voltage Regulation} = \frac{|V_{R,\text{ No Load}}| - |V_{R,\text{ Full Load}}|}{|V_{R,\text{ Full Load}}|} \times 100$$

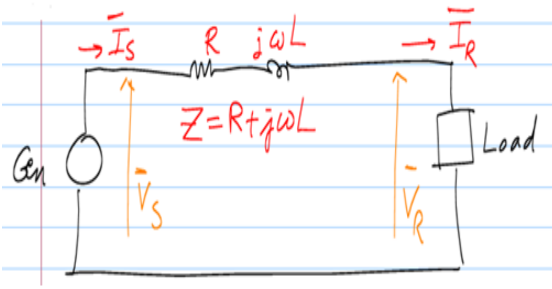
(with $|V_s|$ kept Constant)

Above circuit

$|V_R| = |V_{R,\text{ Full load}}|$ with Load Connected

$|V_S| = |V_{R,\text{ No load}}|$ w/o load Connected

Voltage Regulation

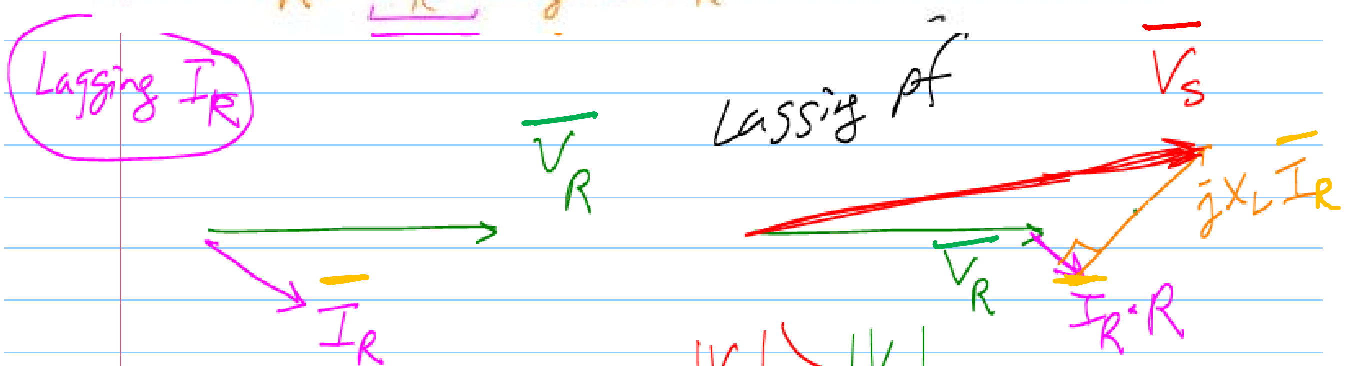


Let's draw a phasor diagram with the same $|V_R|$ but different Load current (leading or lagging)

$$\bar{V}_S = \bar{V}_R + \bar{I} \cdot \bar{Z} = \bar{V}_R + \bar{I}_R(R + jX_L)$$

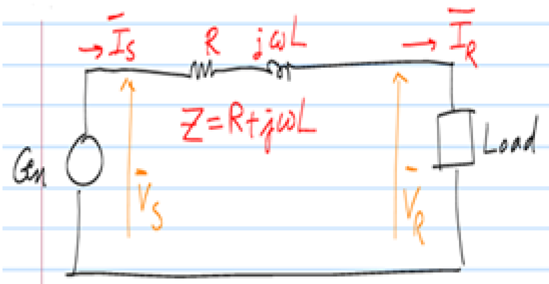
$$= \bar{V}_R + \bar{I}_R R + jX_L \bar{I}_R$$

