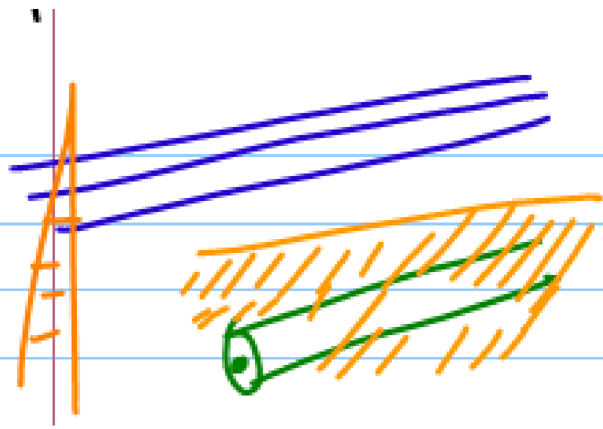
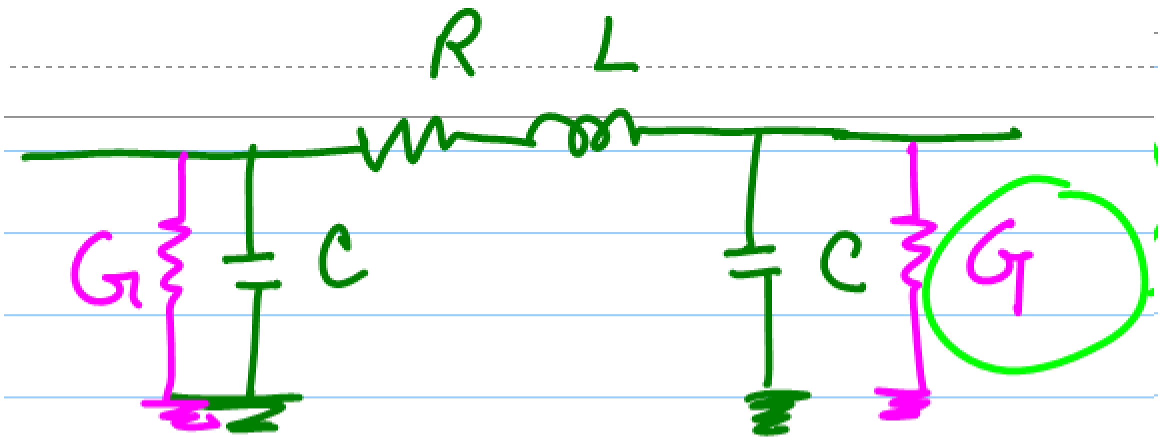
EECE421 Power System Analysis

⌘ Chapter 4: Transmission Line Capacitance



Capacitance

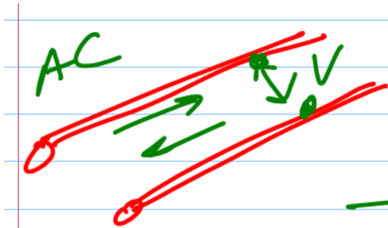
- C: Capacitance
  - Caused by the potential difference between the conductors
  - (Charge) per (unit of potential difference)
  - $C = q / V$ or $q = CV$



- G: Conductance
  - Insignificant
  - Ignored in our discussion

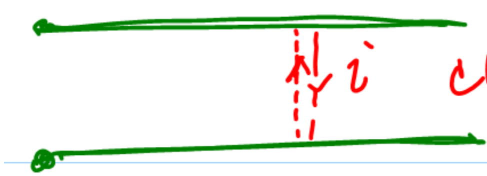
Impact of Capacitance to Circuit

- Charging Current
- Varying voltage causes current to flow between two conductors



V is varying
→ Causes current to flow
($i = C\frac{dv}{dt}$) → "charging current"

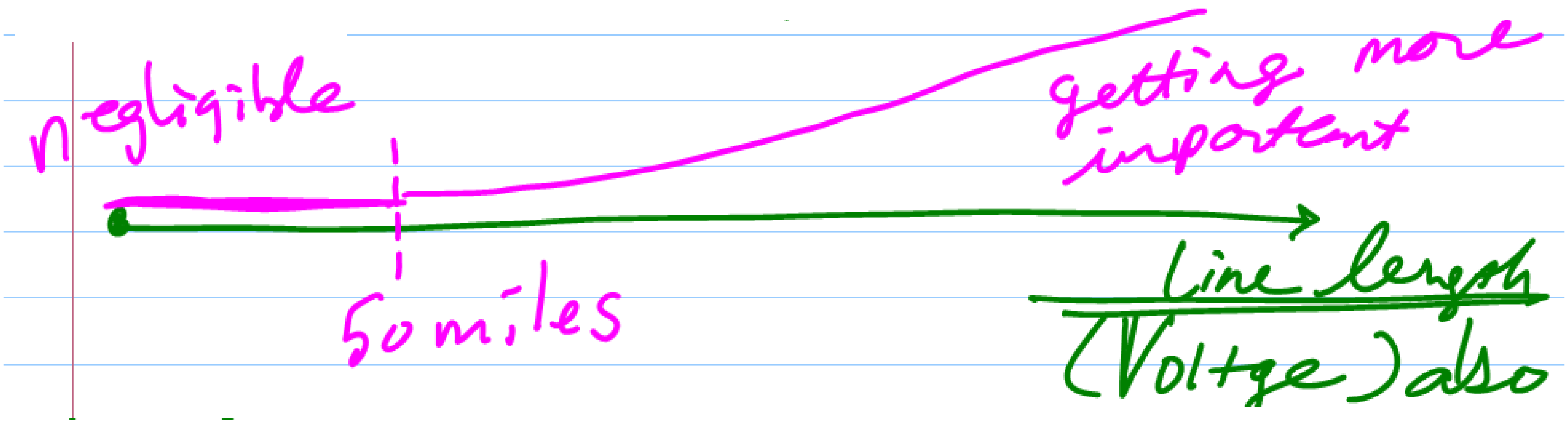
- Impact of Charging Current



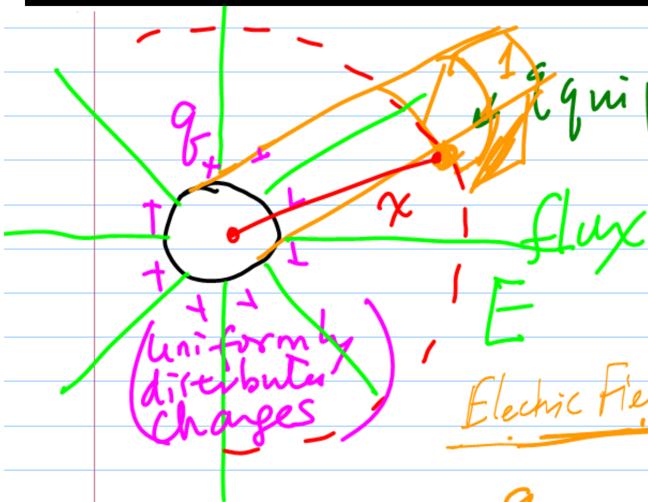
charging current flows!!!
→ affects the voltage drop

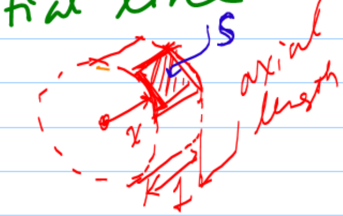
Impact of Capacitance

- Line Length
- Voltage



4.1 E-Field of a long Straight Conductor





Electric Field Density (D) = $\left(\frac{\text{Charge}}{\text{area}}\right)$

$$D = \frac{q}{2\pi x \cdot 1}$$ (per axial length of 1 m)

$[C/m^2]$

Permittivity of Free Space: Farad

$$k_0 = 8.85 \times 10^{-12} \ F/m$$

Relative Permittivity (k_r)

$$k_r = \frac{k}{k_0}$$ actual permittivity

Dry air: $k_r = 1.00054$

≈ 1.0 in calculation for overhead lines.

Electric Field Intensity $(\mathcal{E})$: $D = k\mathcal{E}$

permittivity of the medium

$$\mathcal{E} = \frac{q}{2\pi x \cdot k} \quad [V/m]$$

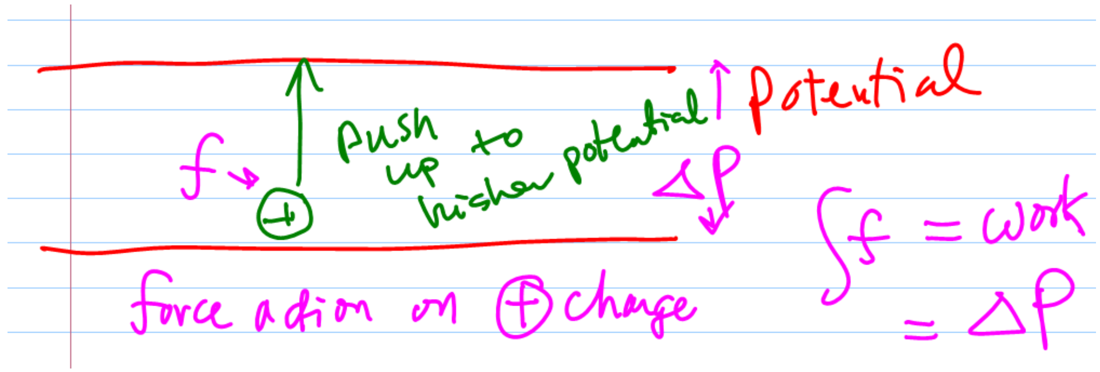
4.2 Potential Difference between 2 Points due to a charge

ΔP

- ⌘ ***Potential difference [V]*** = Work [Joule] per Coulomb, required to move a Coulomb charge between 2 points

$\mathcal{E}$

- ⌘ ***Electric Field Intensity [V/m]:*** (a) Force on a charge in the field; (b) Force [Newton] per Coulomb exerted on a Coulomb charge at a point. f



