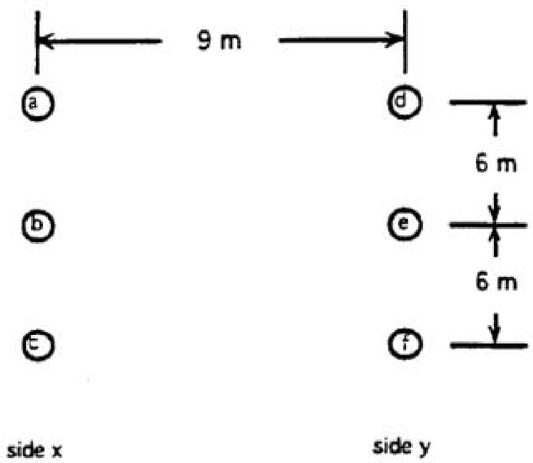
Class Activity XY-side

As illustrated below a single-phase transmission line is composed of 3 solid conductors (X side). The return circuit (Y side) is also composed of 3 wires. The radius of each conductor wire is 0.25 cm. Find (a) the inductance due to the each side of the line (namely, L_x and L_y) and (b) the inductance of the complete circuit (namely L).



$$L_X = \frac{L_{av}}{n} = \frac{L_a + L_b + \cdots + L_n}{n^2}$$

$$= 2\times10^{-7} \times \ln \frac{\sqrt[mn]{(D_{aa'}D_{ab'}\cdots D_{am})(D_{ba'}D_{bb'}\cdots D_{bm})\cdots(D_{na'}D_{nb'}\cdots D_{nm})}}{\sqrt[n^2]{(D_{aa}D_{ab}\cdots D_{an})(D_{ba}D_{bb}\cdots D_{bn})\cdots(D_{na}D_{nb}\cdots D_{nn})}}$$

where $D_{aa} = r_a'$

$D_{bb} = r_b'$

$\vdots$ etc

GMR (Geometric mean radius) of conductor X $= D_s$ ("self GMD")

GMD (Geometric Mean Distance) between conductor X and Conductor Y $= D_m$

Separation of X in to 2 parts

- $X = Xa + Xd$

GMR

$$X_L = 2\pi f L = 2\pi f \cdot 2\times10^{-7} \ln\frac{D_m}{D_s}$$

$$= 4\pi f \times10^{-7} \ln\frac{D_m}{D_s} \quad \Omega/m$$

$$= 2.022\times10^{-3}\cdot f\cdot \ln\frac{D_m}{D_s} \quad \Omega/mi$$

$$= 2.022\times10^{-3}\cdot f \ln\frac{1}{D_s}$$

$$+\ 2.022\times10^{-3}\cdot f\cdot \ln\frac{D_m}{1}$$

Xa
Inductive Reactance at 1-ft Spacing

Xd
inductive reactance spacing factor

Use of Tables for Inductance Determination

- GMR (or Ds) and GMD are usually available
- Xa (inductive Reactance at 1-ft spacing, i.e.,GMD=1) is available
- Xd (inductive reactance for Ds=1, or “spacing factor”) is also available
- Xa and Xd

GMR D_s, ft	Reactance per conductor 1-ft spacing, 60 Hz: Inductive X_a, Ω/mi ($2.022 \cdot 10^{-3} f \ln \frac{1}{D_s}$)	Reactance per conductor 1-ft spacing, 60 Hz: Capacitive X'_a, MΩ·mi
0.0198	0.476	0.1090
0.0217	0.465	0.1074
0.0229	0.458	0.1057
0.0222	0.462	0.1055

Feet	Inches: 0
0	......
1	0
2	0.0841
3	0.1333
4	0.1682
5	0.1953
6	0.2174
7	0.2361
8	0.2523
9	0.2666
10	0.2794
11	0.2910
12	0.3015
13	0.3112
14	0.3202
15	0.3286
16	0.3364
17	0.3438
18	0.3507
19	0.3573
20	0.3636
21	0.3694
22	0.3751
23	0.3805
24	0.3856
25	0.3900
26	0.3953
27	0.3999
28	0.4043
29	0.4086
30	0.4127
31	0.4167
32	0.4205
33	0.4243
34	0.4279
35	0.4314
36	0:4348
37	0.4382
38	0.4414

At 60 Hz, in Ω/mi per conductor
$X_d = 0.2794 \log d$
d = separation, ft
For three-phase lines
$d = D_{eq}$

Use of Tables for Inductance Determination

- X = Xa + Xd

GMR

$$X_L = 2\pi f L = 2\pi f \cdot 2\times10^{-7} \ln\frac{D_m}{D_s}$$

$$= 4\pi f \times 10^{-7} \ln\frac{D_m}{D_s} \ \Omega/m$$

$$= 2.022\times10^{-3}\cdot f \cdot \ln\frac{D_m}{D_s} \ \Omega/mi$$

A

$$= 2.022\times10^{-3}\cdot f \ln\frac{1}{D_s}$$

$$+ 2.022\times10^{-3}\cdot f \cdot \ln\frac{D_m}{1}$$

Xa

Inductive Reactance at 1-ft Spacing

Xd

inductive reactance spacing factor

GMR D_s, ft	Reactance per conductor 1-ft spacing, 60 Hz: Inductive X_a, Ω/mi ($2.022\times10^{-3} f \ln\frac{1}{D_s}$)	Capacitive X'_a, MΩ·mi
0.0198	0.476	0.1090
0.0217	0.465	0.1074
0.0229	0.458	0.1057
0.0222	0.462	0.1055

```
>>> A=2.022*0.001*60
>>> Ds=0.0217
>>> A*math.log(1/Ds)
0.46470934699662192
>>>
```