$$\bar{I}_R = \frac{\bar{V}_R}{Z_{Load}}$$

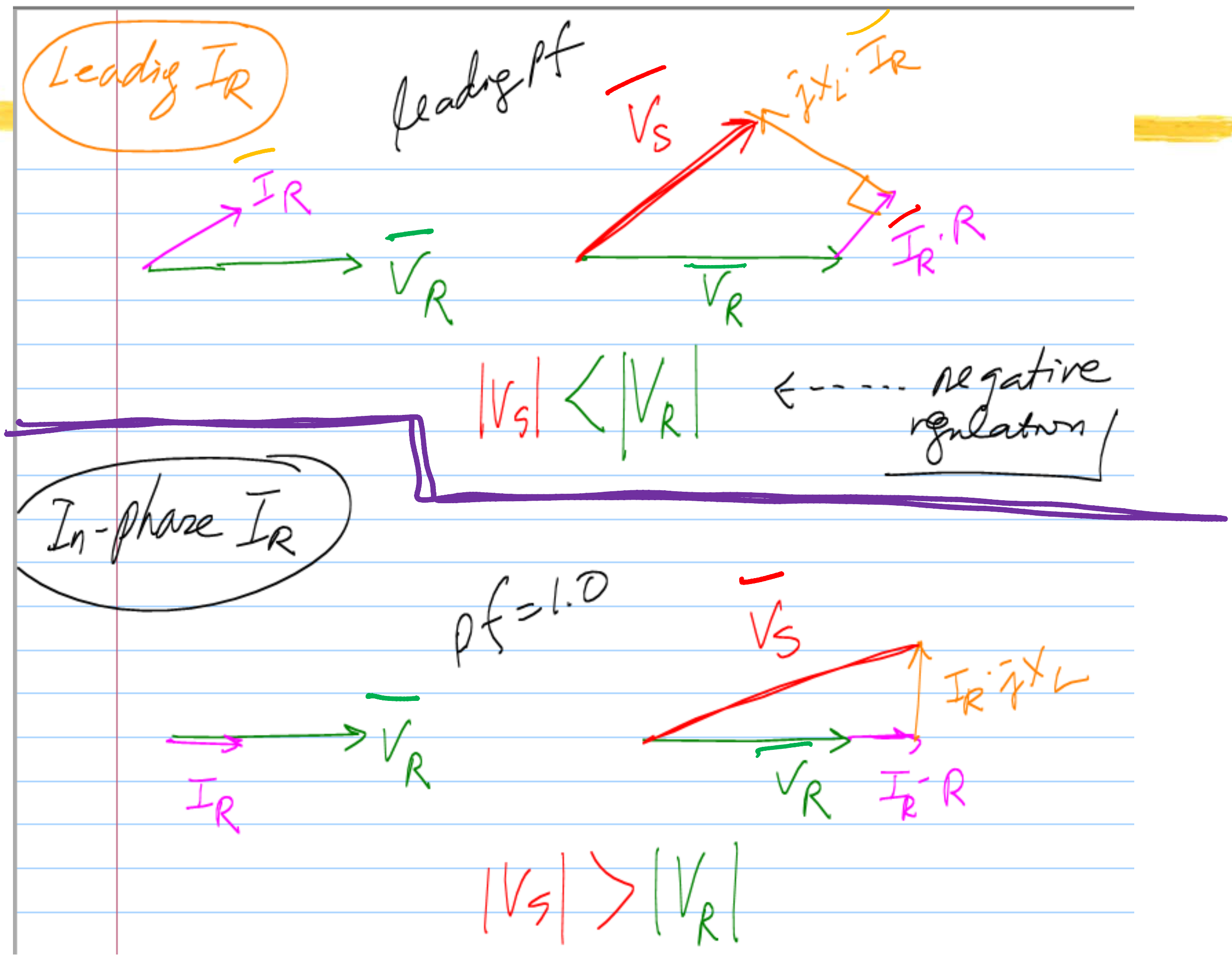


$|V_S| > |V_R|$

Voltage Regulation



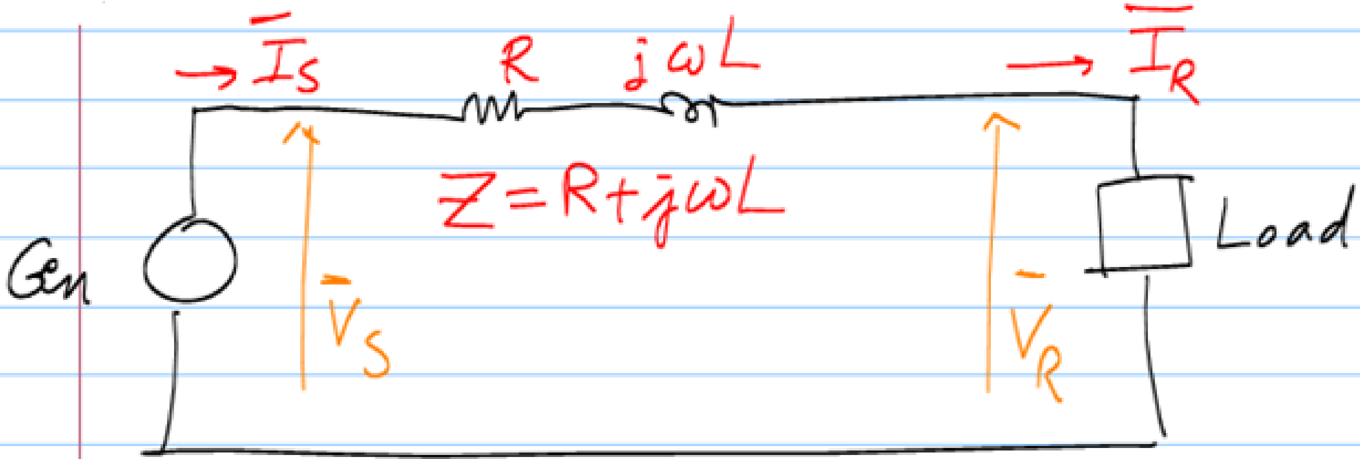
$$\bar{V}_S = \bar{V}_R + \bar{I}\cdot\bar{Z} = \bar{V}_R + \bar{I}_R(R + jX_L)$$
$$= \bar{V}_R + \bar{I}_R R + jX_L \bar{I}_R$$



Class Activity

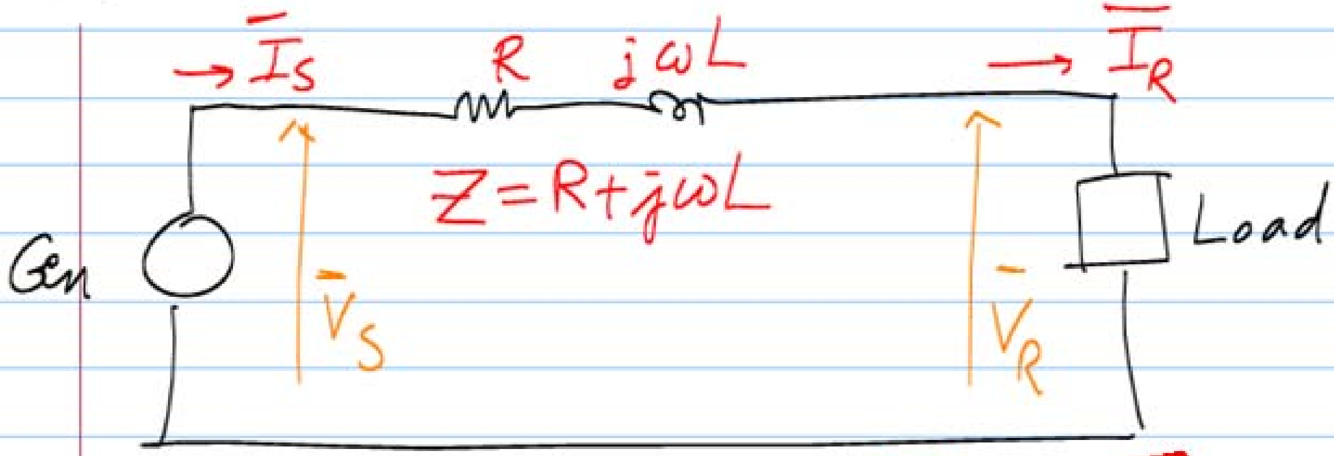
- For a 12 mile long 60Hz single circuit 3-phase line is made of an unknown conductor. The line delivers 2500kW at 11,000 V to a 3-phase balanced load. We assume the receiving side voltage is kept the same. If the load is maintained a unity power factor, the sending-end line voltage is 12,081 V; however, when the load is of 0.8 lagging power factor, the sending end line voltage is changed to 13,268 V. Find the impedance of the conductor per mile.
- Hint: Per-phase analysis and Line-to-neutral voltage

Transmission Line Equation



$$\left\{ \begin{aligned} \bar{I}_R &= \bar{I}_S \\ \bar{V}_S &= \bar{V}_R + \bar{I}_R \bar{Z} \end{aligned} \right.$$

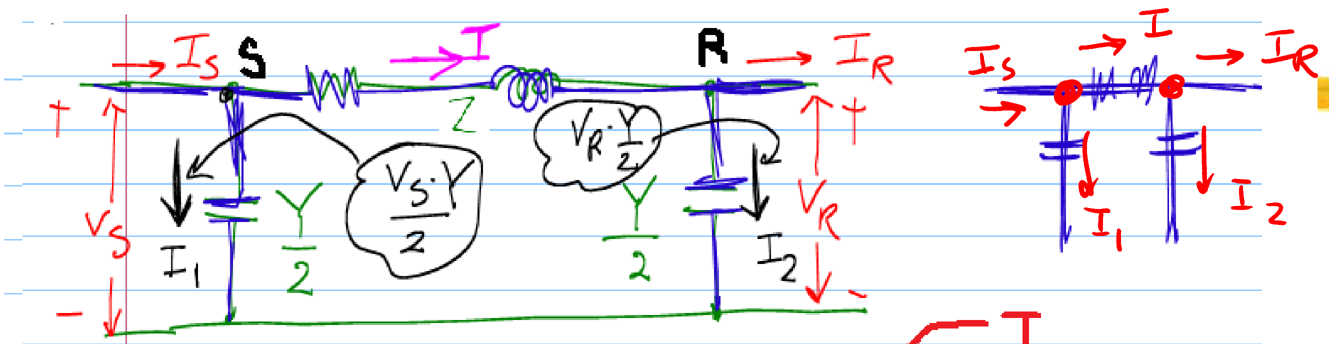
Transmission Line Equation



$$\left\{\begin{aligned} \bar{I}_R &= \bar{I}_S \\ \bar{V}_S &= \bar{V}_R + \bar{I}_R \bar{Z} \end{aligned}\right. \longrightarrow \begin{aligned} \bar{V}_S &= \underset{A}{1}\cdot \underbrace{\bar{V}_R} + \underset{B}{\bar{Z}} \cdot \underbrace{\bar{I}_R} \\ \bar{I}_S &= \underset{C}{0}\cdot \underbrace{\bar{V}_R} + \underset{D}{1}\cdot \underbrace{\bar{I}_R} \end{aligned}$$