Voltage Drop Between 2 Points

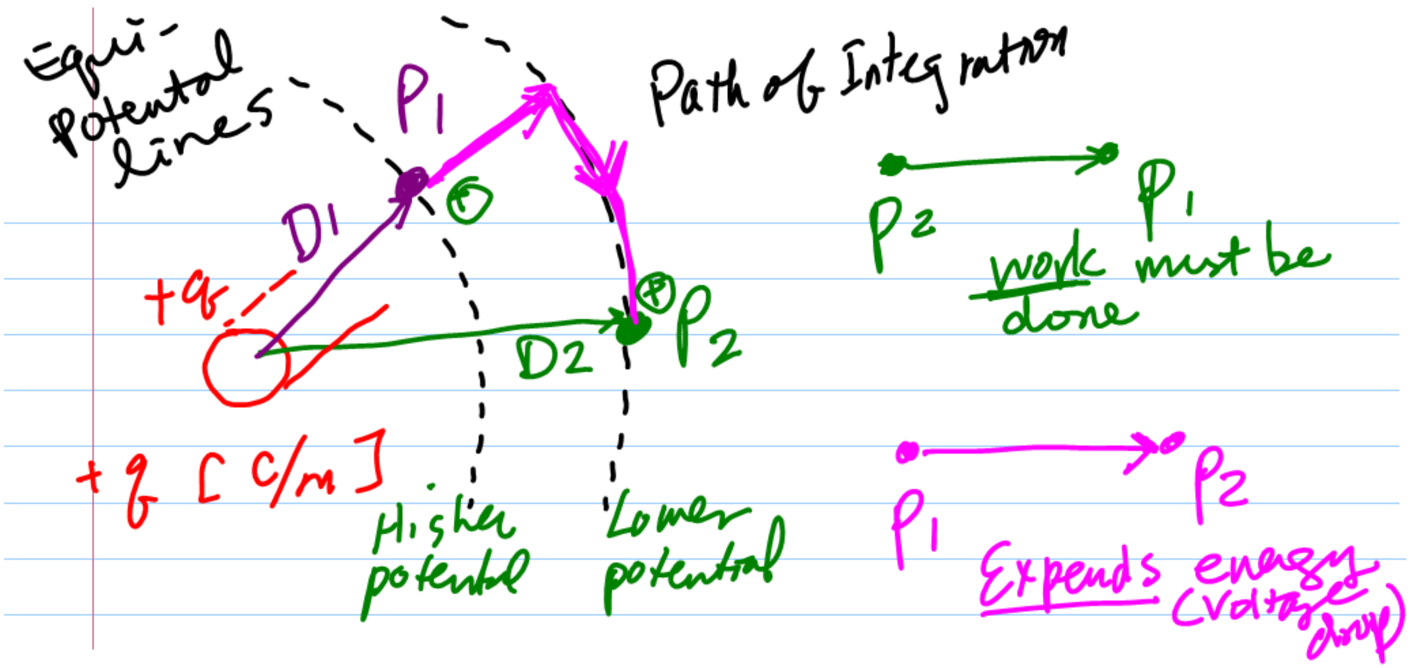
- Equipotential Lines
- Potential Difference is Path Independent
- Energy used (expended) to do work → voltage drop

$\int f = \text{work} = \Delta P$

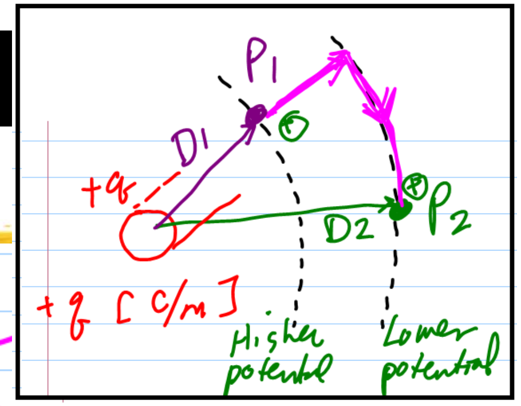
E-field

$V = Ed$

potential difference, distance



Voltage Drop Between 2 Points



- Instantaneous voltage drop

$$v_{12} = \int_{D_1}^{D_2} \mathcal{E}\, dx = \int_{D_1}^{D_2} \frac{q}{2\pi k x}\, dx$$

[per meter of length]

Electric field intensity ("force")

$$= \frac{q}{2\pi k}\int_{D_1}^{D_2} \frac{1}{x}\, dx = \frac{q}{2\pi k}\left[\ln x\right]_{D_1}^{D_2}$$

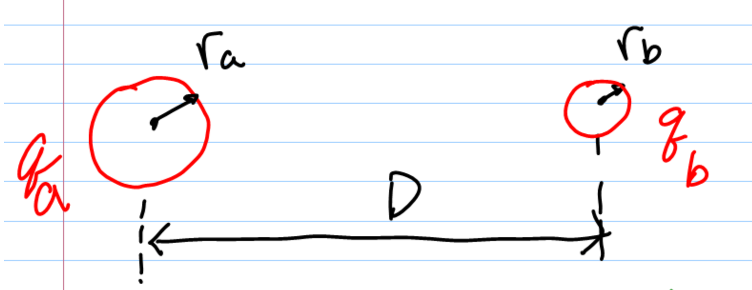
$$= \frac{q}{2\pi k}\ln\frac{D_2}{D_1}$$

Capacitance of a 2-wire line

- Capacitance between 2 conductors = "charge on the conductors per unit of potential difference between them": C = q/V [F/m]

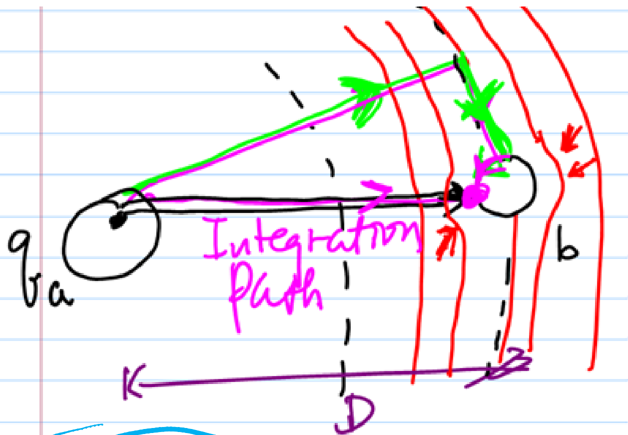
$$C = \frac{q}{v} \ [F/m]$$

- Voltage drop between two: due to q_a and q_b



(Voltage drop due to the charge q_a on conductor a) + (Voltage drop due to the charge q_b on conductor b)

Voltage Drop between 2 conductors

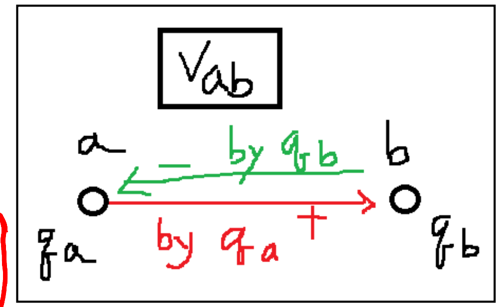


V_{ab} by q_a $\propto \frac{D}{r_a}$

D1 = ra
D2 = D

(path a→b) $\frac{ab}{aa'}$

$$\bar{V}_{ab} = \frac{\bar{q}_a}{2\pi k}\ln\frac{D}{r_a}$$



V_{ba} by q_b $\propto \frac{D}{r_b}$

→ V_{ab} by q_b : $\propto \frac{r_b}{D}$

$$\bar{V}_{ab} = \frac{\bar{q}_b}{2\pi k}\ln\frac{r_b}{D}$$

(path b→a) $\frac{bb'}{ba}$

due to q_b

or

$$\bar{V}_{ab} = -\frac{\bar{q}_b}{2\pi k}\ln\frac{D}{r_b}$$

$\bar{V}_{ba}$

sign

- +: out from *a*
- +: in to *b*
- -: in to *a*
- -: Out from *b*

V_{ab} $\begin{cases} + & V_{ab} \text{ direction} \\ - & V_{ba} \text{ direction} \end{cases}$

Negative Polarity: Toward a
Positive Polarity: Toward b

Capacitance Between 2 conductors

$$\bar{V}_{ab} = \frac{\bar{q}_a}{2\pi k} \ln\frac{D}{r_a} + \frac{\bar{q}_b}{2\pi k} \ln\frac{r_b}{D}$$

$q_a = -q_b$ for 2-wire line,

$$\bar{V}_{ab} = \frac{\bar{q}_a}{2\pi k}\left(\ln\frac{D}{r_a} - \ln\frac{r_b}{D}\right) [V]$$

$$= \frac{\bar{q}_a}{2\pi k} \ln\frac{D^2}{r_a \cdot r_b}$$

or

$$\bar{V}_{ab} = \frac{\bar{q}_a}{2\pi k} \ln\frac{D}{r_a} + \left(-\frac{\bar{q}_b}{2\pi k} \ln\frac{D}{r_b}\right)$$

$q_a = -q_b$ for 2-wire line,

$$\bar{V}_{ab} = \frac{\bar{q}_a}{2\pi k}\left(\ln\frac{D}{r_a} + \ln\frac{D}{r_b}\right) [V]$$

$$= \frac{\bar{q}_a}{2\pi k} \ln\frac{D^2}{r_a \cdot r_b}$$

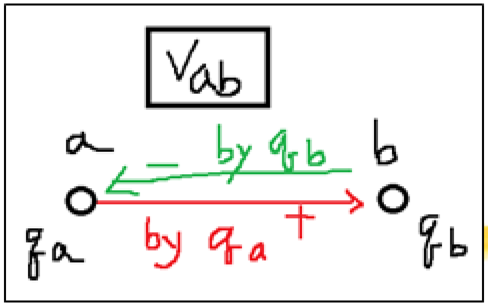
Now,

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{2\pi k}{\ln\left(\frac{D^2}{r_a \cdot r_b}\right)} \quad F/m$$

if $r_a = r_b = r$

$$\rightarrow C_{ab} = \frac{2\pi k}{\ln\left(\frac{D^2}{r^2}\right)} = \frac{\pi k}{\ln\left(\frac{D}{r}\right)} \quad F/m$$

(cf) $L_1 = 2 \times 10^{-7} \ln\frac{D}{r'}$ H/m

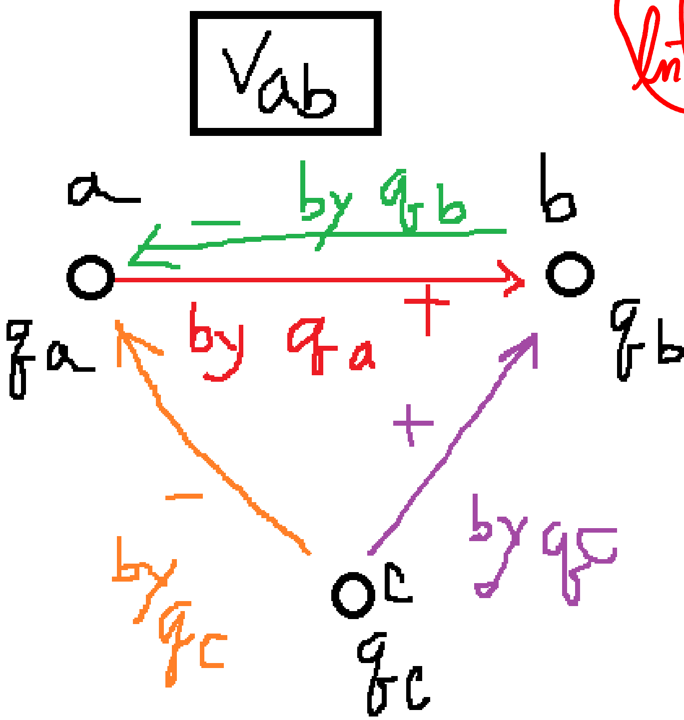


$\frac{\pi k}{\ln\left(\frac{D}{r}\right)}$

Capacitance Between 2 conductors

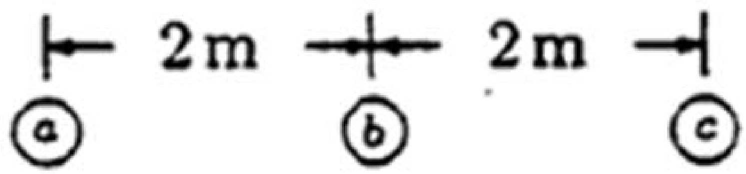
$\ln\left(\frac{\text{Distance}}{\text{radius}}\right)$

- Polarity for ln (D/r) format for V_{ab} when we have the third charge q_c:
  - +: out from *a*
  - +: in to *b*
  - -: in to *a*
  - -: Out from *b*



Class Activity

A 3-phase transmission line has flat horizontal spacing with 2 m between adjacent conductors as illustrated below. At a certain instant the charge on phase *a* conductor is 60 μC/km, and the charge on the *b* and *c* center conductors is -30 μC/km. The radius of each conductor is 0.8 cm. Find the voltage drop between the conductors *b* and *c* at the instant specified.