Use of Tables for Inductance De

- $X = X_a + X_d$

GMR

$$X_L = 2\pi f L = 2\pi f \cdot 2\times10^{-7} \ln \frac{D_m}{D_s}$$

$$= 4\pi f \times 10^{-7} \ln \frac{D_m}{D_s} \quad \Omega/m$$

$$= 2.022 \times 10^{-3} \cdot f \cdot \ln \frac{D_m}{D_s} \quad \Omega/mi$$

$$= 2.022 \times 10^{-3} \cdot f \ln \frac{1}{D_s}$$

$$+ 2.022 \times 10^{-3} \cdot f \cdot \ln \frac{D_m}{1}$$

Xa

Inductive Reactance at 1-ft Spacing

Xd

inductive spacing fac

A

At 60 Hz, in Ω/mi per conductor
$X_d = 0.2794 \log d$
d = separation, ft
For three-phase lines
$d = D_{eq}$

Feet	Inches: 0
0	......
1	0
2	0.0841
3	0.1333
4	0.1682
5	0.1953
6	0.2174
7	0.2361
8	0.2523
9	0.2666
10	0.2794
11	0.2910
12	0.3015
13	0.3112
14	0.3202
15	0.3286
16	0.3364
17	0.3438
18	0.3507
19	0.3573
20	0.3635
21	[illegible]
22	0.3751
23	0.3805
24	0.3856
25	0.3906
26	0.3953
27	0.3999
28	0.4043
29	0.4086
30	0.4127
31	0.4167
32	0.4205
33	0.4243
34	0.4279
35	0.4314
36	0.4348
37	0.4382
38	0.4414

```
>>> A
0.12132
>>> A*math.log(2)
0.08409261594553264
>>> A*math.log(15)
0.3285406503977202
>>> A*math.log(20)
0.36344223942757015
>>> A*math.log(30)
0.41263326634325 27
>>>
```

Example 3.3

EX3.3 Find the inductive reactance per mile of a single-phase line operating at 60 Hz. The conductor's GMR (D_s) is 0.0217 ft, and the spacing between 2 conductor centers is 20 ft.

Example 3.3

EX3.3 Find the inductive reactance per mile of a single-phase line operating at 60 Hz. The conductor's GMR (D_s) is 0.0217 ft, and the spacing between 2 conductor centers is 20 ft.

(Sol) $D_m = 20$ ft

$D_s = 0.0217$ ft

$$X_L = 2.022 \times 10^{-3} \times f \ln \frac{D_m}{D_s}$$

$$= 2.022 \times 10^{-3} \times f \times \ln \frac{20}{0.0217}$$

$$= 0.828\ \Omega/\text{mi}$$

Above value is for one conductor only, therefore 2 conductors,

$$X_L = 2 \times 0.8285 = 1.657\ \Omega/\text{mi}$$

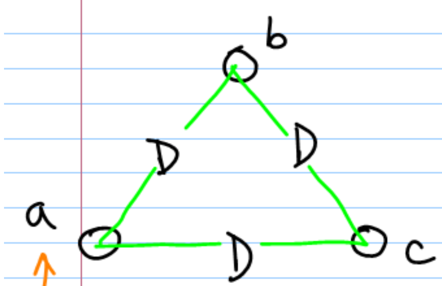
⌘ Alternatively

(Sol) When X_a and X_d given,

$X_a = 0.465\ \Omega/\text{mi}$ & $X_d = 0.3635\ \Omega/\text{mi}$

$$X_L = X_a + X_d = 0.8285\ \Omega/\text{mi}$$

3.11 Inductance of 3-phase lines with equal spacing



Assumptions:

① Balanced system: $I_a + I_b + I_c = 0$

② No neutral wire

From ● 1 ---- ● P, 2, 3, 4

$\psi_1 = 2\times10^{-7} \times \left(I_1 \ln\frac{1}{r_1'} + I_2 \ln\frac{1}{D_{12}} + \dots + I_n \ln\frac{1}{D_{1n}}\right)$

For Conductor a:

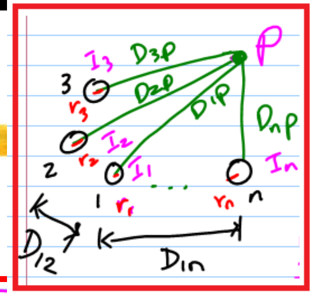
$$\psi_a = 2\times10^{-7} \times \left(I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right) \text{ wbt/m}$$

from $I_a = -(I_b + I_c)$

$$= 2\times10^{-7}\left(I_a \ln\frac{1}{D_s} - I_a \ln\frac{1}{D}\right)$$

$$= 2\times10^{-7}\, I_a \frac{D}{D_s}$$

⌘ **Recall**



Therefore,

$$\psi_1 = 2\times10^{-7}\left[I_1 \ln\frac{1}{r_1'} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + \dots + I_n \ln\frac{1}{D_{1n}}\right]$$

with the assumption that $\sum_{k=1}^{n} I_k = 0 \rightarrow L_1 = \frac{\psi_1}{I_1}$