$A = D = 1, \quad B = \bar{Z}, \quad C = 0$

Medium-Length Line (π-circuit)



$$@S:\quad V_S = I\cdot Z + V_R = \Big(I_R + \overbrace{V_R\cdot\frac{Y}{2}}^{I_2}\Big)Z + V_R$$

$$@R:\quad I = I_R + I_2 = I_R + \underbrace{V_R\cdot\frac{Y}{2}}_{I_2}$$

$$\boxed{V_S = \left(\frac{ZY}{2} + 1\right)V_R + Z\cdot I_R}$$

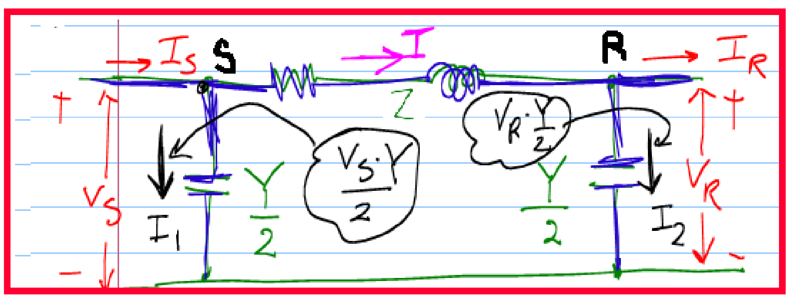
Medium-Length Line (π-circuit)

@ S: $I_S = I + V_S \cdot \frac{Y}{2}$ ← I_1

$\left(I = I_R + V_R \cdot \frac{Y}{2} \right)$

$$I_S = I_R + V_R \cdot \frac{Y}{2} + V_S \cdot \frac{Y}{2}$$

$$= V_S \cdot \frac{Y}{2} + V_R \cdot \frac{Y}{2} + I_R$$



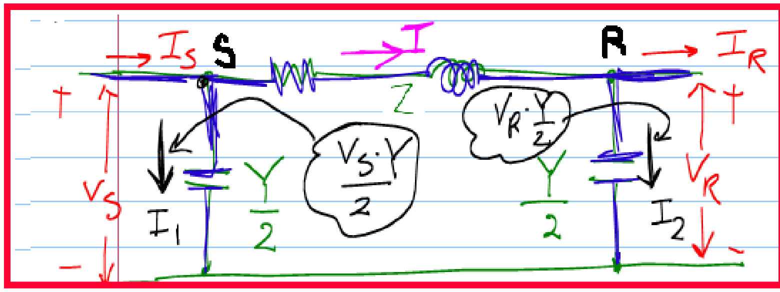
$$\boxed{V_S = \left(\frac{ZY}{2} + 1\right) V_R + Z \cdot I_R}$$

$$= \left[\left(\frac{ZY}{2} + 1\right) V_R + Z \cdot I_R\right] \cdot \frac{Y}{2} + \frac{Y}{2} V_R + I_R$$

$$= \left[\frac{ZY^2}{4} + \frac{Y}{2} + \frac{Y}{2}\right] V_R + \left(\frac{ZY}{2} + 1\right) I_R$$

$$= Y\left(\frac{ZY}{4} + 1\right) V_R + \left(\frac{ZY}{2} + 1\right) I_R$$

Medium-Length Line (π-circuit)



$$V_S = \underbrace{\left(\frac{ZY}{2}+1\right)}_{A} V_R + \underbrace{Z}_{B}\cdot I_R$$

$$I_S = \underbrace{Y\left(\frac{ZY}{4}+1\right)}_{C} V_R + \underbrace{\left(\frac{ZY}{2}+1\right)}_{D}\cdot I_R$$

$$\rightarrow \quad V_S = A\cdot V_R + B\cdot I_R$$

$$I_S = C\cdot V_R + D\cdot I_R$$

$$\left\{\begin{array}{ll} A = D = \frac{ZY}{2}+1 & [\ ] \\ B = Z & [\Omega] \\ C = Y\left(1+\frac{ZY}{4}\right) & [℧] \end{array}\right.$$

ABCD Constants: "generalized circuit constants"

① $I_R = 0$

: Open Circuit (No load)

$$A = \frac{V_S}{V_R} \quad (\text{with } I_R = 0)$$

② $V_R = 0$

: short circuit

$$B = \frac{I_S}{I_R} \quad (\text{with } V_R = 0)$$

Medium-Length Line (π-circuit)

① $I_R = 0$

: Open Circuit (No load)

$$A = \frac{V_S}{V_R} \quad (\text{with } I_R = 0)$$

② $V_R = 0$

: Short circuit

$$B = \frac{I_S}{I_R} \quad (\text{with } V_R = 0)$$

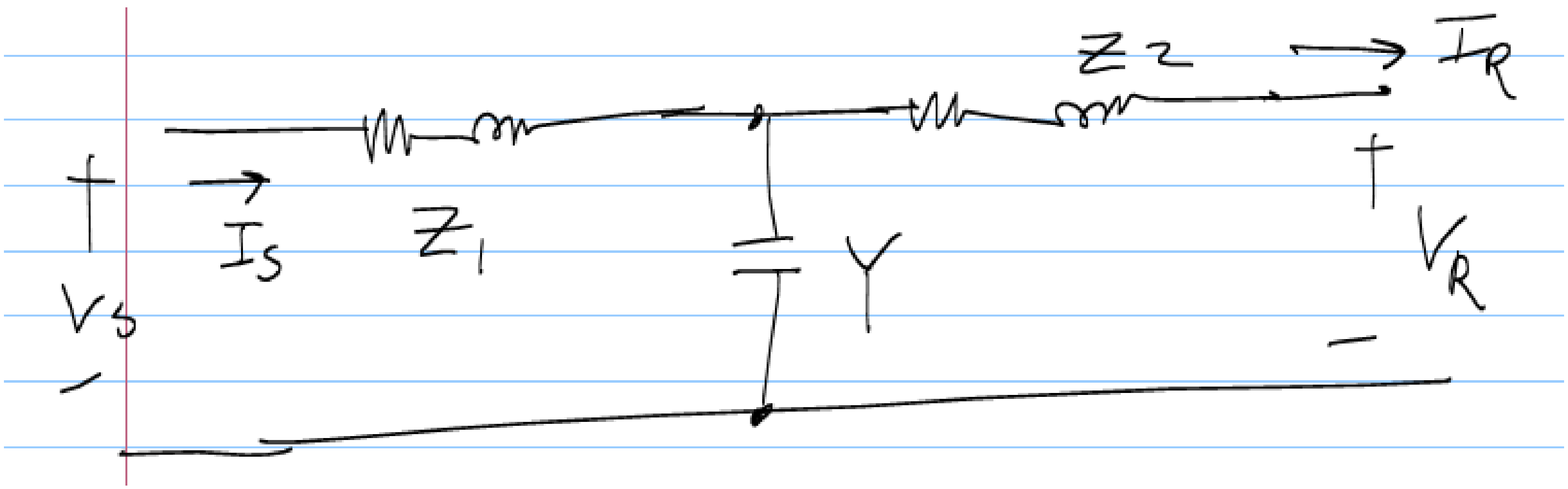
$$\frac{|V_S|}{|A|}$$

$$\%\,\text{Voltage Regulation} = \frac{|V_{R,\,\text{No Load}}| - |V_{R,\,\text{Full Load}}|}{|V_{R,\,\text{Full Load}}|} \times 100$$

