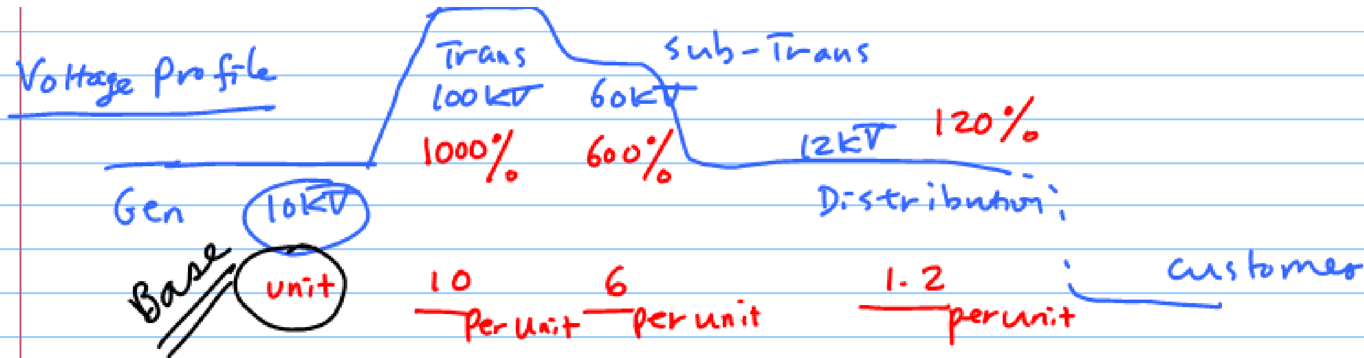
Per-unit (2.10 – 2.11)



kV

kVA

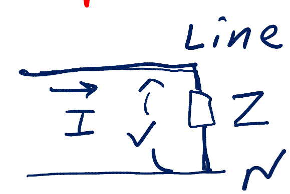
V, I, VA, Z	Inter-related → Base Value	Knowing any 2 would decide the other 2.

$VA = V \cdot I$
$V = I \cdot Z$
$I = \frac{V}{Z}$

- $I_{base}\ [A] = \frac{S_{Base}\ kVA_{1\phi}}{V_{Base}\ kV_{LN}}$ ($I = \frac{VA}{V}$)
- $Z_{base}\ [\Omega] = \frac{V_{Base},\ V_{LN}}{I_{base},\ A}$ ($Z = \frac{V}{I}$)

$VA = |S|$
$kVA = k|S|$

Apparent power



Per-unit system

- $Z_{base}\,[\Omega] = \dfrac{V\ Base,\ V_{LN}}{I_{base},\ A}$

$Z = \dfrac{V}{I}$

$Z_{pu} = \dfrac{Z_{actual}\ \ \Omega}{Z\,Base\ \ \Omega}$

- $Z_{base} = \dfrac{(V\ Base\ \ kV_{LN})^2 \times 10^3}{S\,Base\ kVA_{1\phi}}$

$Z = \dfrac{V}{I} = \dfrac{V}{VA/V} = \dfrac{V^2}{VA}$

$Z_{base} = \dfrac{(V\,Base\ \ kV_{LN})^2}{S\,Base\ MVA_{1\phi}}$

$P_{base}\ \ kW_{1\phi} = S_{base}\ kVA_{1\phi}$

$P_{base}\ \ MW_{1\phi} = S_{base}\ MVA_{1\phi}$

- Usual Base Quantity in Power System Analysis
  - ① S Base MVA
  - ② V Base kV_{LN}

Per-unit system

3ϕ case:

If $S_{Base}\ kVA_{3\phi} = 30{,}000\ kVA$

& $V_{Base}\ kV_{LL} = 120\ kV$,

Then →

$$S_{Base}\ kVA_{1\phi} = \frac{S_{Base}\ kVA_{3\phi}}{3} = 10{,}000\ kVA$$

$$\&\ V_{Base}\ kV_{LN} = \frac{120\ kV}{\sqrt{3}} = 69.2\ kV$$

If an actual line-to-line voltage is

$V_{LL} = 108\ kV$, → $V_{LN} = \frac{108}{\sqrt{3}} = 62.3\ kV$

→ $\text{per-unit Voltage} = \frac{62.3\ kV}{69.2\ kV} = 0.9$

If total 3ϕ power is 18,000 kW

(per phase power is 6000 kW)

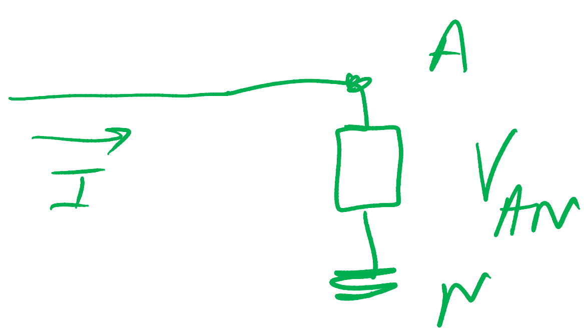
→ $\text{per unit Power} = \frac{18{,}000}{30{,}000} = \frac{6{,}000}{10{,}000} = 0.6$

Per-unit system

$$I_{base}\ A = \frac{S_{Base}\ kVA_{1\phi}}{V_{Base\ Voltage}\ kV_{LN}}$$

$$= \frac{S_{Base}\ kVA_{3\phi}/3}{V_{Base\ Voltage}\ kV_{LL}/\sqrt{3}}$$

$$= \frac{S_{Base}\ kVA_{3\phi}}{\sqrt{3}\cdot V_{Base\ Voltage}\ kV_{LL}}$$



$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

$$Z_{base} = \frac{(V_{base}\ kV_{LN})^2 \cdot 10^3}{S_{Base}\ kVA_{1\phi}}$$

$$= \frac{(V_{base}\ kV_{LL}/\sqrt{3})^2 \cdot 10^3}{S_{Base}\ kVA_{3\phi}/3}$$

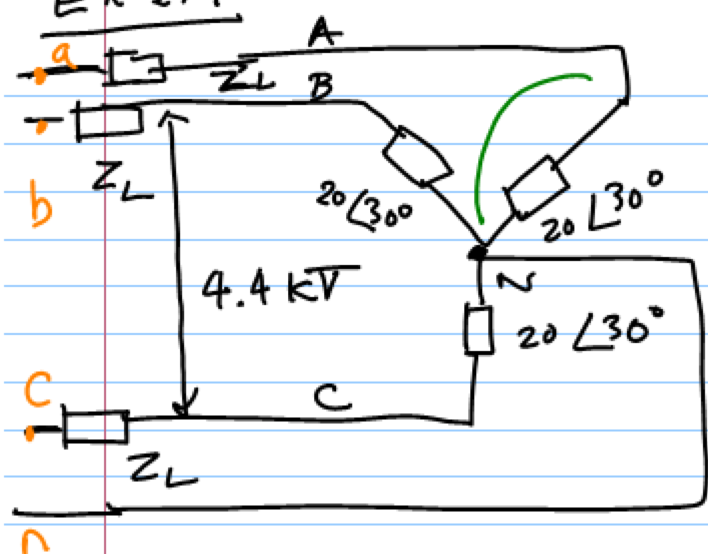
$$= \frac{(V_{base}\ kV_{LL})^2 \cdot 10^3}{S_{Base}\ kVA_{3\phi}}$$

$$= \frac{(V_{base}\ kV_{LL})^2 \cdot 10^3}{S_{base}\ MVA_{3\phi}/10^3}$$

$$= \frac{(V_{base}\ kV_{LL})^2}{S_{Base}\ MVA_{3\phi}}$$

Per-unit system

EX 2.4



$Z_L = 1.4\angle 75^\circ$

Line Impedance Between the Substation and the Load

Q: Find the Line-to-Line Voltage at the Substation by using per-unit (pu) on a base of 4.4 kV and 127 A.