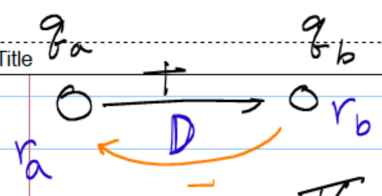
Class Activity

⌘ Recall

Note Title

$$V_{ab}\begin{cases} + & V_{ab} \text{ direction} \\ - & V_{ba} \text{ direction}\end{cases}$$

Negative Polarity: Toward a
Positive Polarity: Toward b

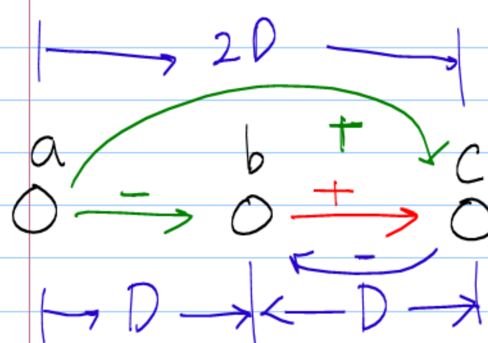


$$V_{ab} = \frac{q_a}{2\pi k}\ln\frac{D}{r_a} - \frac{q_b}{2\pi k}\ln\frac{D}{r_b}$$

If $q_b = -q_b$

$$V_{ab} = \frac{q_a}{2\pi k}\left(\ln\frac{D}{r_a} + \ln\frac{D}{r_b}\right)$$

$$= \frac{q_a}{2\pi k}\ln\frac{D^2}{r_a \cdot r_b}$$

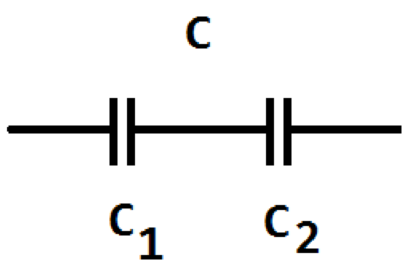


V^1_{bc} by q_b & q_c alone

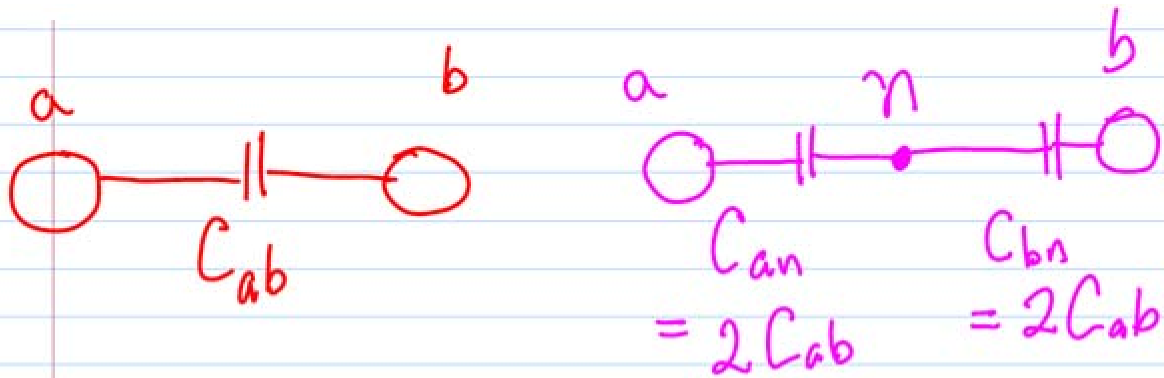
$$V^1_{bc} = \frac{q_b}{2\pi k}\ln\frac{D}{r_b} - \frac{q_c}{2\pi k}\ln\frac{D}{r_c}$$

Negative Polarity: Toward b
Positive Polarity: Toward c

Capacitance between a conductor and the neutral



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$C_n = C_{an} = C_{bn} = \frac{2\pi k}{\ln(D/r)} \quad F/m$$

$$\frac{1}{C_{ab}} = \frac{1}{C_n} + \frac{1}{C_n}$$

$$\boxed{C_n = 2 \cdot C_{ab}}$$

Capacitance between a conductor and the neutral

- Capacitive Reactance to Neutral (Xc):

$$X_C = \frac{1}{2\pi f C} = \frac{2.862}{f} \times 10^9 \ln\frac{D}{r}$$

$(\Omega \cdot m \text{ to neutral})$

$$k = 8.85 \times 10^{-12}\ F/m$$

$$X_C = \frac{2.862}{f} \cdot \frac{1}{1609} \times 10^9 \times \ln\frac{D}{r}$$

$$= \frac{1.779}{f} \times 10^6 \ln\frac{D}{r}$$

$(\Omega \cdot \text{mile})$

Capacitance between a conductor and the neutral

- Separation of 2 terms

$$X_c = \frac{1.779}{f} \times 10^6 \ln \frac{D}{r} \quad (\Omega \cdot \text{mile})$$

$$X_c = \underbrace{\frac{1.779}{f} \times 10^6 \ln \frac{1}{r}}_{X'_a} + \underbrace{\frac{1.779}{f} \times 10^6 \times \ln D}_{X'_d}$$

Capacitive reactance at 1-ft spacing

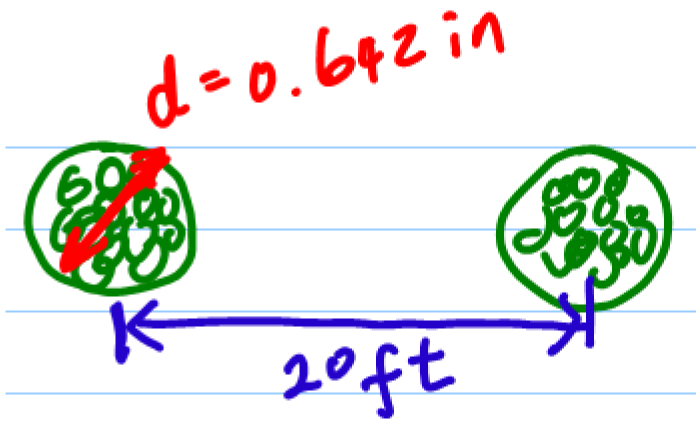
Capacitive reactance spacing factor

Example 4.1

- Find the capacitive susceptance per mile of a single-phase line operating at 60 Hz. The conductor is Partridge, an outside diameter of 0.642 in, and spacing is 20 ft between centers.
- Susceptance (B) = 1/Xc

C_n

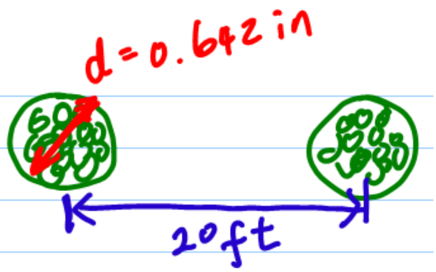
$$X_c = \frac{1.779}{f} \times 10^6 \ln\frac{D}{r} \; (\Omega \cdot \text{mile})$$



$$\boxed{C_n = 2 \cdot C_{ab}}$$

Example 4.1

- Find the capacitive susceptance per mile of a single-phase line operating at 60 Hz. The conductor is Partridge, an outside diameter of 0.642 in, and spacing is 20 ft between centers.



$C_n = 2 \cdot C_{ab}$

$d := 0.6421$ in

$r := \frac{d}{2 \cdot 12} = 0.0268$ ft

$D := 20$

From $X_C = \frac{1.779}{f} \times 10^6 \ln \frac{D}{r}$

$XCan := \frac{1.779}{60} \cdot 10^6 \cdot \ln\left(\frac{D}{r}\right) = 1.9619 \cdot 10^5$

Ω mile to neutral

Line to Line

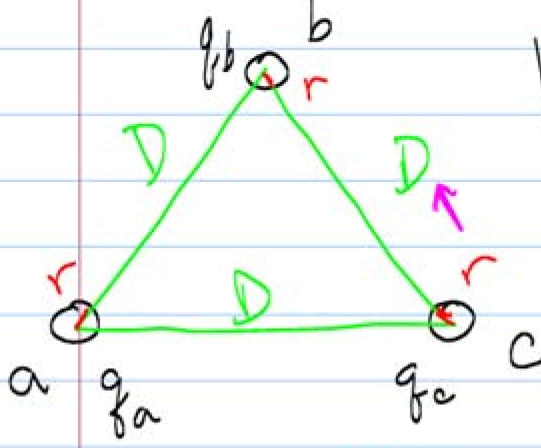
$XCab := \frac{XCan}{2} = 98094.0204$ Ω mile between conductors a and b

Susceptance

$BCan := \frac{1}{XCan} = 5.0972 \cdot 10^{-6}$ mho per mile to neutral

$BCab := \frac{1}{XCab} = 1.0194 \cdot 10^{-5}$ mho per mile between a and b

4.4 Capacitance of a 3-phase line with equivalent Spacing



path a→b path b→a path c→a/b D_{cb} D_{ca}

$$V_{ab} = \frac{1}{2\pi k}\left(\underbrace{q_a \ln\frac{D}{r}}_{\text{due to } q_a} - \underbrace{q_b \ln\frac{D}{r}}_{\text{due to } q_b} + \underbrace{\cancel{q_c \ln\frac{D}{D}}}_{\text{due to } q_c}\right)$$

0

Similarly

$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln\frac{D}{r} + \cancel{q_b \ln\frac{D}{D}} - q_c \ln\frac{D}{r}\right)$$

0

path a→c path b→a/c path c→a

4.4 Capacitance of a 3-phase line with equivalent Spacing

Applying $q_a + q_b + q_c = 0$

$\Rightarrow -q_a = q_b + q_c$

$$V_{ab} + V_{ac} = \frac{1}{2\pi k}\left\{2q_a \ln\frac{D}{r} - (q_b + q_c)\ln\frac{D}{r}\right\}$$

$$= \frac{1}{2\pi k}\left\{2q_a \ln\frac{D}{r} + q_a \ln\frac{D}{r}\right\}$$

$$= \frac{q_a}{2\pi k}\left\{\ln\frac{D^2}{r^2}\cdot\frac{D}{r}\right\} = \frac{q_a}{2\pi k}\ln\left(\frac{D}{r}\right)^3 = \frac{3q_a}{2\pi k}\ln\frac{D}{r}$$

4.4 Capacitance of a 3-phase line with equivalent Spacing

$$V_{ab} + V_{ac} = \frac{q_a}{2\pi k}\left\{ \ln \frac{D^2}{r^2} \cdot \frac{D}{r} \right\} = \frac{q_a}{2\pi k} \ln \left(\frac{D}{r}\right)^3 = \frac{3q_a}{2\pi k} \ln \frac{D}{r}$$

From $V_{ab} = V_{an} - V_{bn}$

$V_{bc} = V_{bn} - V_{cn}$

$V_{ca} = V_{cn} - V_{an}$

$\longrightarrow V_{ac} = -V_{ca} = V_{an} - V_{cn}$

$$V_{ab} + V_{ac} = V_{an} - V_{bn} + V_{an} - V_{cn}$$

$$= 2V_{an} - \underbrace{(V_{bn} + V_{cn})}_{= -V_{an}} = 3V_{an}$$

$$\boxed{V_{an} = \frac{V_{ab} + V_{ac}}{3} = \frac{q_a}{2\pi k} \ln \frac{D}{r}} \quad [V]$$

$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln\left(\frac{D}{r}\right)} \quad F/m \text{ to neutral}$$

3-φ equilateral

"Capacitance to neutral"

Identical!!!