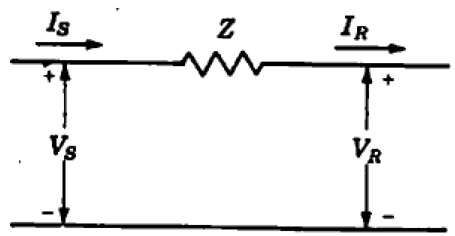
$$= \frac{\frac{|V_S|}{|A|} - |V_R|}{|V_R|} \times 100$$

Class Activity: ABCD-1

⌘ Find the ABCD constants for the following circuit

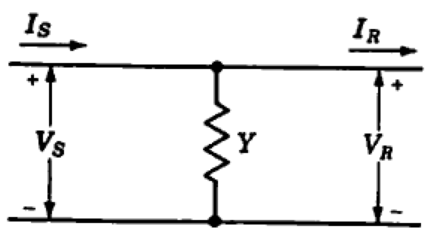


ABCD constants for various circuits



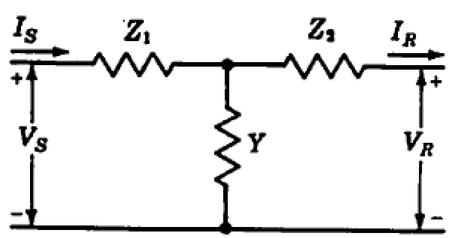
Series impedance

$$A = 1$$
$$B = Z$$
$$C = 0$$
$$D = 1$$



Shunt admittance

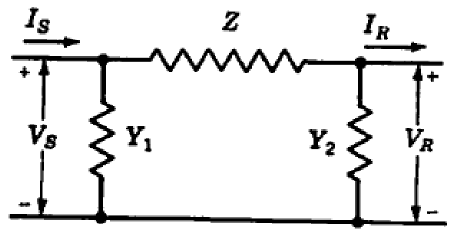
$$A = 1$$
$$B = 0$$
$$C = Y$$
$$D = 1$$



Unsymmetrical T

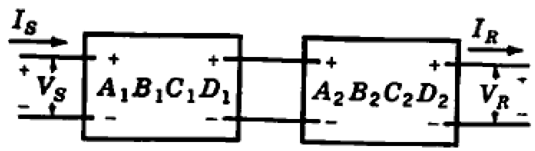
$$A = 1 + YZ_1$$
$$B = Z_1 + Z_2 + YZ_1Z_2$$
$$C = Y$$
$$D = 1 + YZ_2$$

ABCD constants for various circuits



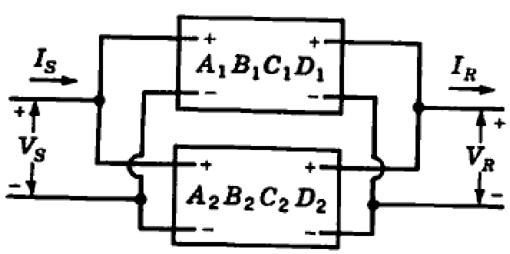
Unsymmetrical π

$$A = 1 + Y_2 Z$$
$$B = Z$$
$$C = Y_1 + Y_2 + Z Y_1 Y_2$$
$$D = 1 + Y_1 Z$$



Networks in series

$$A = A_1 A_2 + B_1 C_2$$
$$B = A_1 B_2 + B_1 D_2$$
$$C = A_2 C_1 + C_2 D_1$$
$$D = B_2 C_1 + D_1 D_2$$



Networks in parallel

$$A = (A_1 B_2 + A_2 B_1)/(B_1 + B_2)$$
$$B = B_1 B_2/(B_1 + B_2)$$
$$C = C_1 + C_2 + (A_1 - A_2)(D_2 - D_1)/(B_1 + B_2)$$
$$D = (B_2 D_1 + B_1 D_2)/(B_1 + B_2)$$

HW #4

A 100-mile, single-circuit, 3-phase transmission line delivers 55MVA at 0.8 power factor lagging to the load at 132 kV (Line-to-line). The line is composed of Drake conductors (with Resistance per mile = 0.1284, and Ds=0.0373 ft) with flat horizontal spacing of 11.9 ft between adjacent conductors.

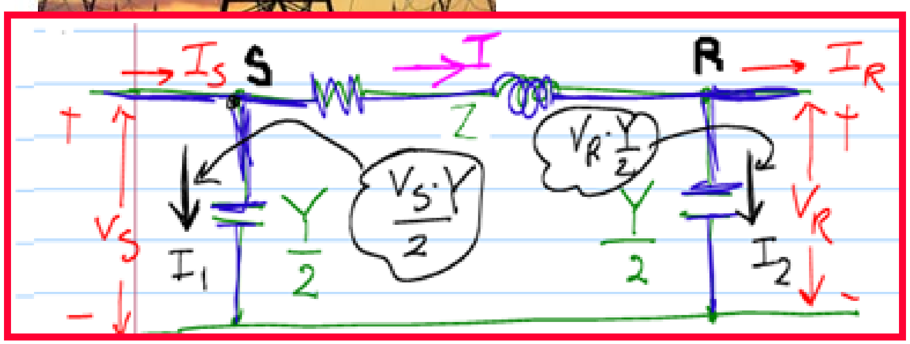
Determine

(a) the series impedance (Z= R+ jXL) and the shunt admittance (Y =1/jXC) of the line.

(b) the ABCD constants of the line.

(c) the sending-end voltage, current, real and reactive power, and the power factor.

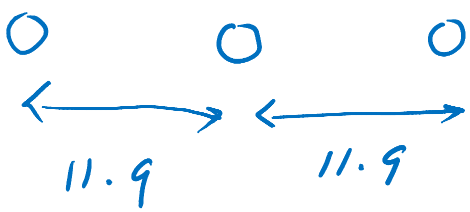
(d) the percent regulation of the line.



$$V_S = \underbrace{\left(\frac{ZY}{2}+1\right)}_{A} V_R + \underbrace{Z}_{B} \cdot I_R$$

$$I_S = \underbrace{Y\left(\frac{ZY}{4}+1\right)}_{C} V_R + \underbrace{\left(\frac{ZY}{2}+1\right)}_{D} \cdot I_R$$

HW#4 - hint



$D_s = r' = 0.0373$ (GMR)