$V_{Base}\,kV_{LL}$ $I_{Base}\,A$

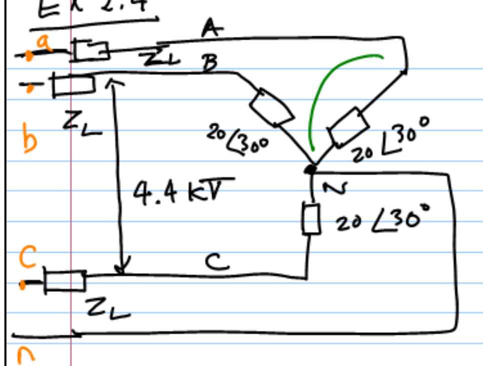
$V_{base}\,kV_{LL} \rightarrow V_{base}\,kV_{LN}$

Note

V_{Base} given as line-to-line voltage.

Per-unit system

EX 2.4



$Z_L = 1.4\angle 75°$ Line Impedance Between the Substation and the Load

Q: Find the Line-to-Line Voltage at the Substation by using per-unit (pu) on a base of 4.4 kV and 127 A.

(V_{Base} kV_{LL}) (I_{Base} A)

$V_{AN} = \frac{4.4K}{\sqrt{3}}$

$V_{AN\,pu} = 1$

$V_{LN\,pu} = \frac{4.4}{\sqrt{3}}$

$$I_{pu} = \frac{V_{LN\,pu}}{Z_{pu}} = \frac{1}{1\angle 30°}$$

(Sol) We need to convert all into pu value.

$$Z_{Base} = \frac{V_{Base}\ V_{LN}}{I_{Base}\ A} = \frac{V_{Base}\ V_{LL}/\sqrt{3}}{I_{Base}\ A}$$

$$= \frac{4400/\sqrt{3}}{127} = 20\ \Omega$$

$$Z_{L\,pu} = \frac{1.4\angle 75°}{20} = 0.07\angle 75°\ \text{pu}$$

$$Z_{load}\ pu = \frac{20\angle 30}{20} = 1\angle 30°\ \text{pu}$$

$$I_{thru\ Load} = \frac{V_{LN}}{Z_{Load}}, \rightarrow I_{pu} = \frac{V_{LN} \cdot pu}{Z_{pu}}$$

$$I_{Base} = \frac{V_{LN\,Base}}{Z_{Base}} = \frac{4400/\sqrt{3}}{20} = \frac{2540}{20} = 127$$

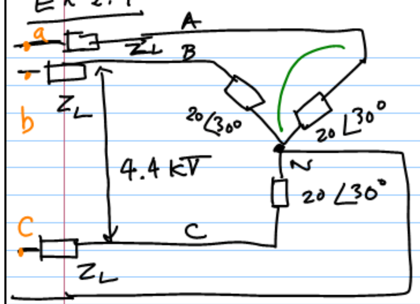
$$I_{pu} = \frac{1.0}{1\angle 30°} = 1\angle -30°\ \text{pu}$$

$V = 1.0$ pu

Z_{pu}

Per-unit system

EX 2.4

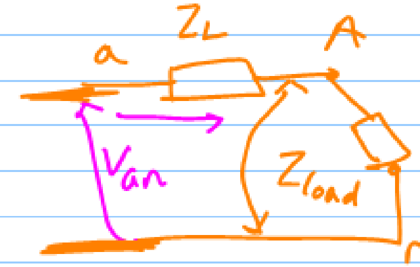


$Z_L = 1.4\angle 75°$ Line Impedance Between the Substation and the Load

Q Find the Line-to-Line Voltage at the Substation by using per-unit (pu) on a base of 4.4 kV and 127 A.