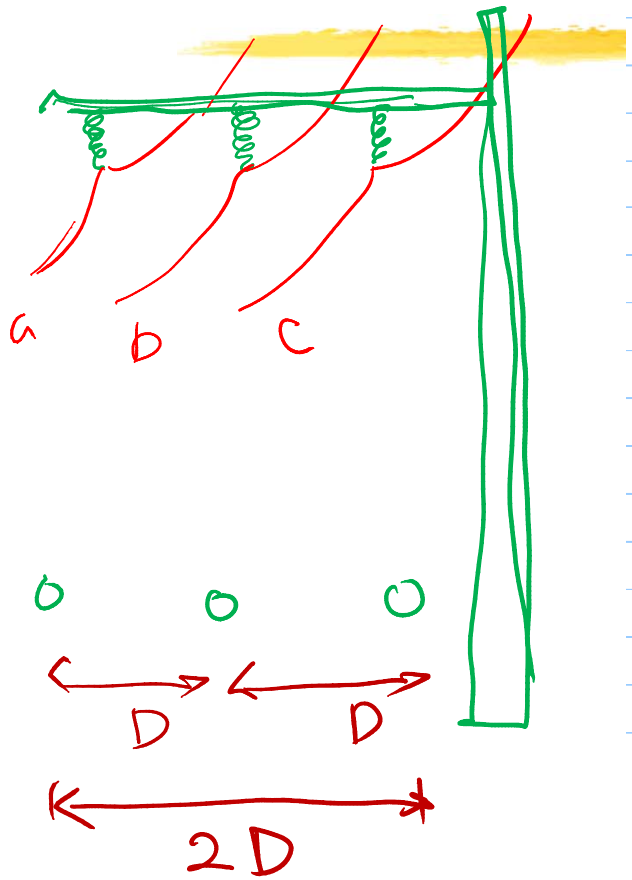
$$\therefore L_a = \frac{\psi_a}{I_a} = \boxed{2\times10^{-7} \ln\frac{D}{D_s}} \ [H/m]$$

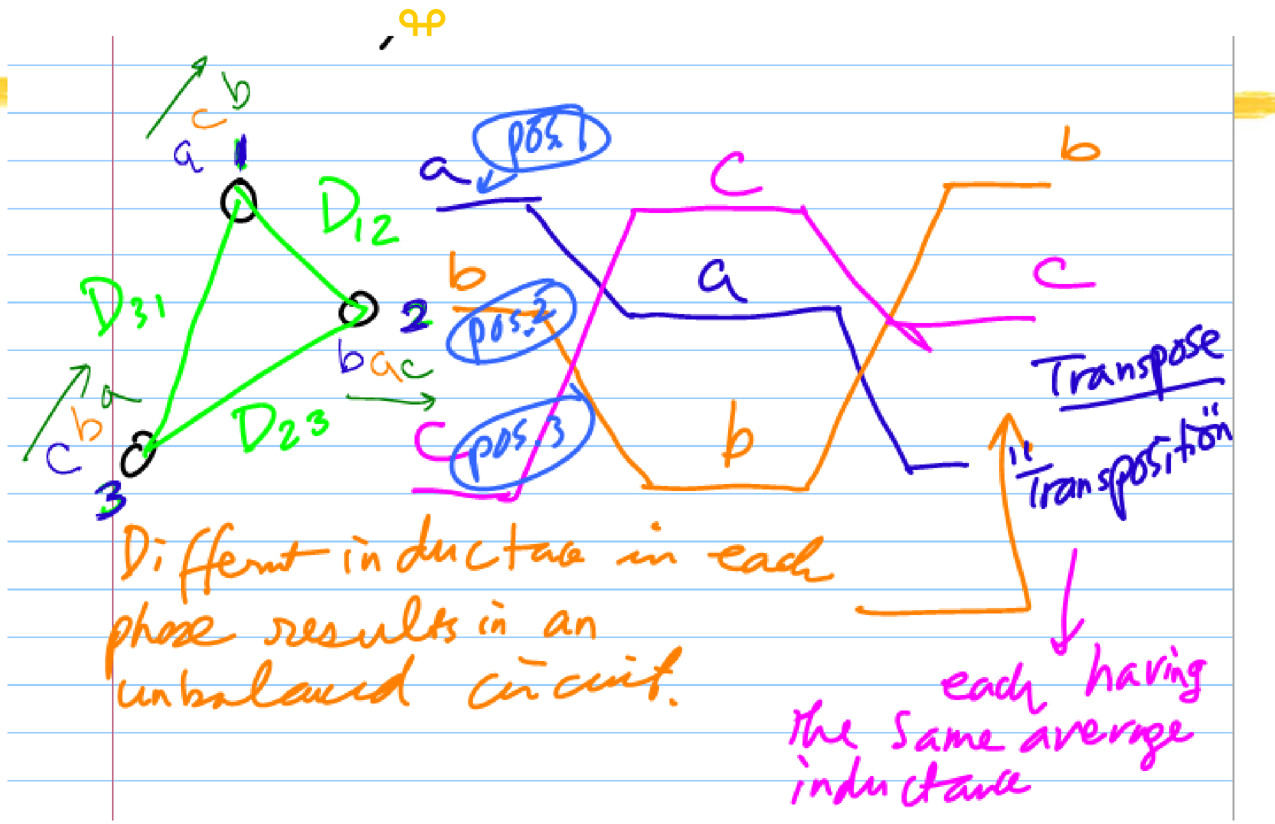
"Between"

"self"

" per-phase inductance of a 3-phase circuit "

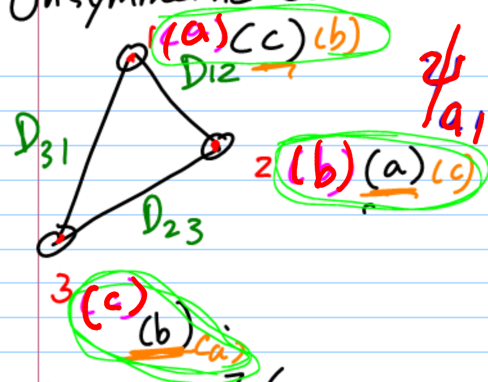
3.12 Inductance of 3-phase lines with unsymmetrical spacing