$r = \dfrac{D_s}{0.7788}$

Z, Y $\rightarrow$ ABCD

$V_S = A\,\boxed{V_R} + B \cdot I_R$

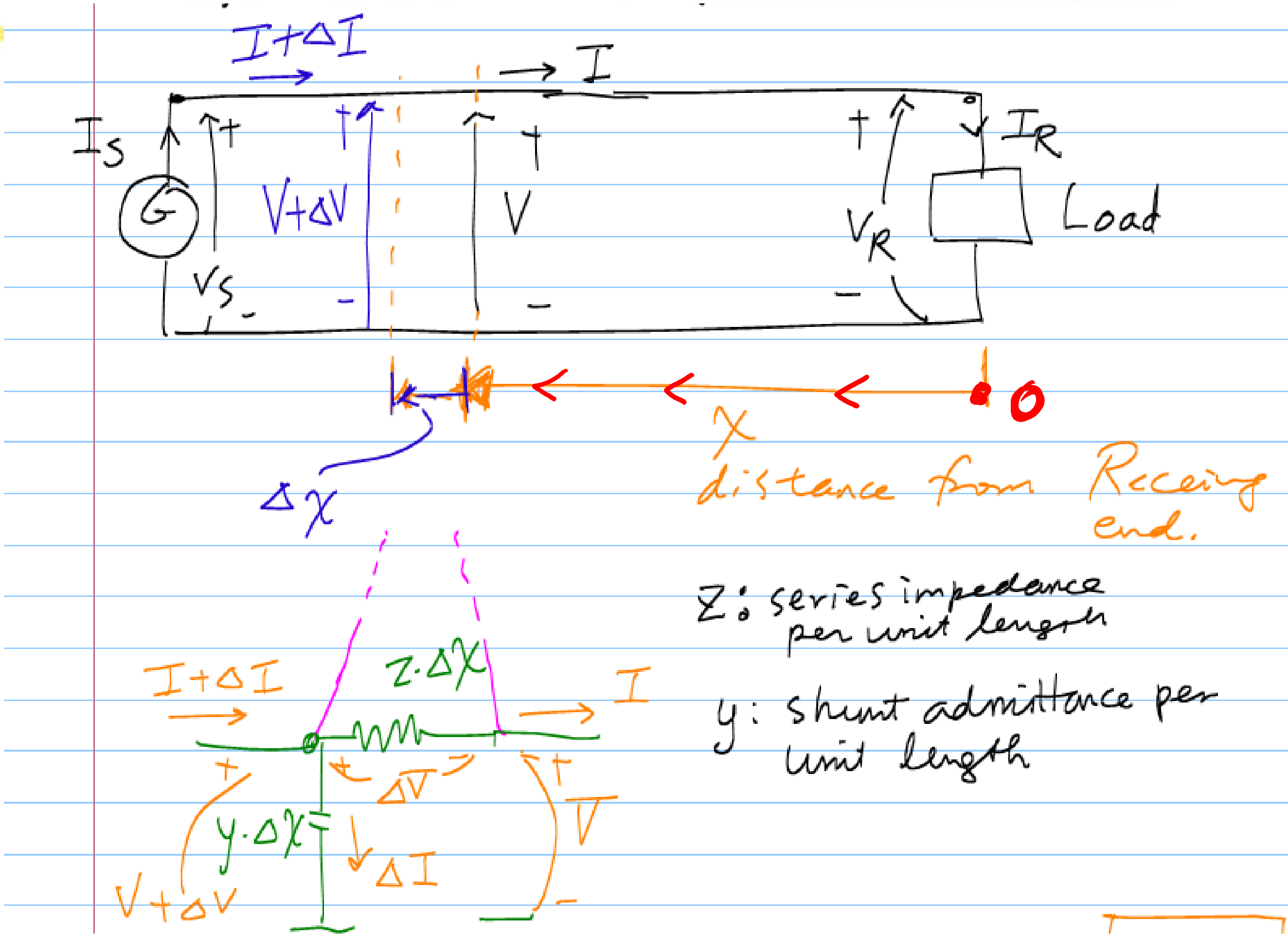
$I_S = C \cdot V_R + D \cdot \underline{I_R}$

$S_{3\phi} \rightarrow S_{1\phi}$ $\qquad V_R = |V_R| \angle 0^\circ$

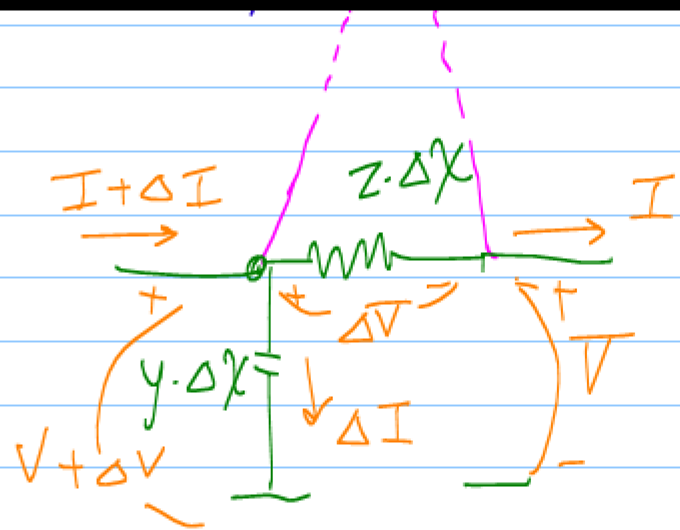
$\bar{S}_R = \bar{V}_R \bar{I}_R^{*}$ $\qquad P_R = |V_R|\,|I_R|\,\underbrace{\cos\theta}_{pf}$

Long Transmission Line (> 150 miles)

- The parameters are not lumped but are distributed uniformly throughout the length of the line



Long Transmission Line (> 150 miles)



z: series impedance per unit length

y: shunt admittance per unit length

① $\Delta V = I \cdot z \cdot \Delta x \longrightarrow \frac{\Delta V}{\Delta x} = I \cdot z \xrightarrow{\Delta x \to 0} \boxed{\frac{dV}{dx} = I \cdot z}$

② $\Delta I = (V + \Delta V)(y \cdot \Delta x) \rightarrow \frac{\Delta I}{\Delta x} = y(V + \Delta V) \xrightarrow[\Delta V \to 0]{\Delta x \to 0} \boxed{\frac{dI}{dx} = yV}$

Long Transmission Line (> 150 miles)

$$\frac{dV}{dx} = IZ \quad \& \quad \frac{dI}{dx} = yV$$

$$\downarrow \left(\frac{d}{dx}\right) \qquad \downarrow \left(\frac{d}{dx}\right)$$

$$\frac{d^2V}{dx^2} = Z\frac{dI}{dx} \qquad \frac{d^2I}{dx^2} = y\frac{dV}{dx}$$

$$\frac{d^2V}{dx^2} = ZyV \qquad \frac{d^2I}{dx^2} = yzI$$

function of (V, x) — function of (I, x)

$$\frac{d^2V}{dx^2} = Zy\,V \rightarrow$$ Solution Form(?) : (double derivative ≈ itself)? exp function.

$$V = A_1 e^{\sqrt{yz}\cdot x} + A_2 \cdot e^{-\sqrt{yz}\cdot x}$$ (A form of Solution)

Take the second derivative:

$$\left(\frac{d^2V}{dx^2} = yz\,V\right)$$

$$\frac{dV}{dx} = A_1 \sqrt{yz}\, e^{\sqrt{yz}\cdot x} - A_2 \cdot \sqrt{yz} \cdot e^{-\sqrt{yz}\cdot x}$$

$$\frac{d^2V}{dx^2} = A_1 yz\, e^{\sqrt{yz}\cdot x} + A_2\, yz\, e^{-\sqrt{yz}\cdot x}$$

$$= yz\,\underbrace{\left(A_1 \cdot e^{\sqrt{yz}\cdot x} + A_2 \cdot e^{-\sqrt{yz}\cdot x}\right)}_{V}$$

$\leftarrow$ indeed, the Solution for V.