$V_{Base\,kV_{LL}}$ $I_{Base\,A}$

$$\overline{V}_{Substation} = \overline{I} \cdot Z_L + \overline{V}_{AN}$$



$$\overline{V}_{an\,pu} = \overline{V}_{AN\,pu} + \overline{I}_{pu} \cdot Z_{L\,pu}$$

$$= 1.0\angle 0° + (1.0\angle -30°)(0.07\angle 75°)$$

$$= 1.0\angle 0° + 0.07\angle 45°$$

$$= 1.051\angle 2.70°\ pu$$

Actual Voltage?

$$V_{an} = V_{an\,pu} \cdot V_{Base\,LN}$$

$$= 1.051 \cdot \left(\frac{4400}{\sqrt{3}}\right) = 2.67\ kV$$

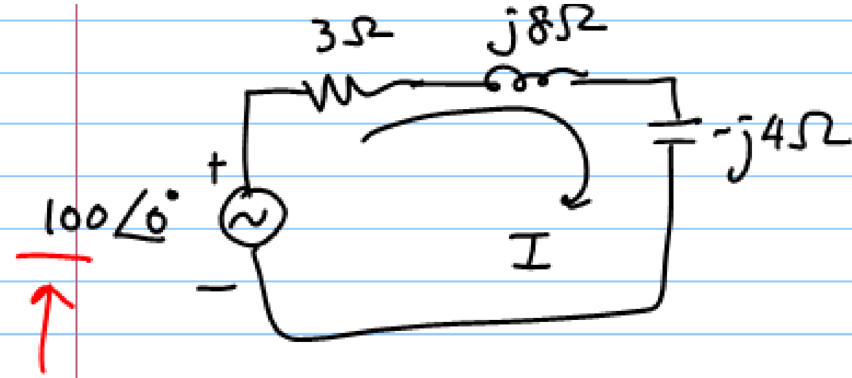
$$V_{LL} = V_{an\,pu} \cdot V_{Base\,LL}$$

$$= 1.051 \cdot (4400) = 4.62\ kV$$

$$\left(\frac{4400}{\sqrt{3}}\right)\sqrt{3}$$

LN → LL

Example



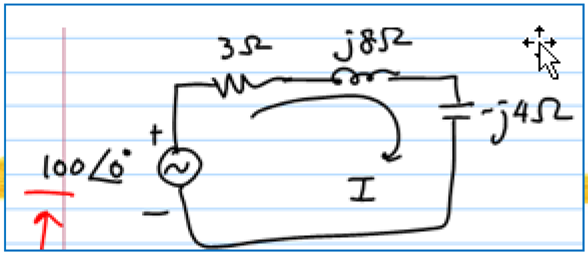
① Draw circuit Diagram in pu with $V_{base} = 100$ $S_{base} = 500$ VA

② Compute Series Impedance in pu

③ Compute the Current I in pu

④ Calculate the Complex Power (in pu) delivered to each of the 4 elements in the circuit

Example - Solution



Solution From $V_{base} = 100\ V$, $S_{base} = 500\ VA$

$\rightarrow I_{base} = \frac{S_{base}}{V_{base}} = \frac{500}{100} = 5\ A$

$$Z_{base} = \begin{cases} \frac{V_{base}}{I_{base}} = \frac{100}{5} = 20\ \Omega \\ \frac{V_{base}^2}{S_{base}} = \frac{(100)^2}{500} = 20\ \Omega \end{cases}$$

① ckt Diagram

Actual	Base	pu
$100\angle 0°$	100	$\frac{100\angle 0°}{100} \rightarrow 1\angle 0°$ pu
$3\ \Omega$	$20\ \Omega$	$\frac{3}{20} \rightarrow 0.15$ pu
$j8\ \Omega$	$20\ \Omega$	$\frac{j8}{20} \rightarrow j0.4$ pu
$-j4\ \Omega$	$20\ \Omega$	$\frac{-j4}{20} \rightarrow -j0.2$ pu