(cf.) Single-phase capacitance

$$C_n = C_{an} = C_{bn} = \frac{2\pi k}{\ln(D/r)} \quad F/m$$

ALSO Remember that: Inductance per conductor is the same for 1-phase and 3-phase (equilateral)

Charging current between conductors

$$\textit{Charging current } I = \frac{V}{X_c}$$

Between Conductors (1ϕ)

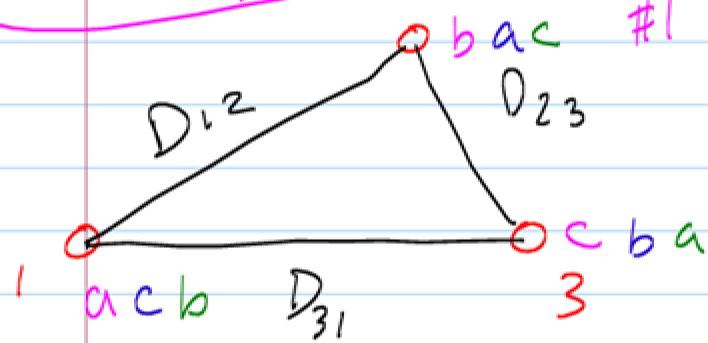
$$I_{chg} = j\omega C_{ab} V_{ab} \quad [A/mile]$$

Phasor charging current in phase a (3-ϕ)

$$I_{chg} = j\omega C_n C_{an} \quad [A/mile]$$

4.5 Capacitance of a 3-phase line with unsymmetrical spacing

Assumed: Cyclic transposition



Cycle 1

Cycle 2

Cycle 3

$$\#1 \quad V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{12}}{r} - q_b \ln\frac{D_{12}}{r} + q_c \ln\frac{D_{23}}{D_{31}}\right)$$

path 1–2   path 2–1   path 2–3/1

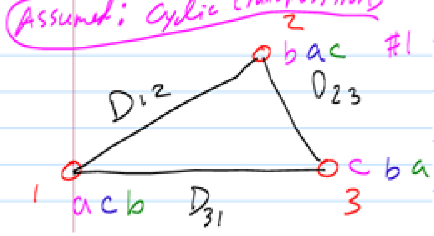
$$\#2 \quad V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{23}}{r} - q_b \ln\frac{D_{23}}{r} + q_c \ln\frac{D_{31}}{D_{12}}\right)$$

$$\#3 \quad V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{31}}{r} - q_b \ln\frac{D_{31}}{r} + q_c \ln\frac{D_{12}}{D_{23}}\right)$$

Average Voltage between a & b: $\dfrac{\{\text{Sum of above 3}\}}{3}$

4.5 Capacitance of a 3-phase line with unsymmetrical spacing

Assumed: Cyclic transposition



Average Voltage between a & b: $\dfrac{\{\text{Sum of above } 3\}}{3}$

$$\therefore V_{ab} = \frac{1}{6\pi k}\left(q_a \ln \frac{D_{12}D_{23}D_{31}}{r^3} - q_b \ln \frac{D_{12}D_{23}D_{31}}{r^3} + \cancel{q_c \ln \frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}}\right)$$

$$= \frac{1}{6\pi k}\left(q_a \ln \frac{\left(\sqrt[3]{D_{12}D_{23}D_{31}}\right)^3}{r^3} - q_b \ln \frac{\left(\sqrt[3]{D_{12}D_{23}D_{31}}\right)^3}{r^3}\right), \quad \sqrt[3]{D_{12}D_{23}D_{31}} = D_{eq}$$

$$= \frac{1}{6\pi k}\left(q_a \ln \frac{D_{eq}^3}{r^3} - q_b \ln \frac{D_{eq}^3}{r^3}\right)$$

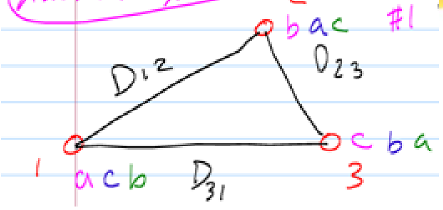
$$= \frac{1}{2\pi k}\left(q_a \ln \frac{D_{eq}}{r} - q_b \ln \frac{D_{eq}}{r}\right) \quad [V]$$

Similarly

$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln \frac{D_{eq}}{r} - q_c \ln \frac{D_{eq}}{r}\right) \quad [V]$$

4.5 Capacitance of a 3-phase line with unsymmetrical spacing

Assumed: Cyclic transposition



Using the relationship of $V_{ab}+V_{ac}=3V_{an}$

$$\rightarrow 3V_{an}=V_{ab}+V_{ac}=\frac{1}{2\pi k}\left(2q_a\ln\frac{D_{eq}}{r}-q_b\ln\frac{D_{eq}}{r}-q_c\ln\frac{D_{eq}}{r}\right)$$

Using $q_a+q_b+q_c=0 \rightarrow q_b+q_c=-q_a$

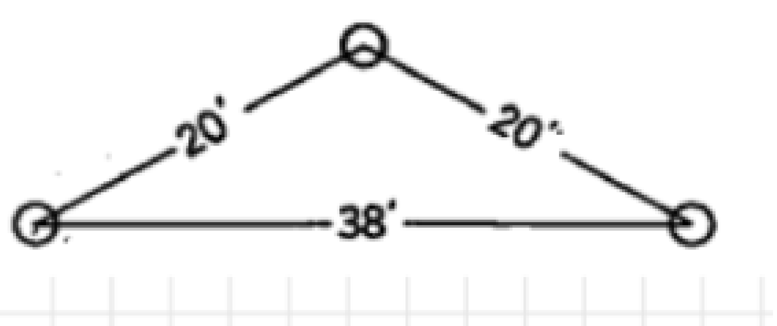
$$\rightarrow 3V_{an}=\frac{1}{2\pi k}\left(2q_a\ln\frac{D_{eq}}{r}+q_a\ln\frac{D_{eq}}{r}\right)$$

$$=\frac{1}{2\pi k}\left(q_a\ln\frac{D_{eq}^2\cdot D_{eq}}{r^2\cdot r}\right)=\frac{3}{2\pi k}q_a\cdot\ln\frac{D_{eq}}{r}$$

$$\Rightarrow C_n=\frac{q_a}{V_{an}}=\frac{2\pi k}{\ln\left(\frac{D_{eq}}{r}\right)}$$

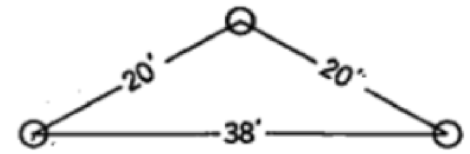
Example 4.2

- For a single-circuit 3-phase line, (a) find the capacitance (C) and the capacitive reactance (Xc) for 1 mile of the line configured as below with ACSR Drake (diameter of each conductor is 1.108 inches). (b) If the length of the line is 175 miles and the normal operating voltage is 220 kV, find (b-1) the capacitive reactance to neutral for the entire length of the line, (b-2) the charging current per mile, and (b-3) the total charging Volt-Amperes (VA or Q) for the entire length of the line.



Example 4.2

- For a single-circuit 3-phase line, (a) find the capacitance (C) and the capacitive reactance (Xc) for 1 mile of the line configured as below with ACSR Drake (diameter of each conductor is 1.108 inches). (b) If the length of the line is 175 miles and the normal operating voltage is 220 kV, find (b-1) the capacitive reactance to neutral for the entire length of the line, (b-2) the charging current per mile, and (b-3) the total charging volt-Amperes (VA or Q) for the entire length of the line.



PLEASE note here (comapred with Ex 3.4), in C calculation we use r (outside diameter) as opposed to GMR (which is r') for L calc.

In Ex3.4 we use, for Drake, Ds=0.0373 as GMR, but here we need to use r (outside diameter) instead.

$L := 175$ mile $\quad VL := 220 \cdot 10^{3}$

$d := 1.108$ in $\quad$ From Table A1. $\quad D12 := 20 \quad D23 := 38 \quad D31 := 20$

Radius $\quad r := \dfrac{d}{2 \cdot 12} = 0.0462$ ft $\quad Deq := \sqrt[3]{D12 \cdot D23 \cdot D31} = 24.7712$

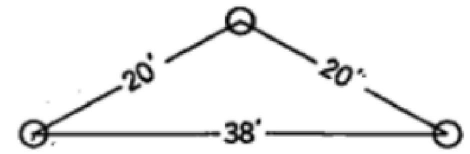
$k := 8.85 \cdot 10^{-12}$

$Cn := \dfrac{2 \cdot \pi \cdot k}{\ln\left(\dfrac{Deq}{r}\right)} = 8.8472 \cdot 10^{-12}$ F / m

$Xc1 := \dfrac{1}{2 \cdot \pi \cdot 60 \cdot Cn} = 2.9982 \cdot 10^{8}$ Ω m

Example 4.2

⌘ For a single-circuit 3-phase line, (a) find the capacitance (C) and the capacitive reactance (Xc) for 1 mile of the line configured as below with ACSR Drake (diameter of each conductor is 1.108 inches). (b) If the length of the line is 175 miles and the normal operating voltage is 220 kV, find (b-1) the capacitive reactance to neutral for the entire length of the line, (b-2) the charging current per mile, and (b-3) the total charging volt-ampere (VA or Q) for the entire length of the line.



$$Xc2 := \frac{Xc1}{1609} = 1.8634 \cdot 10^{5} \quad \Omega \text{ mile}$$

$$Xct := \frac{Xc2}{L} = 1064.8034 \quad \Omega$$

$$Ichg1 := 2 \cdot \pi \cdot 60 \cdot Cn \cdot \frac{VL}{\sqrt{3}} = 0.0004 \quad A / m$$

$$Ichg2 := Ichg1 \cdot 1609 = 0.6816 \quad A / mi$$

$$IchgTotal := Ichg2 \cdot L = 119.2869 \quad A$$

$$IL := IchgTotal = 119.2869$$

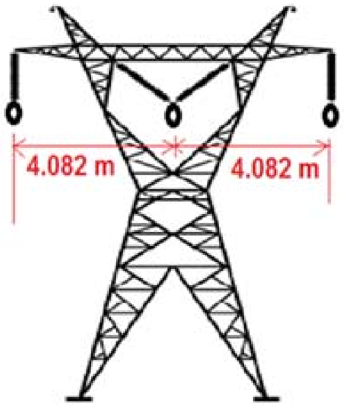
$$Q := \sqrt{3} \cdot VL \cdot IL = 4.5454 \cdot 10^{7} \quad Var$$

$$\frac{Q}{10^{6}} = 45.4544 \quad MVar$$

Positive Q --- Generated by the distributed capacitance of the line

Class Activity --- 3-Phase L and Y

As illustrated in a transmission system map below, the transmission line between Pickering NGS and Cherrywood TS is 100 km long with 230 kV. The structure of the transmission line and the data for conductor data are also shown the figure below.