Long Transmission Line (> 150 miles)

$$\underbrace{\left(A_1 \cdot e^{\sqrt{yz}\cdot x} + A_2 \cdot e^{-\sqrt{yz}\cdot x}\right)}_{V}$$

From $\quad \dfrac{dV}{dx} = Iz$

$$Iz = A_1 \sqrt{yz}\, e^{\sqrt{yz}\cdot x} - A_2 \sqrt{yz} \cdot e^{-\sqrt{yz}\cdot x}$$

$$I = A_1 \sqrt{\frac{y}{z}}\, e^{\sqrt{yz}\cdot x} - A_2 \sqrt{\frac{y}{z}} \cdot e^{-\sqrt{yz}\cdot x}$$

$$= A_1 \frac{1}{\sqrt{z/y}}\, e^{\sqrt{yz}\cdot x} - A_2 \frac{1}{\sqrt{z/y}} \cdot e^{-\sqrt{yz}\cdot x}$$

Z_c

Long Transmission Line (> 150 miles)

Q? How do find A_1 and A_2?

A. Evaluation of V and I at the Receiving end (where $x=0$, $V=V_R$, $I=I_R$)

$$V = A_1 e^{\sqrt{yz}\cdot x} + A_2 e^{-\sqrt{yz}\cdot x}$$

$$I = A_1 \frac{1}{\sqrt{z/y}} e^{\sqrt{yz}\cdot x} - A_2 \frac{1}{\sqrt{z/y}} \cdot e^{-\sqrt{yz}\cdot x}$$

$$V(x=0) = V_R = A_1 + A_2$$

$$I(x=0) = I_R = \frac{1}{\sqrt{z/y}}(A_1 - A_2)$$

Z_C

$$A_1 + A_2 = V_R$$

$$A_1 - A_2 = Z_C \cdot I_R$$

$$Z_C = \sqrt{\frac{z}{y}}$$

characteristic impedance, a complex number

Long Transmission Line (> 150 miles)

$$2A_1 = V_R + I_R \cdot Z_C \longrightarrow A_1 = \frac{V_R + I_R \cdot Z_C}{2}$$

$$A_2 = V_R - A_1 = \frac{2V_R}{2} - \frac{V_R + I_R \cdot Z_C}{2} = \frac{V_R - I_R \cdot Z_C}{2}$$

$$\gamma = \sqrt{yz}$$ ← "propagation constant" complex number

Finally,

$$\begin{cases} V(x) = \dfrac{V_R + I_R \cdot Z_C}{2} e^{\gamma x} + \dfrac{V_R - I_R Z_C}{2} e^{-\gamma x} \\ I(x) = \dfrac{V_R/Z_C + I_R}{2} e^{\gamma x} - \dfrac{V_R/Z_C - I_R}{2} e^{-\gamma x} \end{cases}$$

Line (> 150 miles)

$$V(x) = \frac{V_R + I_R \cdot Z_C}{2} e^{\gamma x} + \frac{V_R - I_R Z_C}{2} e^{-\gamma x}$$

$$I(x) = \frac{V_R/Z_C + I_R}{2} e^{\gamma x} - \frac{V_R/Z_C - I_R}{2} e^{-\gamma x}$$

γ : propagation constant $(= \sqrt{yz})$, complex

Z_c : characteristic impedance $(= \sqrt{z/y})$, complex

$$\gamma = \alpha + j\beta$$

β: phase constant [radian per unit length]

α: attenuation constant [Neper per unit length]

$e^{\alpha x}$

$e^{j\beta x} = \cos\beta x + j\sin\beta x = \angle\beta$

$$V(x) = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \frac{V_R - I_R Z_C}{2} e^{-\alpha x} e^{-j\beta x}$$

$$I(x) = \frac{V_R/Z_C + I_R}{2} e^{\alpha x} e^{j\beta x} - \frac{V_R/Z_C - I_R}{2} e^{-\alpha x} e^{-j\beta x}$$

Long Transmission Line (> 150 miles)

$$V(x) = \frac{V_R + I_R \cdot Z_c}{2} e^{\alpha x} e^{j\beta x} + \frac{V_R - I_R Z_c}{2} e^{-\alpha x} e^{-j\beta x}$$

Long Transmission Line (> 150 miles)

$$V(x) = \underbrace{\frac{V_R + I_R Z_c}{2} e^{\alpha x} e^{j\beta x}}_{\text{increases in mag. \& advances in phase as distance from Receiver (i.e., } x\text{) increases}} + \underbrace{\frac{V_R - I_R Z_c}{2} e^{-\alpha x} e^{-j\beta x}}_{\text{diminishes in mag. \& retards in phase as the distance from receiver (} x\text{) increases}}$$

S

R

increases in mag. & advances in phase as distance from Receiver (i.e., x) increases

diminishes in mag. & retards in phase as the distance from receiver (x) increases

R

S

L … les)

$$V(x) = \underbrace{\frac{V_R + I_R Z_c}{2} e^{\alpha x} e^{j\beta x}}_{} + \underbrace{\frac{V_R - I_R Z_c}{2} e^{-\alpha x} e^{-j\beta x}}_{}$$

S ← R

increases in mag. & advances in phase as distance from receiver (i.e., x) increases

diminished at as it moves from the sending end toward receiving end.

" Incidence Voltage "

S → R

diminishes in mag. & retards in phase as the distance from receiver (x) increases

R ↙ S

diminishes as it moves toward the sending end from the receiver end

" Reflected Voltage "

R → S

Long Transmission Line (> 150 miles)

$$V(x) = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \frac{V_R - I_R Z_C}{2} e^{-\alpha x} e^{-j\beta x}$$

$V(x)$ = Incidence Voltage(x) + Reflected Voltage(x)

Similary,

$$I(x) = \frac{V_R/Z_C + I_R}{2} e^{\alpha x} e^{j\beta x} - \frac{V_R/Z_C - I_R}{2} e^{-\alpha x} e^{-j\beta x}$$

$I(x)$ = Incident Current(x) + Reflected Current(x)

Long Transmission Line (> 150 miles)

- Example
  - 230 mile long line
  - $z=0.1603+j0.8277$
  - $y= j5.105\times10^{-6}$