3.12 Inductance of 3-phase lines with unsymmetrical spacing

Unsymmetric conductor case ⇒ Transposed



$$\psi_{a1} = 2\times10^{-7}\left(I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right)$$

$$\psi_{a2} = 2\times10^{-7}\left(I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right)$$

$$\psi_{a3} = 2\times10^{-7}\left(I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right)$$

wbt/m

Average value for a

$$\psi_a = \frac{\psi_{a1} + \psi_{a2} + \psi_{a3}}{3}$$

$$= \frac{2\times10^{-7}}{3}\left(3 I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + I_c \ln\frac{1}{D_{12}D_{23}D_{31}}\right)$$

From $I_a = -(I_b + I_c)$

$$= \frac{2\times10^{-7}}{3}\left(3 I_a \ln\frac{1}{D_s} - I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right)$$

$$= 2\times10^{-7}\left(I_a \ln\frac{1}{D_s} - \frac{1}{3}\cdot I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right)$$

$$= 2\times10^{-7}\cdot I_a \cdot \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_s} \quad \text{wbt/m}$$

$$= 2\times10^{-7}\cdot I_a \cdot \ln\frac{D_{eq}}{D_s} \qquad D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

"Between"

$$L_a = 2\times10^{-7}\cdot \ln\frac{D_{eq}}{D_s}$$

GMR (self)

3.12 Inductance of 3-phase lines with unsymmetrical spacing

EXAMPLE

Example 3.4 A single-circuit three-phase line operated at 60 Hz is arranged as shown in Fig. 3.12. The conductors are ACSR *Drake*. Find the inductive reactance per mile per phase.

$D_s = 0.0373$ ft

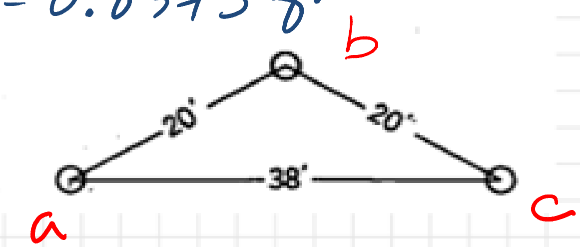


Figure 3.12 Arrangement of conductors for Example 3.4.

transposed.

3.12 Inductance of 3-phase lines with unsymmetrical spacing

E X A M P L E

Example 3.4 A single-circuit three-phase line operated at 60 Hz is arranged as shown in Fig. 3.12. The conductors are ACSR *Drake*. Find the inductive reactance per mile per phase.

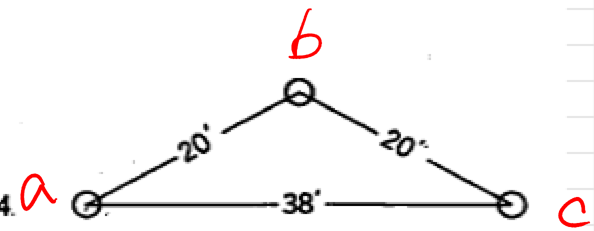


Figure 3.12 Arrangement of conductors for Example 3.4.

Fram Table A: ACSR Drake

$Ds := 0.0373$ ft (m)

$Deq := \sqrt[3]{20 \cdot 20 \cdot 38} = 24.7712$ ft (m)

$L := 2 \cdot 10^{-7} \cdot \ln\left(\frac{Deq}{Ds}\right) = 1.2997 \cdot 10^{-6}$ H/m

$$\frac{\Omega}{m} \Rightarrow \frac{\Omega}{\frac{1}{1609}\,mi} \qquad \frac{\Omega \cdot 1609}{mi}$$

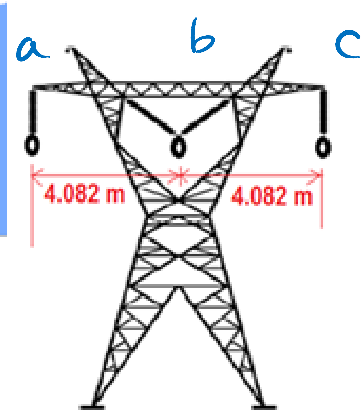
$XL := \underbrace{(2 \cdot \pi \cdot 60 \cdot L)}_{\omega L} \cdot \frac{1609}{1} = 0.7884$ Ω / mile

1 mile = 1609 meter

Class Activity – 3Phase TLZ

As illustrated in a transmission system map below, the transmission line between Pickering NGS and Cherrywood TS is 100 km long with 230 kV. The structure of the transmission line and the data for conductor data are also shown the the figure below.

3 phase line



Phase Conductor Data 795 MCM SCSR

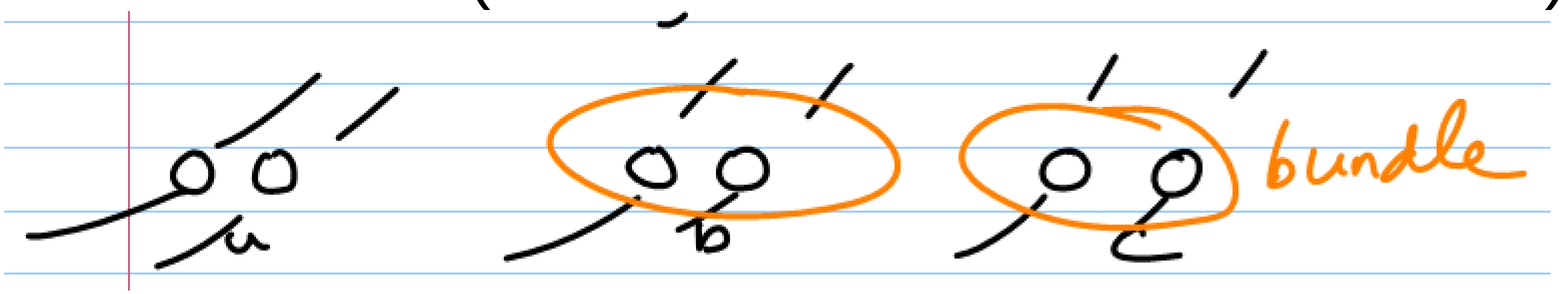
r: radius = 1.350 cm
r' = GMR = 1.073 cm
R = Resistance = 0.08 Ω/km