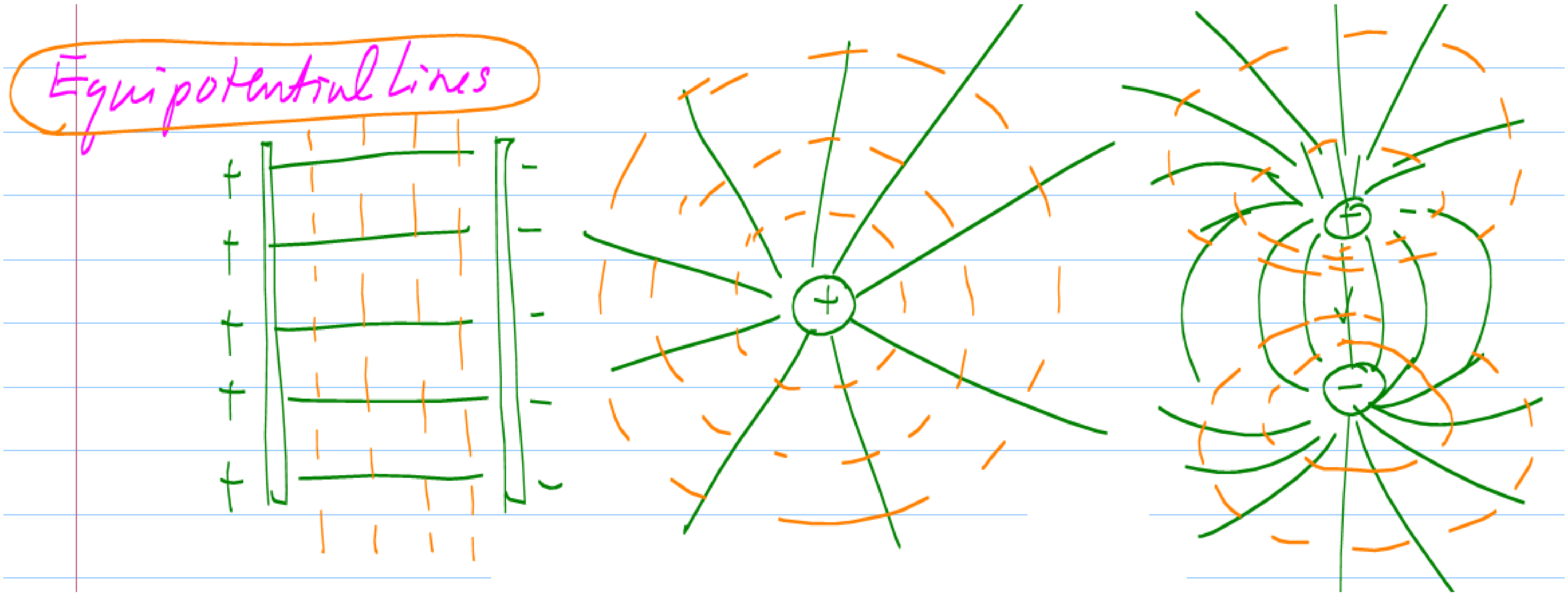
Phase Conductor Data 795 MCM SCSR

r: radius = 1.350 cm
r' = GMR = 1.073 cm
R = Resistance = 0.08 Ω/km

Line completely transposed

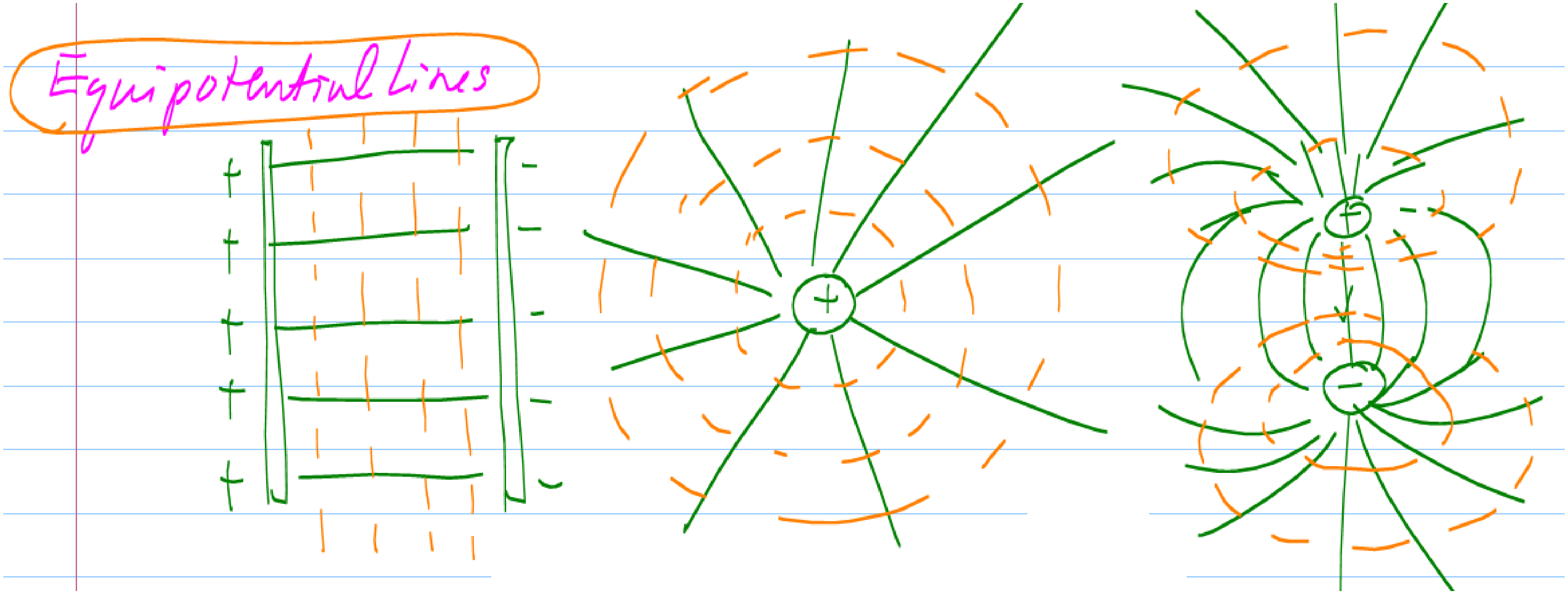
(Q) Determine the shunt admittance (i.e., Y) in S/km at 60Hz for the transmission line between the Pickering NGS and Cherrywood TS.

4.6 Effect of Earth on the Capacitance of 3-phase lines



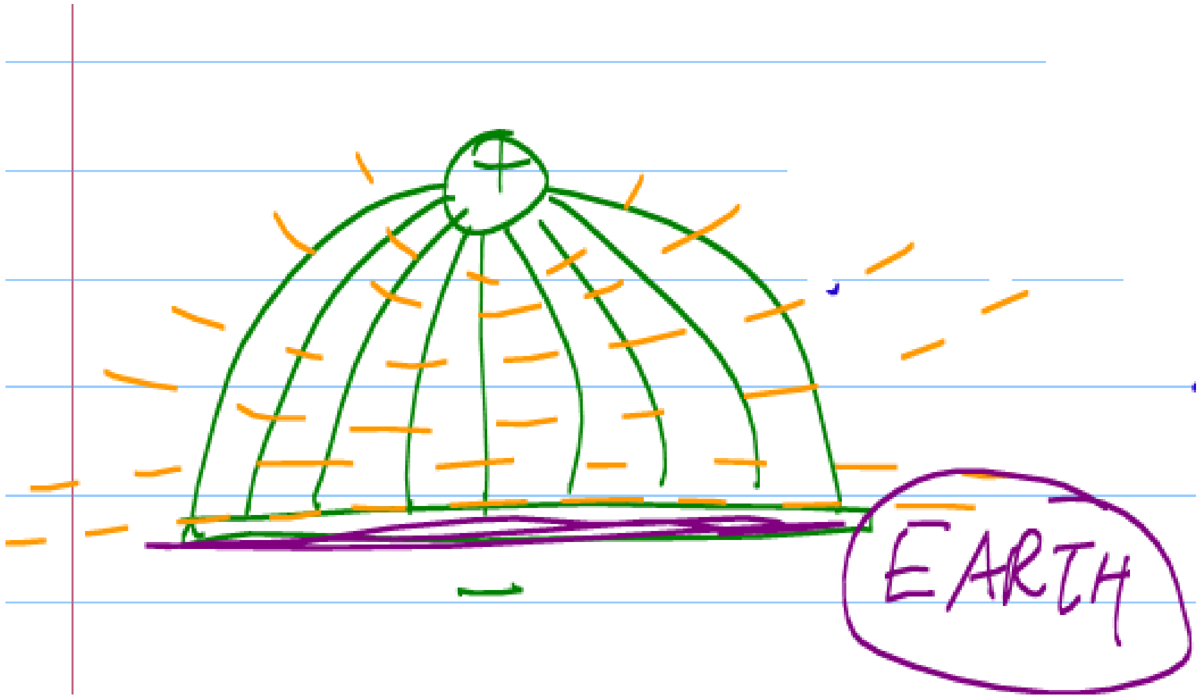
4.6 Effect of Earth on the Capacitance of 3-phase lines

- Normal E-Field Lines



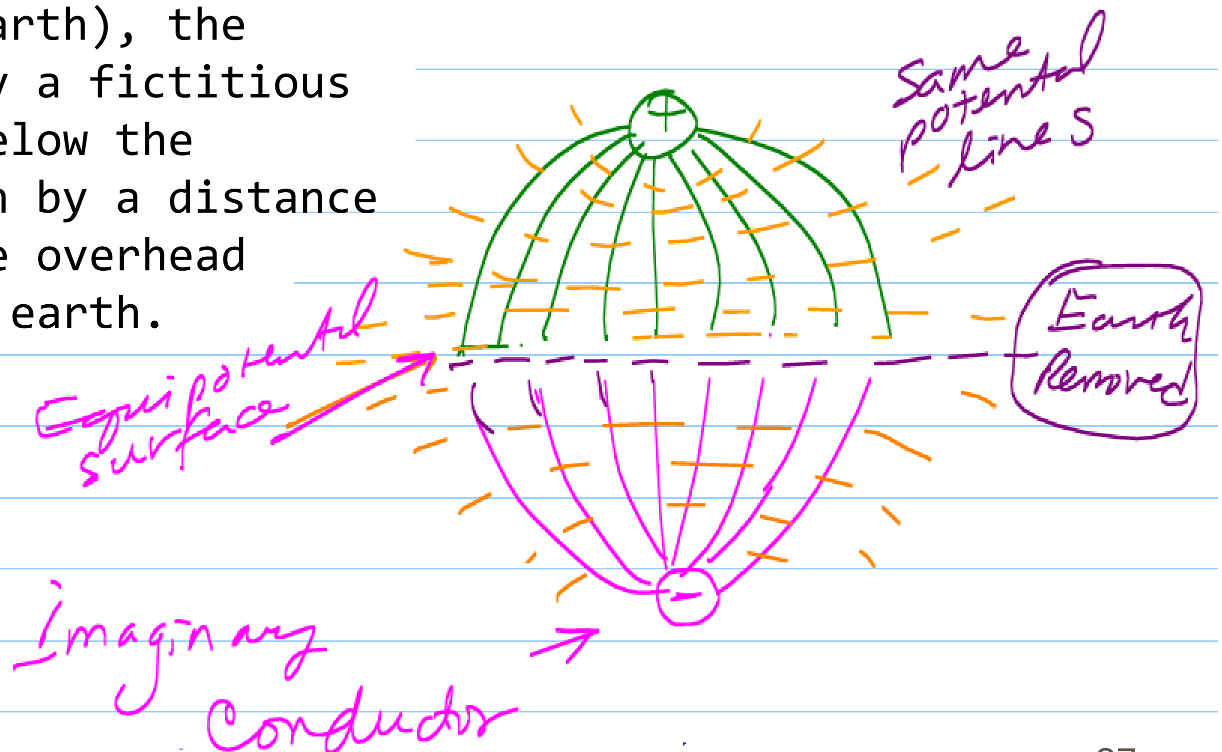
4.6 Effect of Earth on the Capacitance of 3-phase lines