$$\gamma := \sqrt{z \cdot y} = 1.981 \times 10^{-4} + 2.065i \times 10^{-3}$$

$$Zc := \sqrt{\frac{z}{y}} = 404.526 - 38.812i$$

$$VR := 124130$$

$$IR := 335.7$$

Z_{load} = Resistance of $\frac{VR}{IR}$

$$|Zc| = 406.384 \qquad \arg(Zc) = -0.096$$

$$\Delta L := \frac{L}{100} = 2.3$$

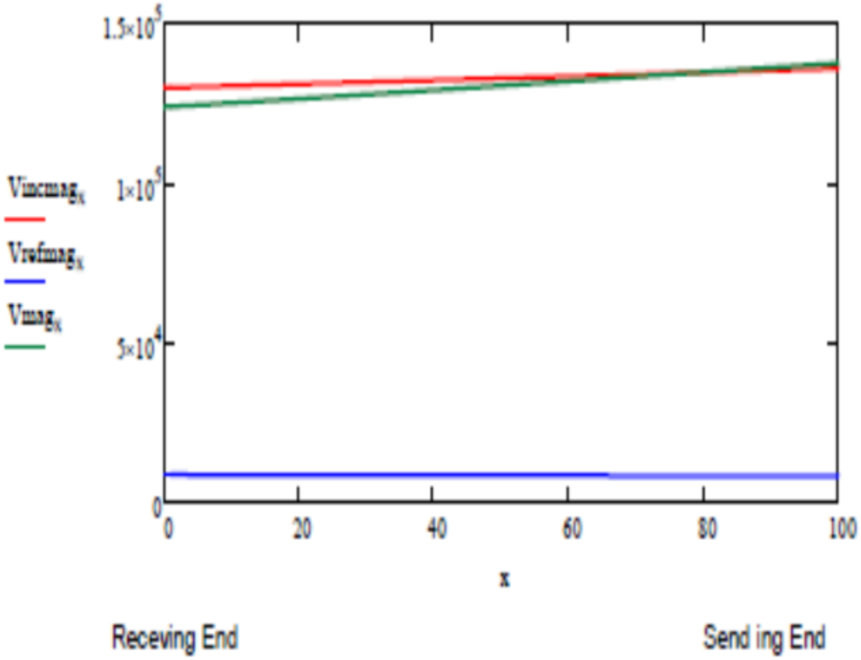
$$x := 0..100$$

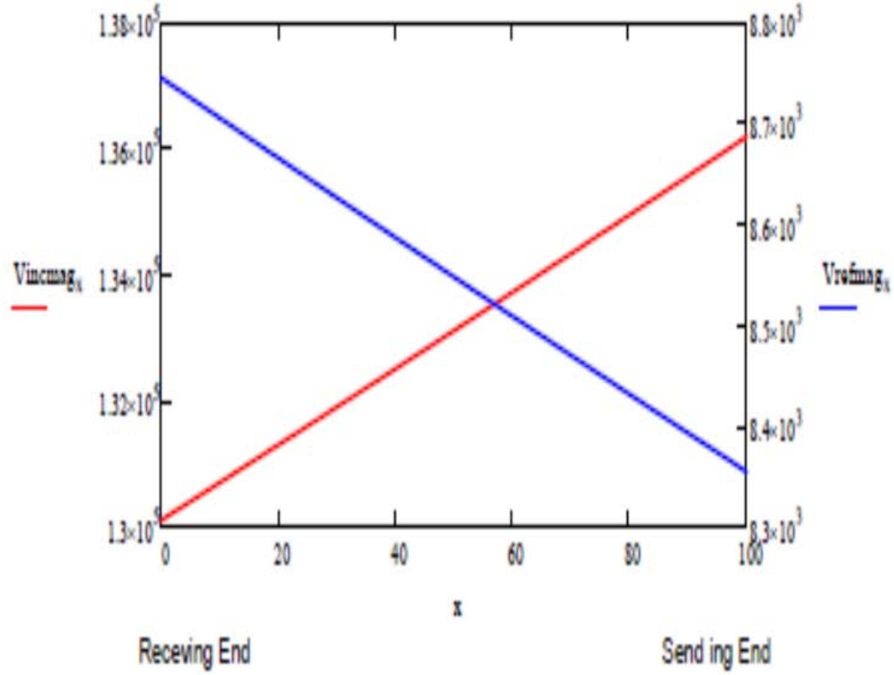
$$Vinc_x := \frac{VR + IR \cdot Zc}{2} \cdot e^{\gamma \cdot x \cdot \Delta L}$$

$$Vref_x := \frac{VR - IR \cdot Zc}{2} \cdot e^{-\gamma \cdot x \cdot \Delta L}$$

$$V_x := Vinc_x + Vref_x$$

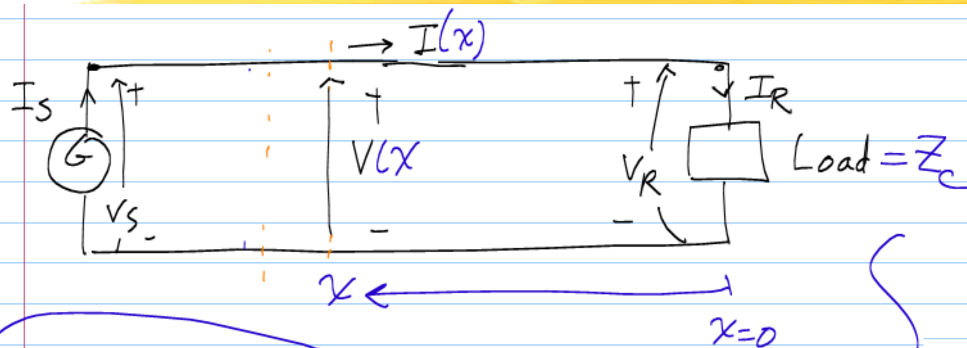
Long Transmission Line (> 150 miles)





Flat Line or “Infinite Line”

⌘ Load at the receiver side = Z_c (Characteristic Impedance) case



$$V_R = I_R \cdot Z_C$$

$$V(x) = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \underbrace{\frac{V_R - I_R Z_C}{2}}_{=0} e^{-\alpha x} e^{-j\beta x}$$

No Reflected Voltage !!

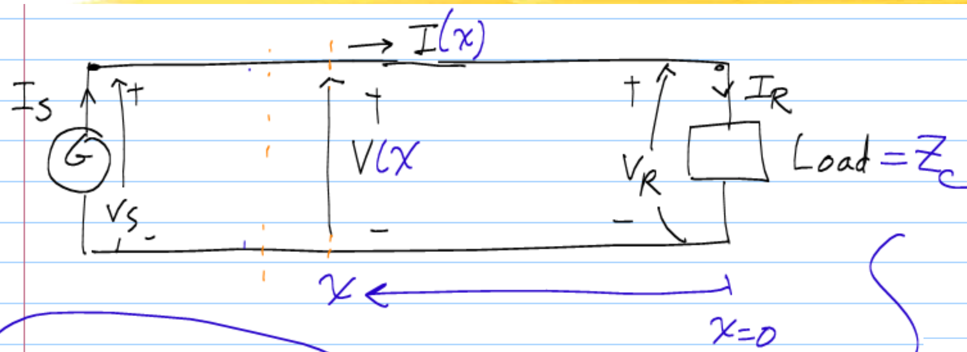
$$I(x) = \frac{V_R/Z_C + I_R}{2} e^{\gamma x} - \underbrace{\frac{V_R/Z_C - I_R}{2}}_{=0} e^{-\gamma x}$$

No Reflected Current !!

(Load = Z_C) → "Flat Line", "Infinite Line"

Flat Line or “Infinite Line”

⌘ Load at the receiver side = Z_c (Characteristic Impedance) case



$$V_R = I_R \cdot Z_C$$

$$V(x) = \frac{V_R + I_R Z_C}{2} e^{\alpha x} e^{j\beta x} + \underbrace{\frac{V_R - I_R Z_C}{2}}_{0} e^{-\alpha x} e^{-j\beta x}$$

No Reflected Voltage!!