Example - Solution

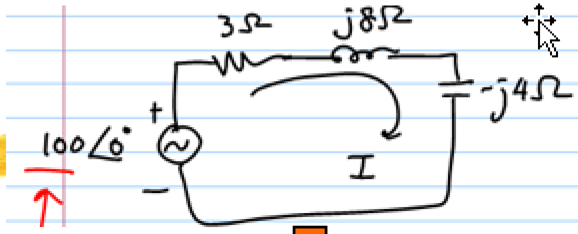
Solution: From $V_{base} = 100\ V$, $S_{base} = 500\ VA$

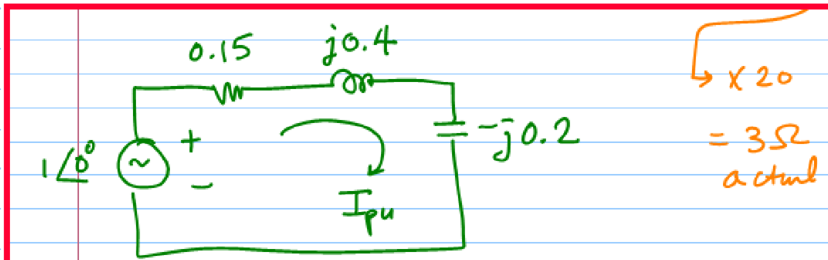
$\rightarrow I_{base} = \frac{S_{base}}{V_{base}} = \frac{500}{100} = 5\ A$

$$Z_{base} = \begin{cases} \frac{V_{base}}{I_{base}} = \frac{100}{5} = 20\ \Omega \\ \frac{V_{base}^2}{S_{base}} = \frac{(100)^2}{500} = 20\ \Omega \end{cases}$$

① ckt Diagram

Actual	Base	pu
$100\angle 0^\circ$	100	$\frac{100\angle 0^\circ}{100} \rightarrow 1\angle 0^\circ$ pu
$3\ \Omega$	$20\ \Omega$	$\frac{3}{20} \rightarrow 0.15$ pu
$j8\ \Omega$	$20\ \Omega$	$\frac{j8}{20} \rightarrow j0.4$ pu
$-j4\ \Omega$	$20\ \Omega$	$\frac{-j4}{20} \rightarrow -j0.2$ pu





② Total Series impedance in pu

$Z_{pu} = 0.15 + j(0.4 - 0.2) = 0.15 + j0.2$

$= 0.25\angle 53.1^\circ$

Example - Solution

③ Current $\bar{I}_{pu}$

$$\bar{I}_{pu} = \frac{\bar{V}_{pu}}{Z_{pu}} = \frac{1.0\angle 0^\circ}{0.25\angle 53.1^\circ} = 4.0\angle -53.1^\circ$$

↓ ×5 = 20 [A] Actual

④ Complex Power in pu

$$\bar{S}_{R_{pu}} = I_{pu}^2 R_{pu} = 4^2 \cdot (0.15) = 2.4 + j0$$

$$\bar{S}_{L_{pu}} = I_{pu}^2 (jX_L) = 4^2 \cdot (j0.4) = 0 + j6.4$$

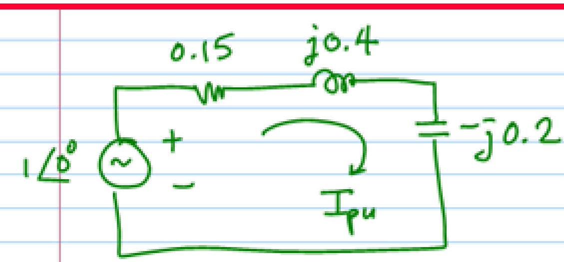
$$\bar{S}_{C_{pu}} = I_{pu}^2 (-jX_C) = 4^2 \cdot (-j0.2) = 0 - j3.2$$

$$\bar{S}_{pu} = \bar{S}_{R_{pu}} + \bar{S}_{L_{pu}} + \bar{S}_{C_{pu}}$$

$$= 2.4 + j6.4 - j3.2$$

$$= 2.4 + j3.2$$

$$= 4.0\angle 53.1^\circ$$ → ×500 → 2000 VA (actual)