- Actual E-Field Lines

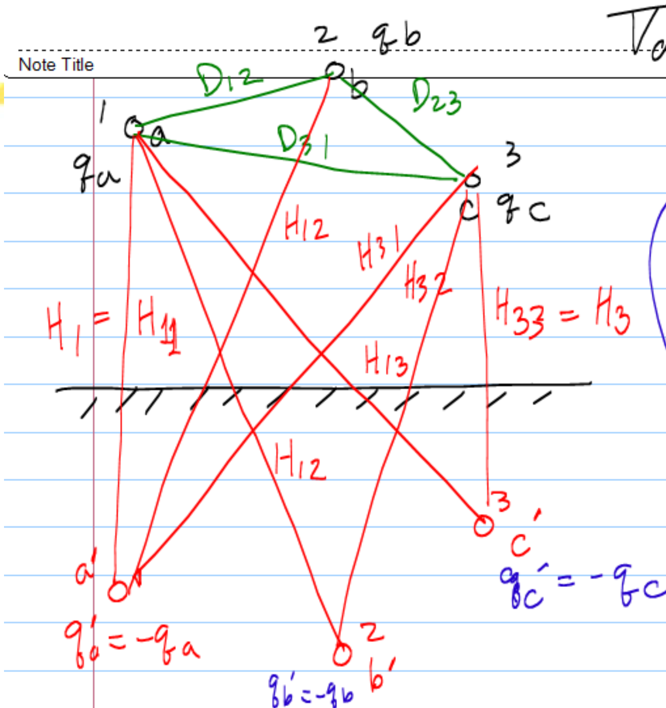


4.6 Effect of Earth on the Capacitance of 3-phase lines

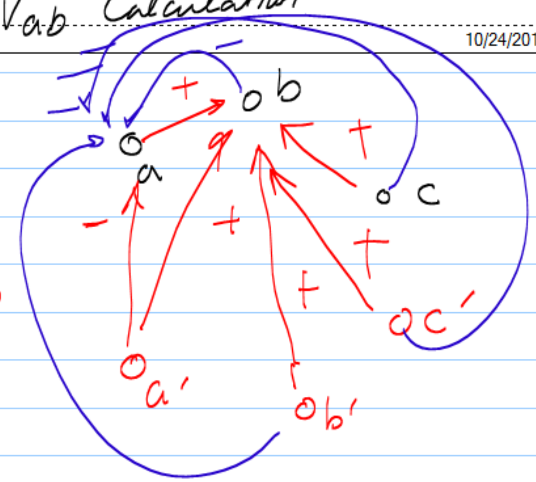
- “Imaginary Conductor”: for the purpose of capacitance calculation (on the effect of Earth), the earth is replaced by a fictitious charged conductor below the surface of the earth by a distance equal to that of the overhead conductor above the earth.

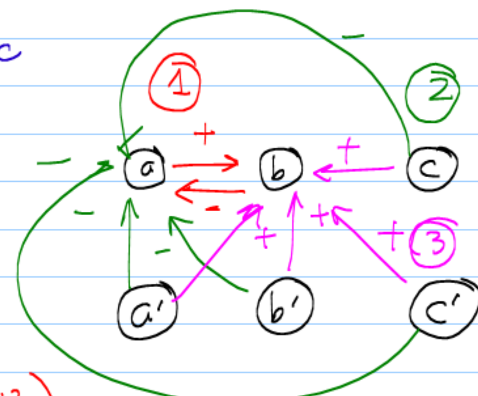


3-Phase line and its image



V_{ab} Calculation

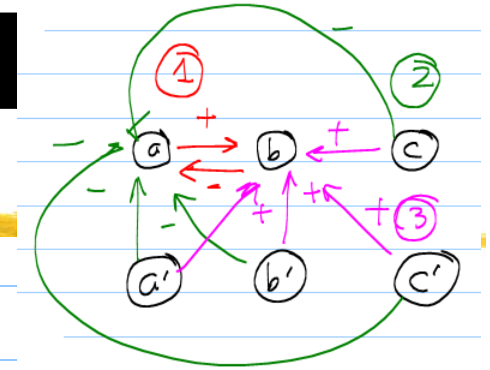




① by q_a & q_b

$$V_{ab}' = \frac{1}{2\pi k}\left(q_a \ln \frac{D_{12}}{r_a} - q_b \ln \frac{D_{12}}{r_b}\right)$$

3-Phase line and its image



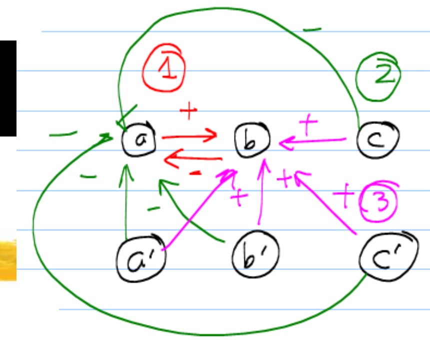
② by q_a', q_b', q_c', and q_c toward a

$$V_{ab}^2 = -\frac{1}{2\pi k}\left(q_a' \ln\frac{H_1}{r_a'} + q_b' \ln\frac{H_{12}}{r_b'} + q_c' \ln\frac{H_{31}}{r_c'} + q_c \ln\frac{D_{31}}{r_c}\right)$$

③ by q_a', q_b', q_c', and q_c toward b

$$V_{ab}^3 = \frac{1}{2\pi k}\left(q_a' \ln\frac{H_{12}}{r_a'} + q_b' \ln\frac{H_2}{r_b'} + q_c' \ln\frac{H_{22}}{r_c'} + q_c \ln\frac{D_{23}}{r_c}\right)$$

3-Phase line and its image



$$\bar{V}_{ab} = \bar{V}_{ab}^{1} + \bar{V}_{ab}^{2} + \bar{V}_{ab}^{3}$$

$$q_a' = -q_a, \quad q_b' = -q_b, \quad q_c' = -q_c$$

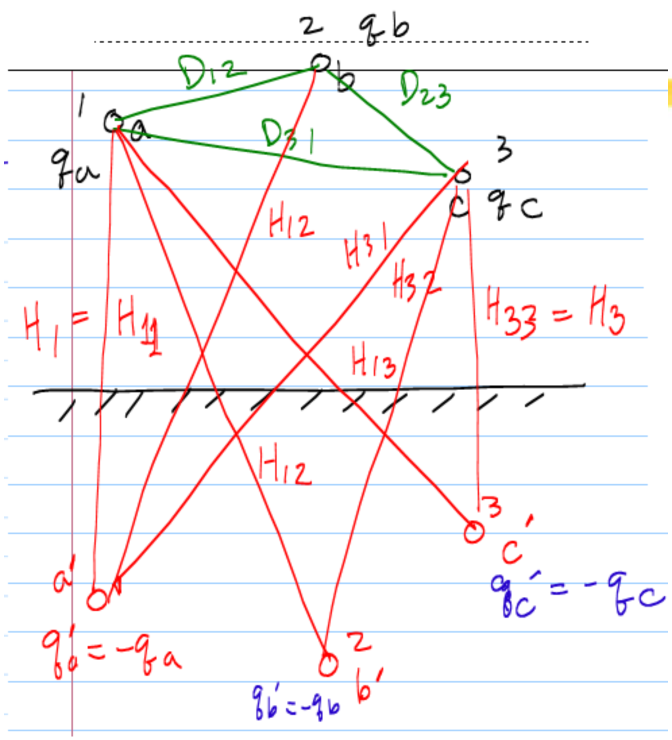
$$r_a = r_b = r_c = r_a' = r_b' = r_c' = r$$

$$= \frac{1}{2\pi k}\Bigg[\left(q_a \ln\frac{D_{12}}{r} + q_a \ln\frac{H_1}{r} - q_a \ln\frac{H_{12}}{r}\right)$$

$$+ \left(-q_b \ln\frac{D_{12}}{r} + q_b \ln\frac{H_{12}}{r} - q_b \ln\frac{H_2}{r}\right)$$

$$+ \left(-q_c \ln\frac{H_{13}}{r} - q_c \ln\frac{H_{23}}{r} - q_c \ln\frac{D_{31}}{r} + q_c \ln\frac{D_{23}}{r}\right)\Bigg]$$

3-Phase line and its image



$$V_{ab} = \frac{1}{2\pi k}\Big[q_a\left(\ln\frac{D_{12}}{r} - \ln\frac{H_{12}}{H_1}\right) - q_b\left(\ln\frac{D_{23}}{r} - \ln\frac{H_{12}}{H_2}\right) - q_c\left(\ln\frac{D_{31}}{D_{23}} - \ln\frac{H_{13}}{H_{23}}\right)\Big]$$

Similarly, we get V_{ac}

and $V_{an} = \dfrac{V_{ab}+V_{ac}}{3} \rightarrow C_n = \dfrac{q_a}{V_{an}}$

3-Phase line and its image

$$C_n = \frac{2\pi k}{\ln\left(D_{eq}/r\right) - \ln\left(\dfrac{\sqrt[3]{H_{12}H_{23}H_{31}}}{\sqrt[3]{H_1 H_2 H_3}}\right)}$$