Line completely transposed

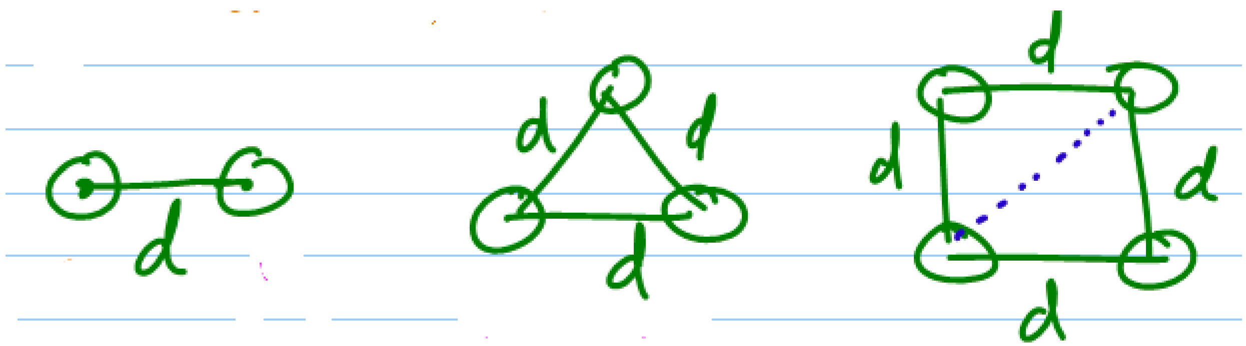
(Q) Determine the line impedance (namely, $Z = R + jX$) of the transmission line per phase between the Pickering NGS and Cherrywood TS.

3.13 Bundled Conductors

- 2 or more conductors per phase
- Increases D_s (denominator part)
- Reduces corona (and thus communication interference)



- Usual Bundling Practice



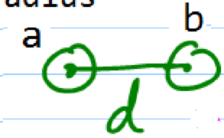
3.13 Bundled Conductors (GMR --- “SELF” or (per-phase))

GMR

Ds: Self - its own radius

"Bundle"

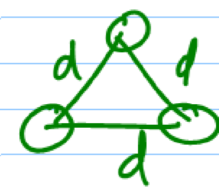
$$D_s^b = \sqrt[4]{D_s \cdot d \cdot D_s \cdot d} = \sqrt{D_s \cdot d}$$



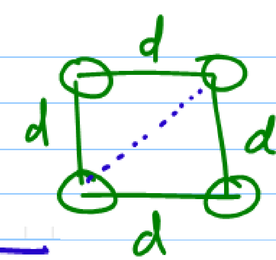
Ds d d Ds

(aa, ab, ba, bb)

$$D_s^b = \sqrt[9]{(D_s \cdot d \cdot d)^3} = \sqrt[3]{D_s \cdot d^2}$$



$$D_s^b = \sqrt[16]{(D_s \cdot d \cdot d \cdot \sqrt{2}\, d)^4} = \sqrt[4]{D_s \cdot d^3 \cdot \sqrt{2}}$$



$$\sqrt[4]{\sqrt{2}} = 1.0905077$$

$$\sqrt[8]{2} = 1.0905077$$

$$= 1.09 \sqrt[4]{D_s \cdot d^3}$$

Example

Example 3.5 Each conductor of the bundled-conductor line shown in Fig. 3.14 is ACSR, 1,272,000-cmil *Pheasant*. Find the inductive reactance in ohms per km (and per mile) per phase for $d = 45$ cm. Also find the per-unit series reactance of the line if its length is 160 km and base is 100 MVA, 345 kV.

$D_s = 0.0466$ ft

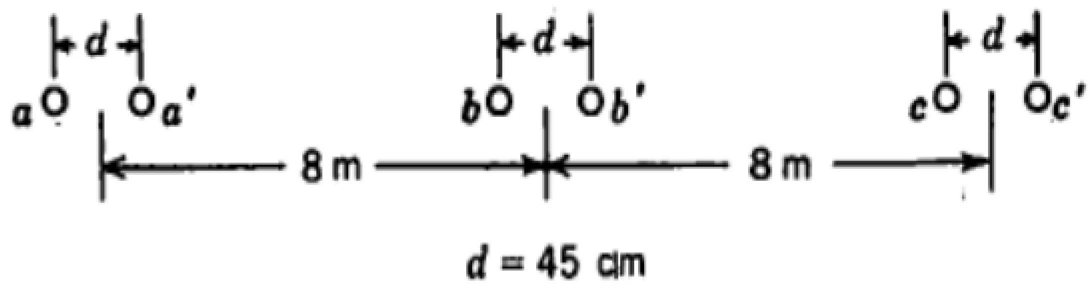
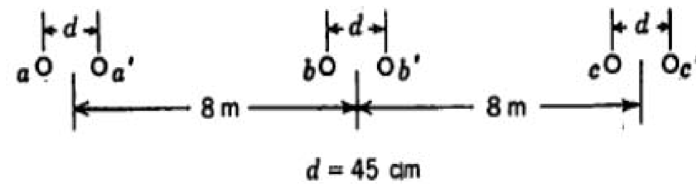


Figure 3.14 Spacing of conductors of a bundled-conductor line.