↳ ×20 = 3Ω actual

② Total Series impedance in pu

$$Z_{pu} = 0.15 + j(0.4 - 0.2) = 0.15 + j0.2$$

$$= 0.25\angle 53.1^\circ$$

Example

A 2400-V 3ϕ Source Supplies 2 parallel loads:

load 1 : 300 KVA pf = 0.8 lagging

load 2 : 240 KVA pf = 0.6 leading

If $V_{an} = 1386\angle 0^\circ$ V, $S_{base\,3\phi} = 300$ KVA

and $V_{base\,LL} = 2.4$ KV

1. Draw a 1ϕ equivalent circuit
2. Determine all 3 Source Line currents
3. Find System bases for 1ϕ equivalent circuit
4. Draw 1ϕ equivalent circuit in pu
5. Determine Source Line current in pu

Sol

1ϕ; phase a

① $\bar{S}_{load\,1\,3\phi} = 300\angle\cos^{-1}0.8 \rightarrow S_{a_1} = \frac{1}{3} S_{load,3\phi}$

$= 100\angle\cos^{-1}0.8$ kVA

$\bar{S}_{load2\,3\phi} = 240\angle\cos^{-1}0.6$

$\rightarrow S_{a_2} = 80\angle-\cos^{-1}0.6$ kVA

load 1 : 300 kVA pf = 0.8 lagging

load 2 : 240 kVA pf = 0.6 leading

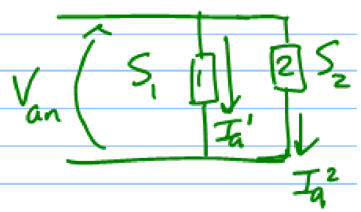
$S_{Base\,3\phi} = 300$ kVA

$V_{base\,LL} = 2.4$ kV

$\rightarrow \begin{cases} \bar{S}_{a_1} = 100\angle 36.9^\circ \\ \bar{S}_{a_2} = 80\angle -53.1^\circ \end{cases}$

From $\bar{S} = \bar{V}\bar{I}^*$

$\bar{I}_{a_1} = \left[\frac{S_{a_1}}{\bar{V}_{an}}\right]^*$



current through load 1

$= \left[\frac{100\angle 36.9}{1.386\angle 0}\right]^* \approx 72.2\angle -36.9$ A

$\bar{I}_{a_2} = \left[\frac{S_{a_2}}{V_{an}}\right]^* = \left(\frac{80\angle -53.1}{1.386\angle 0^\circ}\right)^* = 57.7\angle 53.1^\circ$ A

Load Impedance

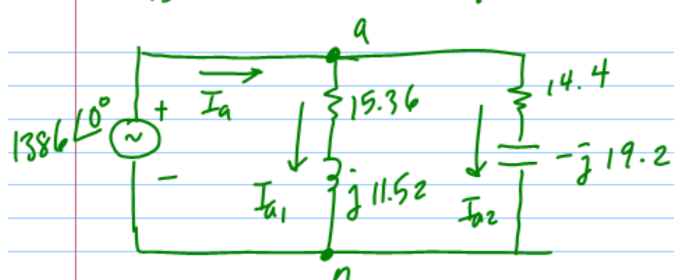
$Z_1 = \frac{\bar{V}_{an}}{\bar{I}_{a_1}} = \frac{1386\angle 0^\circ}{72.2\angle -36.9^\circ} = 19.2\angle 36.9^\circ$