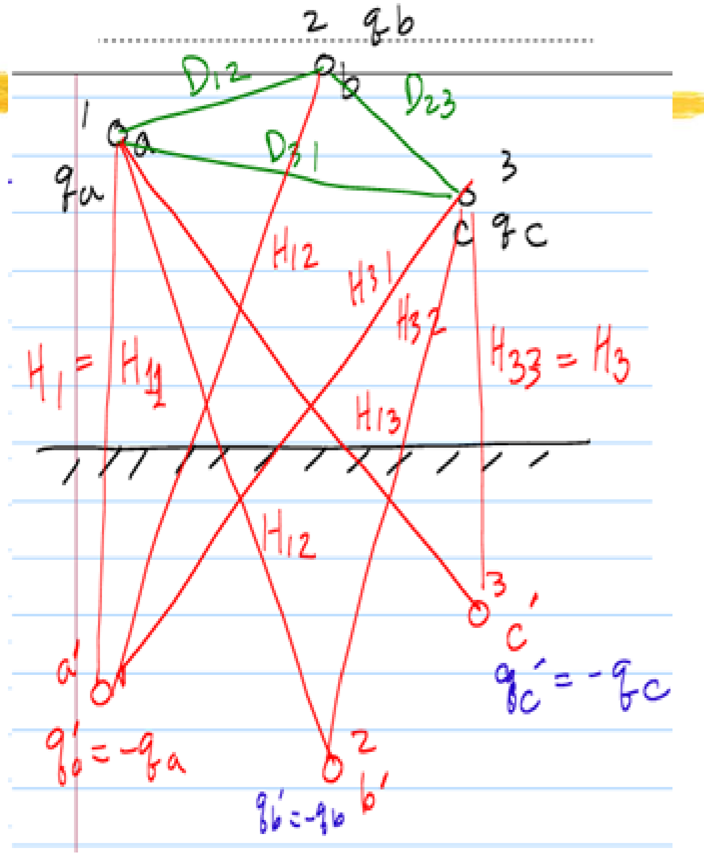
$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$

factor

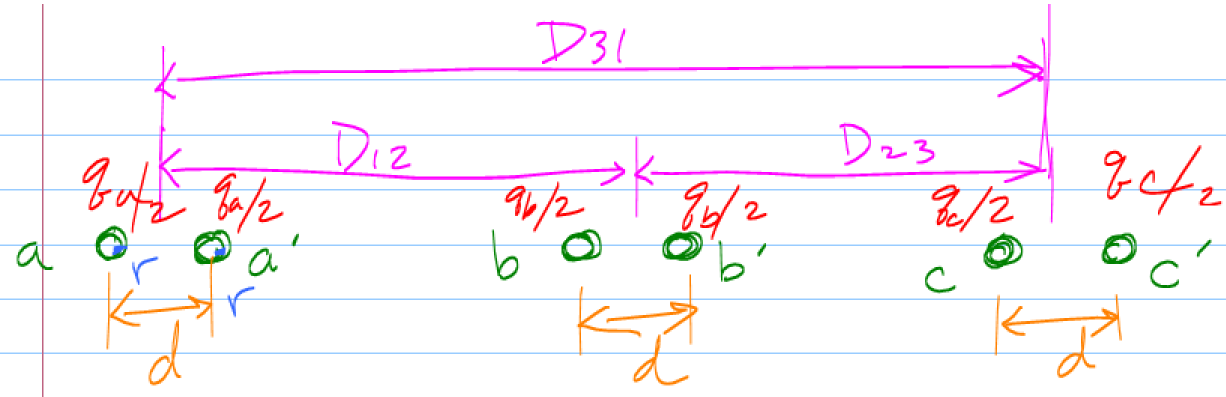
cf. $\boxed{C_n = \dfrac{2\pi k}{\ln\left(\frac{D_{eq}}{r}\right)}}$ (without Earth effect)

C_n (with earth effect) > C_n (without Earth effect)

If conductors are very high, then $H_1 \doteq H_{12}$, $H_2 \doteq H_{23}$ & $H_{31} \doteq H_3$, the factor disappears



4.7 Bundled 3-phase conductors



Since $D >> d$,

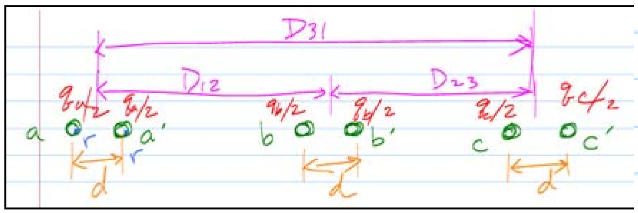
$$D_{12} - d \approx D_{12}$$

$$D_{12} + d \approx D_{12}$$

① Each phase has q_a, q_b, and q_c charge

$$\rightarrow \left\{ \begin{array}{lll} a \rightarrow q_a/2 & b' \rightarrow q_b/2 & c' \rightarrow q_c/2 \\ a' \rightarrow q_a/2 & b \rightarrow q_b/2 & c \rightarrow q_c/2 \end{array} \right\}$$

4.7 Bundled 3-phase conductors



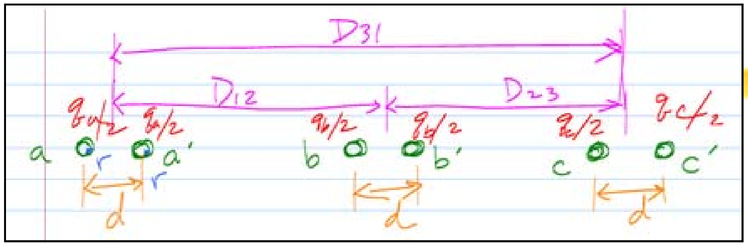
$$② \; V_{ab} = \frac{1}{2\pi k}\left(\frac{q_a}{2}\left(\underbrace{\ln\frac{D_{12}}{r}}_{a} + \underbrace{\ln\frac{D_{12}}{d}}_{a'}\right)\right. \qquad \ln\frac{D_{12}^2}{rd} = 2\ln\frac{D_{12}}{\sqrt{rd}}$$

$$- \frac{q_b}{2}\left(\underbrace{\ln\frac{D_{12}}{r}}_{b} + \underbrace{\ln\frac{D_{12}}{d}}_{b'}\right) \qquad \ln\frac{D_{12}^2}{rd} = 2\ln\frac{D_{12}}{\sqrt{rd}}$$

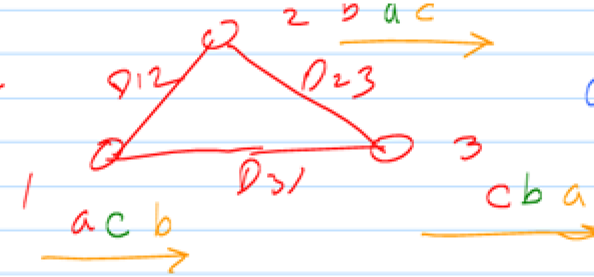
$$\left. + \frac{q_c}{2}\left(\underbrace{\ln\frac{D_{23}}{D_{31}}}_{c} + \underbrace{\ln\frac{D_{23}}{D_{31}}}_{c'}\right)\right) \qquad \ln\frac{D_{23}^2}{D_{31}^2} = 2\ln\frac{D_{23}}{D_{31}}$$

$$= \frac{1}{2\pi k}\left(q_a \ln\frac{D_{12}}{\sqrt{rd}} - q_b \ln\frac{D_{12}}{\sqrt{rd}} + q_c \ln\frac{D_{23}}{D_{31}}\right)$$

4.7 Bundled 3-phase conductors



As before



cyclic transposition

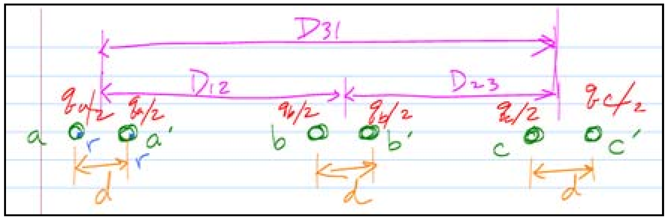
#2 c c' a a' b b'

#3 b b' c c' a a'

#2 $V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{23}}{\sqrt{rd}} - q_b \ln\frac{D_{23}}{\sqrt{rd}} + q_c \ln\frac{D_{31}}{D_{12}}\right)$

#3 $V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{31}}{\sqrt{rd}} - q_b \ln\frac{D_{31}}{\sqrt{rd}} + q_c \ln\frac{D_{12}}{D_{23}}\right)$

4.7 Bundled 3-phase conductors



$\rightarrow$ Average $V_{ab} = \dfrac{{}^{\#1}V_{ab} + {}^{\#2}V_{ab} + {}^{\#3}V_{ab}}{3}$

$$= \frac{1}{2\pi k}\left(q_a \ln\frac{D_{eq}}{\sqrt{rd}} - q_b \ln\frac{D_{eq}}{\sqrt{rd}}\right) \quad [V]$$

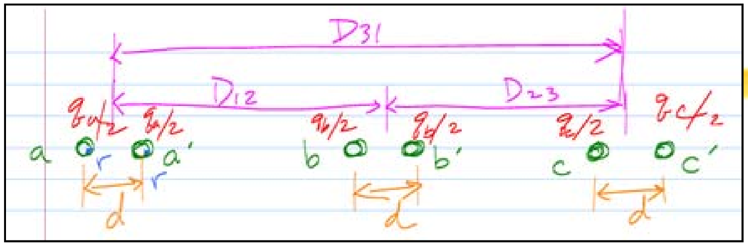
where $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

Similarly

$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln\frac{D_{eq}}{\sqrt{rd}} - q_c \ln\frac{D_{eq}}{\sqrt{rd}}\right) \quad [V]$$

$$3V_{an} = V_{ab} + V_{ac} = \frac{1}{2\pi k}\left(2q_a \ln\frac{D_{eq}}{\sqrt{rd}} - q_b \ln\frac{D_{eq}}{\sqrt{rd}} - q_c \ln\frac{D_{eq}}{\sqrt{rd}}\right)$$

4.7 Bundled 3-phase conductors



$$3V_{an} = V_{ab}+V_{ac} = \frac{1}{2\pi k}\left(2q_a \ln\frac{D_{eq}}{\sqrt{rd}} - q_b \ln\frac{D_{eq}}{\sqrt{rd}} - q_c \ln\frac{D_{eq}}{\sqrt{rd}}\right)$$

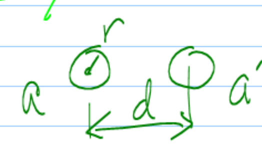
Using $q_a+q_b+q_c=0 \rightarrow (q_b+q_c)=-q_a$

$$3V_{an} = \frac{3}{2\pi k}\, q_a \cdot \ln\frac{D_{eq}}{\sqrt{rd}}$$

$$\Rightarrow C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln(D_{eq}/\sqrt{rd})}$$

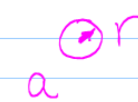
$\boxed{cf}$ $C_n = \dfrac{2\pi k}{\ln\frac{D_{eq}}{r}}$

Bundle line 3-φ



$$\ln\frac{D_{eq}}{\sqrt{rd}}$$

Single-line 3φ phase



$$\ln\frac{D_{eq}}{r}$$

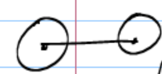
4.7 Bundled 3-phase conductors

$$\boxed{C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln(D_{eq}/\sqrt{rd})}}$$

$\sqrt{rd} = D_S^b$ Bundled GMR in inductance calculative

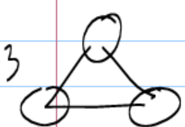
D_{SC}^b Bundled GMR in capacitance calculation

b2



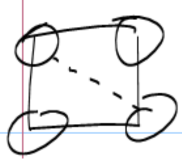
$$D_{SC}^b = \sqrt[4]{r \cdot r \cdot d \cdot d} = \sqrt{rd}$$

b3



$$D_{SC}^b = \sqrt[9]{r^3 \cdot d^3 \cdot d^3} = \sqrt[3]{rd^2}$$

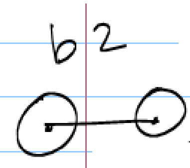
b4



$$D_{SC}^b = \sqrt[16]{r^4 \cdot d^3 \cdot (\sqrt{2})^4} = (1.09)\sqrt[4]{rd^3}$$