Example

Example 3.5 Each conductor of the bundled-conductor line shown in Fig. 3.14 is ACSR, 1,272,000-cmil *Pheasant*. Find the inductive reactance in ohms per km (and per mile) per phase for $d = 45$ cm. Also find the per-unit series reactance of the line if its length is 160 km and base is 100 MVA, 345 kV.

Figure 3.14 Spacing of conductors of a bundled-conductor line.



Given from Table for Pheasant conductor:

$Ds := 0.0466$ ft

$Ds := Ds \cdot 0.3048 = 0.0142$ meter

$d := 0.45$ meter

$Dsb := \sqrt{Ds \cdot d} = 0.0799$ meter

$Sbase := 100 \cdot 10^{6}$

$Vbase := 345 \cdot 10^{3}$

$Dm := 8$

$Zbase := \dfrac{Vbase^{2}}{Sbase} = 1190.25$

$Deq := \sqrt[3]{Dm \cdot Dm \cdot (2 \cdot Dm)} = 10.0794$

$$XL := \underbrace{2 \cdot \pi \cdot 60}_{\omega} \cdot \underbrace{2 \cdot 10^{-7} \cdot \ln\left(\frac{Deq}{Dsb}\right)}_{L} \cdot 10^{3} = 0.3647 \ \Omega \ / \ \text{km}$$

m → km

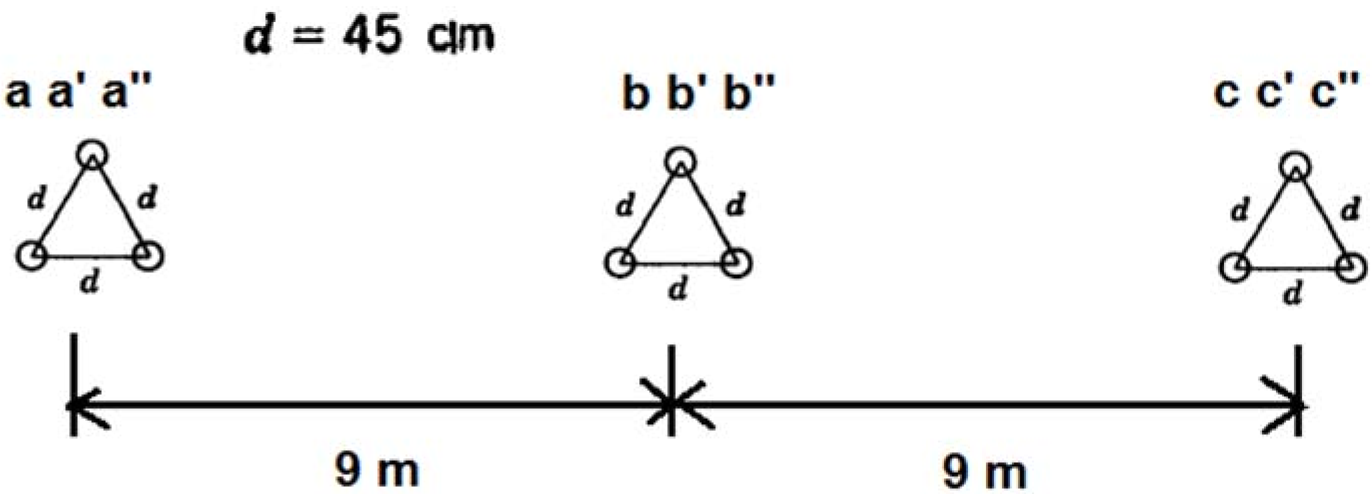
$line := 160$ km

$X := XL \cdot 160 = 58.3506$ Ω

$Xpu := \dfrac{X}{Zbase} = 0.049$ pu

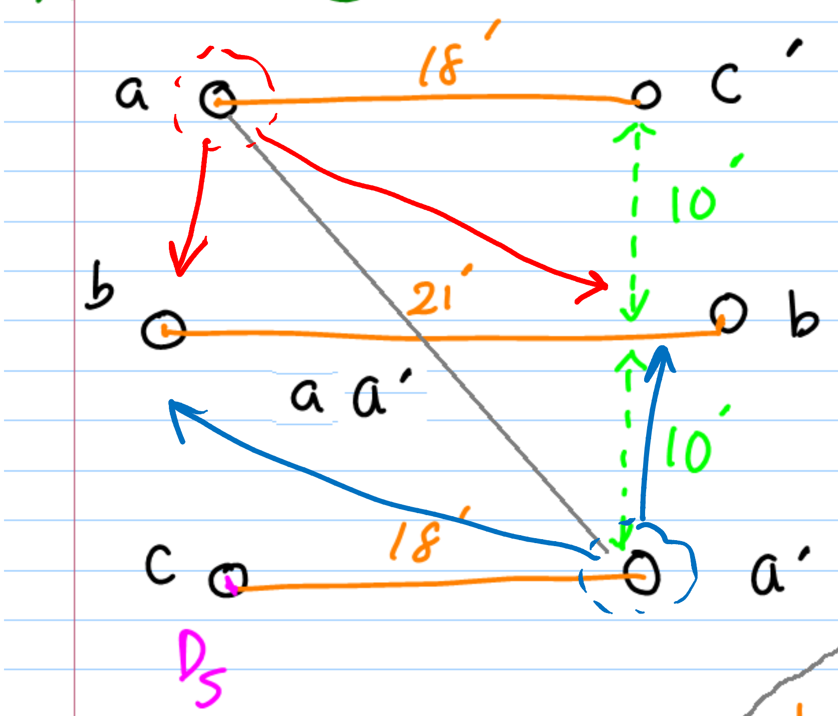
Class Activity – Bundled Conductor

As illustrated below, a bundled 3-phase transmission line has 3 ACSR *Rail* conductors per bundle with 45 cm between conductors pf the bundle.
The spacing between bundle centers is 9, 9, and 18 m.
Calculate the inductive reactance per km in 60 Hz.
D_s of the Rail conductor is known to be 0.0386 ft.