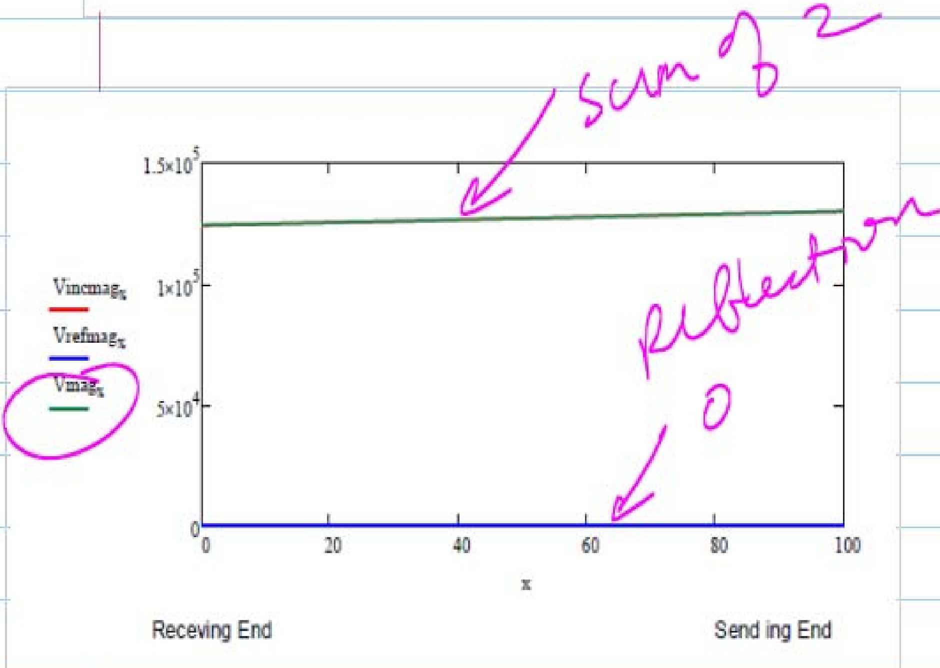
$$I(x) = \frac{V_R/Z_C + I_R}{2} e^{\gamma x} - \underbrace{\frac{V_R/Z_C - I_R}{2}}_{=0} e^{-\gamma x}$$

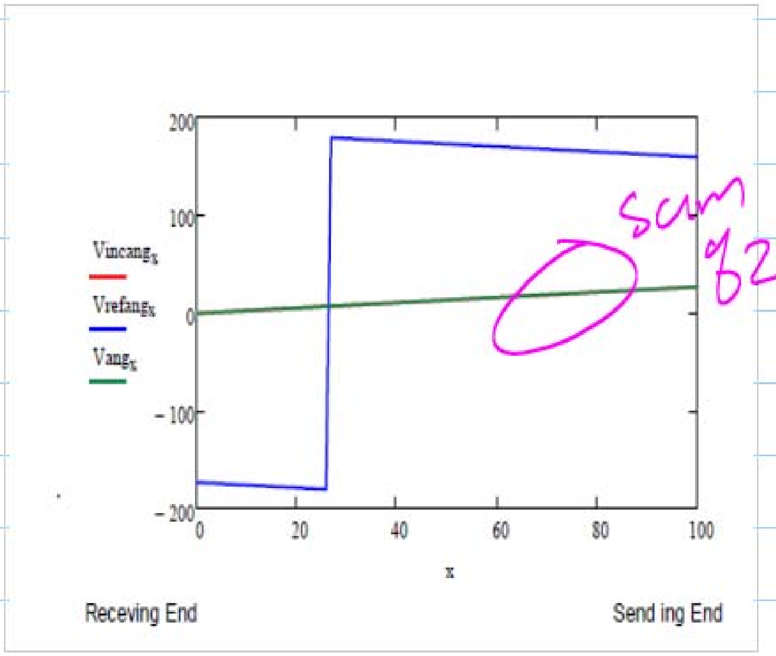
No Reflected Current!!

(Load = Z_C) → "Flat Line", "Infinite Line"

Long Transmission Line (> 150 miles)

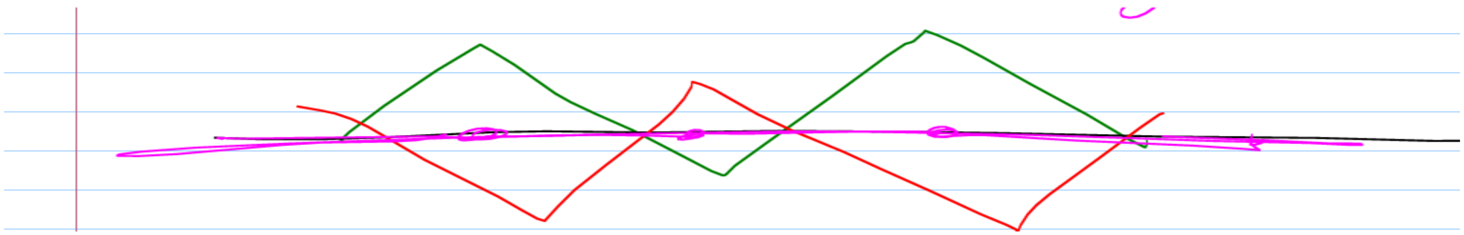
$$IR := \frac{VR}{Zc} = 304.054 + 29.172i$$





Observation

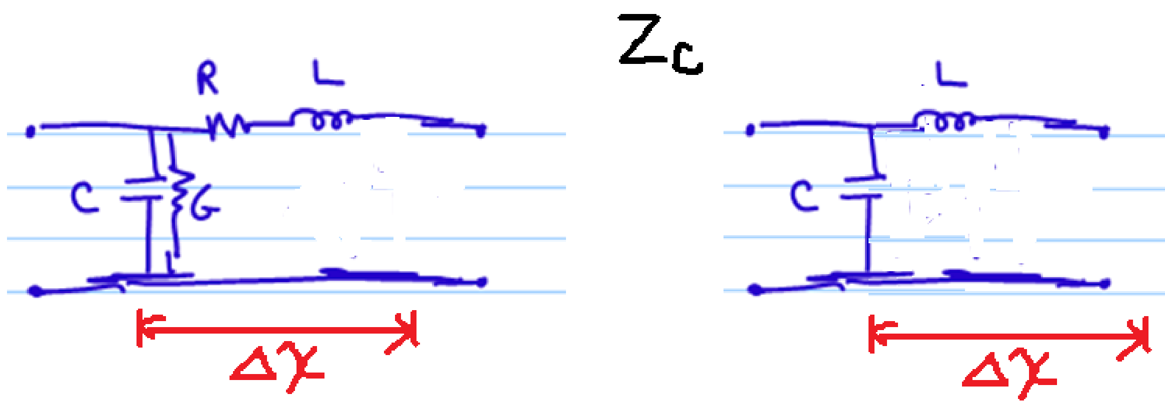
- Termination of a line with the characteristic impedance of the transmission line eliminates the reflection wave
- Reflective wave can interfere with the incident wave and produces interference, and leads to standing waves which do not propagate along the line



- Power lines are difficult to terminate with the characteristic impedance
- Communication lines are frequently terminated with characteristic impedance to eliminate the reflected wave and thus any interference

Zc and “Surge Impedance”

- Characteristic impedance (Zc) $Z_c = \sqrt{\dfrac{Z}{y}}$
- Surge Impedance: Zc with the **condition** that r (series resistance) = 0 and G (shunt conductance)=0 [this condition is called **Lossless Line**]



- Surge Impedance $Z_c = \sqrt{\dfrac{L}{C}}$ [Ω]