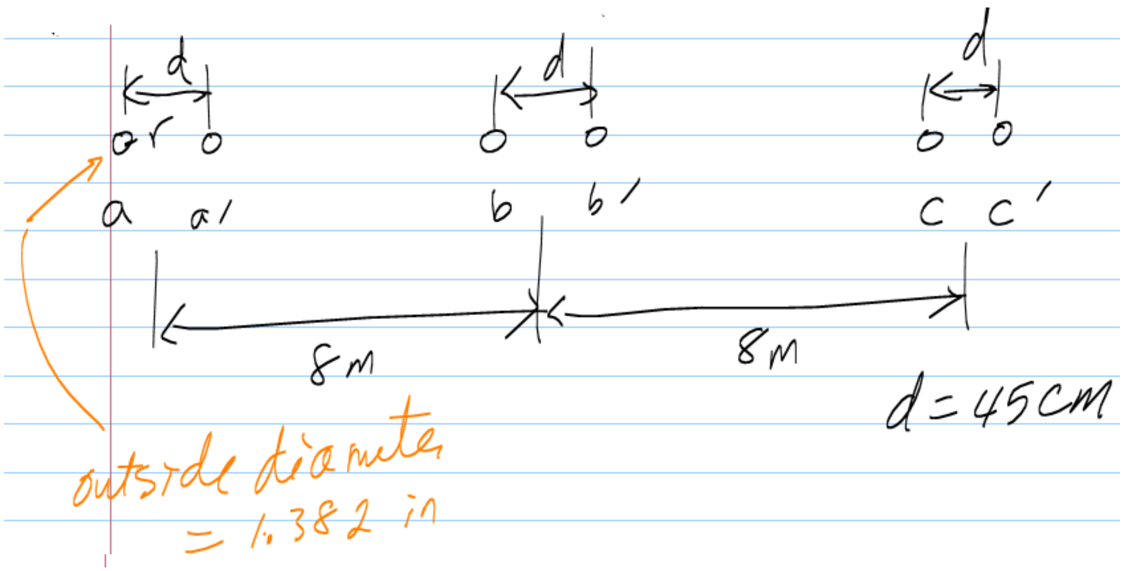
Example 4.3

⌘ Find the capacitive reactance to neutral of the line show below. The outside diameter of each conductor is 1.382 inches, and the distance of each bundled conductor is 45 cm.



$$D_{SC}^{b} = \sqrt[4]{r \cdot r \cdot d \cdot d} = \sqrt{rd}$$

$$\boxed{C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln\left(D_{eq}/\sqrt{rd}\right)}}$$



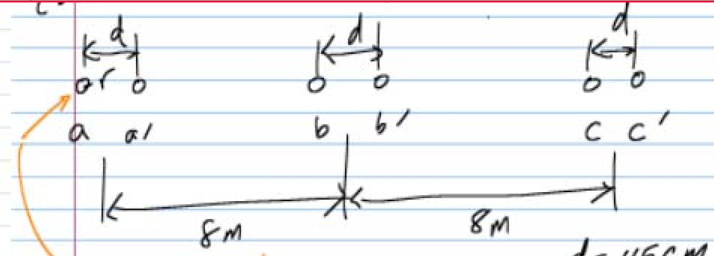
$$\boxed{r = \frac{1.382}{2} \times (2.54) = 1.7551 \text{ cm}}$$

Example 4.3

- Find the capacitive reactance to neutral of the line show below. The outside diameter of each conductor is 1.382 inches, and the distance of each bundled conductor is 45 cm.

b2

$$D_{SC}^{b} = \sqrt[4]{r\cdot r\cdot d\cdot d} = \sqrt{rd}$$



outside diameter = 1.382 in

$$\rightarrow r = \frac{1.382}{2}(2.54) = 1.7551 \text{ cm}$$

$$r := \frac{1.382}{2}\cdot\frac{2.54}{100} = 0.0176 \quad \text{m}$$

$$d := \frac{45}{100} = 0.45 \quad \text{m}$$

$$DsCb2 := \sqrt{r\cdot d} = 0.0889 \quad \text{m}$$

$$D12 := 8 \qquad D23 := 8 \qquad D31 := 16$$

$$Deq := \sqrt[3]{D12\cdot D23\cdot D31} = 10.0794$$

$$k := 8.85\cdot 10^{-12}$$

$$Cm := \frac{2\cdot\pi\cdot k}{\ln\left(\frac{Deq}{DsCb2}\right)} = 1.1753\cdot 10^{-11} \quad \text{F / m}$$

$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi k}{\ln\left(D_{eq}/\sqrt{rd}\right)}$$

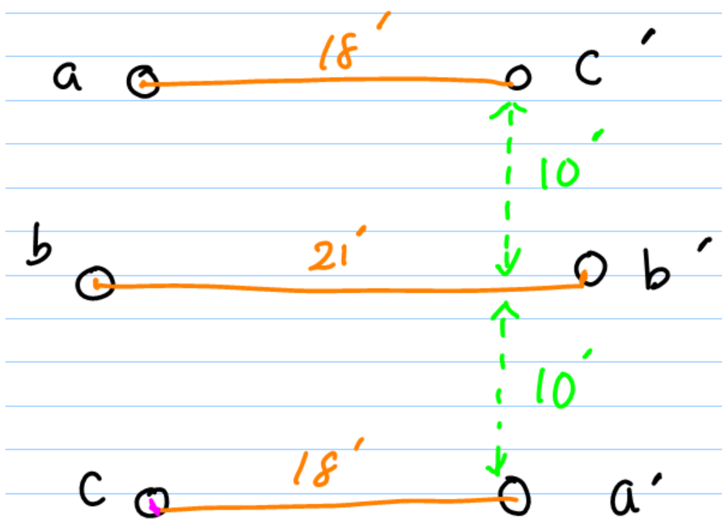
$$Xc := \frac{1}{2\cdot\pi\cdot 60\cdot Cm} = 2.2569\cdot 10^{8} \quad \Omega\ .\ \text{m per phase to neutral}$$

$$Xc\cdot 10^{-3} = 2.2569\cdot 10^{5} \quad \Omega\ .\ \text{km per phase to neutral}$$

$$\frac{Xc\cdot 10^{-3}}{1.609} = 1.4026\cdot 10^{5} \quad \Omega\ .\ \text{mile per phase to neutral}$$

4.8 Parallel-Circuit 3-Phase Lines

- Example: Find the 60-Hz Capacitive Susceptance to Neutral per mile per phase of the double-circuit 3-phase lines as constructed below. The outside diameter of each conductor is 0.68 inches.



$$d_{aa'} = \sqrt{20^2 + 18^2} = 26.9$$

$$d_{bb'} = 21 \qquad d_{cc'} = d_{aa'} = 26.9$$

$$D_{SC}^{P} = \sqrt[3]{\sqrt{r\,d_{aa'}} \cdot \sqrt{r \cdot d_{bb'}} \cdot \sqrt{r \cdot d_{cc'}}} = 0.837\ \text{ft}$$

$$D_{abp} = \sqrt[4]{D_{ab} \cdot D_{ab'} \cdot D_{a'b} \cdot D_{a'b'}}\ , \quad \text{etc}$$

$$D_{eq} = \sqrt[3]{D_{abp} \cdot D_{bcp} \cdot D_{cap}} = 16.14\ \text{ft}$$

$$C_n = \frac{2\pi k}{\ln\left(D_{eq}/D_{SC}^{P}\right)} = 18.81 \times 10^{-12}\ F/m$$

$$B_c = 2\pi f\, C_n \cdot 1609 = 11.41 \times 10^{-6}\ \mho/mi$$

4.8 Parallel-Circuit 3-Phase Lines

$$Deq := \sqrt[3]{Dabp \cdot Dbcp \cdot Dcap} = 16.1401$$ We use this one!!

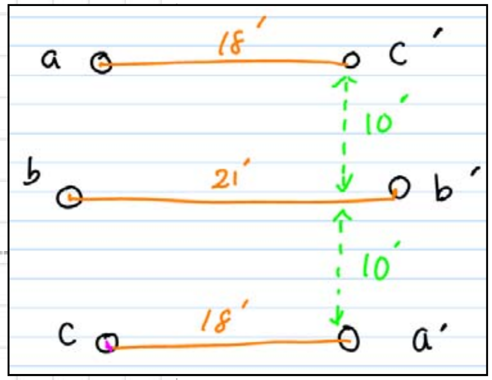
Now let's work on Modified GMD method for Capacitance Calculation

$$r := \frac{0.68}{2} \cdot \frac{1}{12} = 0.0283 \quad \text{ft} \qquad k := 8.85 \cdot 10^{-12}$$

$d_{aa'} = \sqrt{20^2 + 18^2} = 26.9$

$d_{bb'} = 21 \qquad d_{cc'} = d_{aa'} = 26.9$

$$daa' := \sqrt{18^2 + 20^2} = 26.9072 \qquad dbb' := 21 \qquad dcc' := daa' = 26.9072$$



$$DsCp := \sqrt[3]{\sqrt{r \cdot daa'} \cdot \sqrt{r \cdot dbb'} \cdot \sqrt{r \cdot dcc'}} = 0.8378 \quad \text{ft}$$

$D_{abp} = \sqrt[4]{D_{ab} \cdot D_{ab'} \cdot D_{a'b} \cdot D_{a'b'}}$, etc

$$Cn := \frac{2 \cdot \pi \cdot k}{\ln\left(\frac{Deq}{DsCp}\right)} = 1.8797 \cdot 10^{-11} \quad \text{F / m}$$

$$Xc1 := \frac{1}{2 \cdot \pi \cdot 60 \cdot Cn} = 1.4112 \cdot 10^{8} \quad \Omega \ \text{m} \qquad Xc2 := \frac{1}{2 \cdot \pi \cdot 60 \cdot Cn \cdot 1609} = 87705.868 \quad \Omega \ \text{mile}$$

$$Bc1 := \frac{1}{Xc1} = 7.0862 \cdot 10^{-9} \quad \text{mho / m} \qquad Bc2 := \frac{1}{Xc2} = 1.1402 \cdot 10^{-5} \quad \text{mho / mile}$$

Summary

① Capacitance to Neutral

$$C_n = \frac{2\pi k}{\ln\left(\frac{D_{eq}}{D_{sC}}\right)} \quad F/m$$

$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$

$\begin{cases} r : \text{outside radius} \\ \sqrt{rd} : \text{Bundle} \end{cases}$

$$k = 8.85 \times 10^{-12}$$

$$X_C = \frac{1}{\omega \cdot C_n} \quad \Omega \cdot m = \frac{1}{\omega \cdot C_n \cdot 1000} \quad \Omega \cdot km$$

$$= \frac{1}{\omega \cdot C_n \cdot 1609} \quad \Omega \cdot mile$$

$$B_C = \frac{1}{X_C} \quad \text{susceptance} \quad \mho/km \quad \mho/mile$$

"mho" or "Siemens"