3.14 Parallel Circuit 3-Phase Lines

Typical arrangement



ab, bc, ca
ab', ac', bc'
Between phases

$$\frac{GMD}{GMR}$$

(a-b) : ab, ab', a'b, a'b'

(b-c) : bc, bc', b'c, b'c'

(c-a) : ca, c'a, ca', c'a'

$$D_{eq} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}}$$

a-a' phase a

b-b' phase b

c-c' phase c

$$\sqrt{D_{aa} \, d_{aa'} \cdot D_{a'a'} \cdot d_{a'a}}$$

$$\sqrt{D}$$

Bundle

3.14 Parallel Circuit 3-Phase Lines

Ex 3.6 $D_s = 0.0229$ ft (given)

D_{ab}^{P} ← "Parallel"

$D_{ab}^{P} \leftarrow (\underbrace{D_{ab}, D_{ab'}}, \underbrace{D_{a'b}, D_{a'b'}})$

$D_{bc}^{P} \leftarrow (\underbrace{D_{bc}, D_{bc'}}, \underbrace{D_{b'c}, D_{b'c'}})$

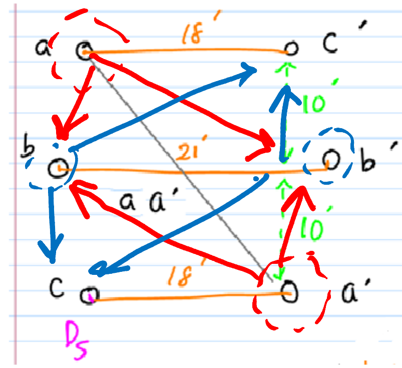
$D_{ca}^{P} \leftarrow (D_{ca}, D_{ca'}, D_{c'a}, D_{c'a'})$

$$D_{ab}^{P} = D_{bc}^{P} = \sqrt[4]{D_{ab} \cdot D_{ab'} \cdot D_{a'b} \cdot D_{a'b'}}$$

$$D_{ca}^{P} = \sqrt[4]{D_{ca} \cdot D_{ca'} \cdot D_{c'a} \cdot D_{c'a'}}$$

$$\rightarrow D_{eq} = \sqrt[3]{D_{ab}^{P} \cdot D_{bc}^{P} \cdot D_{ca}^{P}}$$

<-- **GMD**

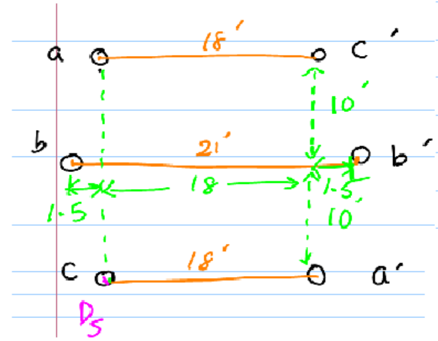


$D_{ab} = \sqrt{10^2 + 1.5^2}$

$D_{ab'} = \sqrt{10^2 + 19.5^2}$

$D_{a'b} = D_{ab'}$

$D_{a'b'} = D_{ab}$



$D_{ca} = 20$

$D_{ca'} = 18$

$D_{c'a} = 18$

$D_{c'a'} = 20$

3.14 Parallel Circuit 3-Phase Lines

For GMR

$d_{a-a'} = \sqrt{20^2+18^2}$

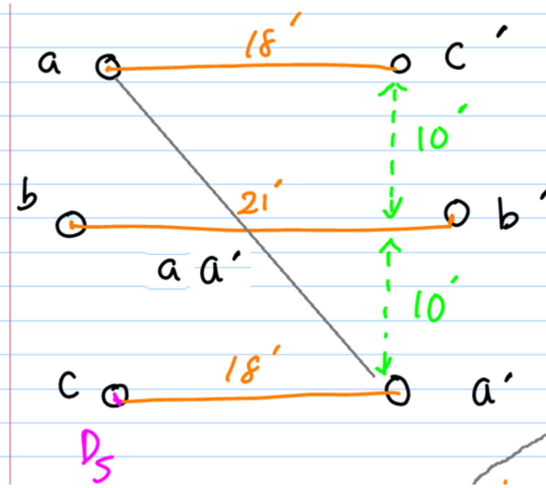
$a-a'$: $\sqrt{D_s \cdot d_{a-a'}}$ $(= D_{sa})$

$d_{b-b'} = 21$

$b-b'$: $\sqrt{D_s \cdot d_{b-b'}}$ $(= D_{sb})$

$c-c'$: $\sqrt{D_s \cdot d_{c-c'}}$ $(= D_{sc})$ $d_{c-c'} = d_{a-a'}$

$\rightarrow D_s^P = \sqrt[3]{D_{sa} \cdot D_{sb} \cdot D_{sc}}$



$\rightarrow L = 2\times10^{-7} \ln \frac{D_{eq}}{D_s^P}$ [H/m]

$\rightarrow X_L = 2\pi\, 60 \cdot L \cdot 1609$ [Ω/mile]

Example 3.6

Example 3.6 A three-phase double-circuit line is composed of 300,000-cmil 26/7 ACSR *Ostrich* conductors arranged as shown in Fig. 3.15. Find the 60-Hz inductive reactance in ohms per mile per phase.

From Table A.1 for *Ostrich* $D_s = 0.0229$ ft

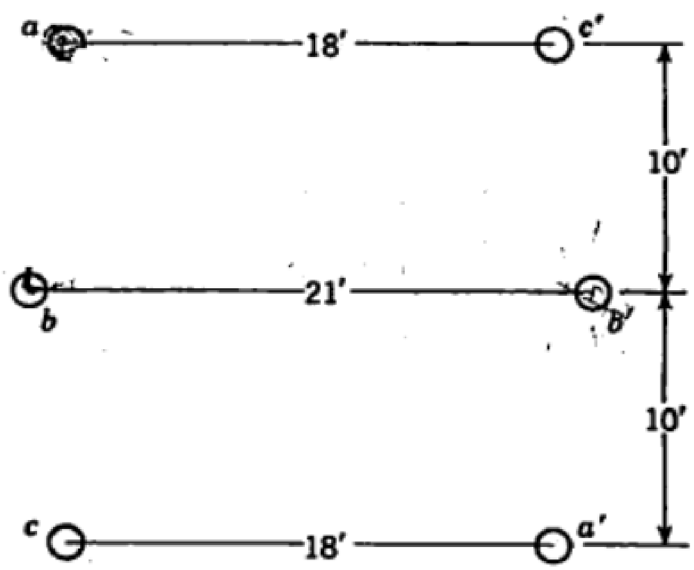


Figure 3.15 Typical arrangement of conductors of a parallel-circuit three-phase line.

Example 3.6

⌘GMD

From Table A.1 for *Ostrich* $D_s = 0.0229$ ft

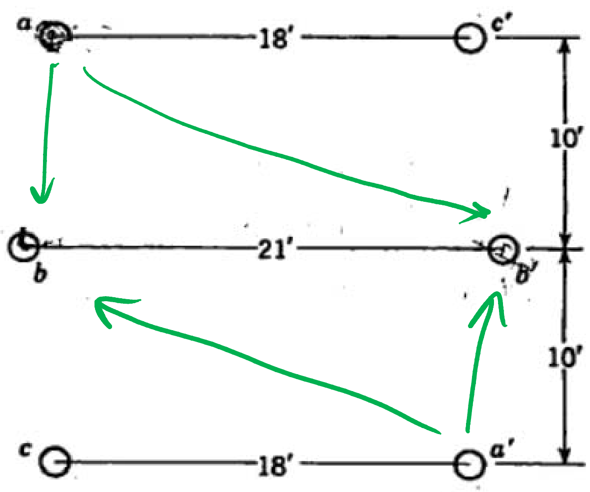


Figure 3.15 Typical arrangement of conductors of a parallel-circuit three-phase line.

For GMD --- distance between phases

$$dab := \sqrt{10^2 + 1.5^2} = 10.1119$$

$$dab' := \sqrt{10^2 + 19.5^2} = 21.9146$$

$$da'b := dab'$$

$$da'b' := dab$$

$$dca := 20 \qquad dca' := 18$$

$$dc'a := 18$$

$$dc'a' := dca$$

$$Dabp := \sqrt[4]{dab \cdot dab' \cdot da'b \cdot da'b'} = 14.8862 \quad \text{ft}$$

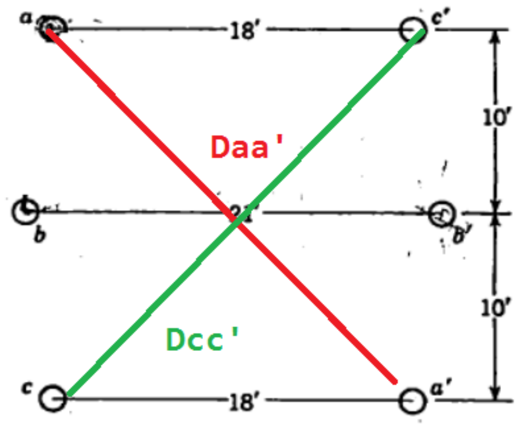
$$Dbcp := Dabp$$

$$Dcap := \sqrt[4]{dca \cdot dca' \cdot dc'a \cdot dc'a'} = 18.9737 \quad \text{ft}$$

Then The equivalent distance (which is GMD) between phases is:

$$Deq := \sqrt[3]{Dabp \cdot Dbcp \cdot Dcap} = 16.1401$$

Example 3.6



- ⌘GMR
- ⌘L
- ⌘XL

Now let's work on GMR --- for each phase with $Ds := 0.0229$ ft

Phase a conductors are composed of concutor a and a' bundled

$$daa' := \sqrt{18^2 + 20^2} = 26.9072 \qquad dbb' := 21 \qquad dcc' := daa'$$

$$Daa' := \sqrt{Ds \cdot daa'} = 0.785 \qquad Dbb' := \sqrt{Ds \cdot dbb'} = 0.6935 \qquad Dcc' := \sqrt{Ds \cdot dcc'} = 0.785$$

$$Dsp := \sqrt[3]{Daa' \cdot Dbb' \cdot Dcc'} = 0.7532$$

Finally

$$L := 2 \cdot 10^{-7} \cdot \ln\left(\frac{Deq}{Dsp}\right) = 6.1295 \cdot 10^{-7}$$ H / m per phase

$$XL := 2 \cdot \pi \cdot 60 \cdot L \cdot 1609 = 0.3718$$ Ω /mile per phase

ω

L

meter → mile