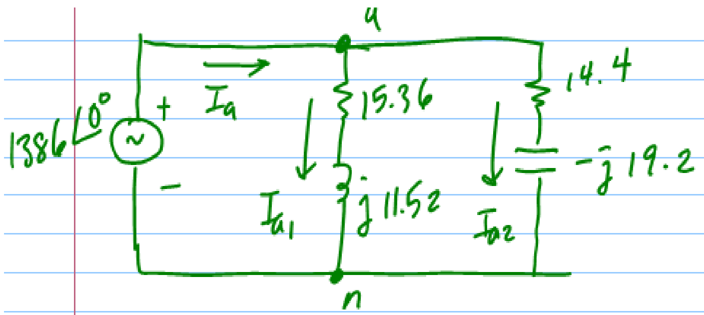
$= 15.36 + j11.52\ \Omega$

$Z_2 = \frac{\bar{V}_{an}}{\bar{I}_{a_2}} = \frac{1386\angle 0^\circ}{57.7\angle 53.1^\circ} = 24\angle -53.1^\circ$

$= 14.4 - j19.2\ \Omega$

Finally, we have 1ϕ equivalent circuit.





② Source Current I_a

$$\bar{I}_a = \bar{I}_{a_1} + \bar{I}_{a_2} = 72.2\angle -36.9° + 57.7\angle 53.1°$$

$$= 57.7 - j43.3 + 34.6 + j46.2$$

$$= 92.3 + j2.9$$

$$= 92.4\angle 1.8°$$

3ϕ Symmetry →

$$\bar{I}_b = 92.4\angle 1.8° - 120° = 92.4\angle -118.2°$$

$$\bar{I}_c = 92.4\angle 1.8° + 120° = 92.4\angle 121.8°$$

③ System bases for 1ϕ circuit

($S_{base\,3\phi} = 300\,kVA$ & $V_{base\,LL} = 2.4\,kV$)

$$S_{base} = \frac{S_{base\,3\phi}}{3} = 100\,kVA$$

$$V_{base\,LN} = \frac{V_{base\,LL}}{\sqrt{3}} = \frac{2.4}{\sqrt{3}} = 1.386\,kV$$

(default)

$$I_{base} = \frac{S_{base}}{V_{base}} = \frac{100}{1.386} = 72.2\,A$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{1386}{72.2} = 19.2\,\Omega$$

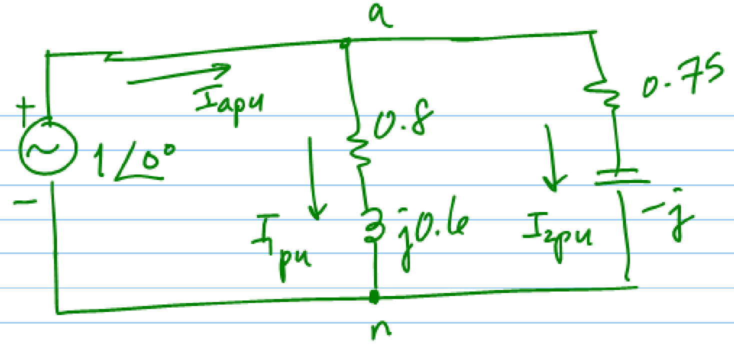
④ 1ϕ equivalent circuit in pu.

$$Z_{a1\,pu} = \frac{19.2\angle 36.9°}{19.2} = 1\angle 36.9° = 0.8 + j0.6$$

$$Z_{a2\,pu} = \frac{24\angle -53.1°}{19.2} = 1.25\angle -53.1° = 0.75 - j1.0$$

$$\bar{V}_{a\,pu} = \frac{1386\angle 0°}{1386} = 1\angle 0°$$

Example



⑤ Source current $\bar{I}_{a\,pu}$

$$\bar{I}_{apu} = \bar{I}_{a1pu} + \bar{I}_{a2pu}$$

$$= \frac{1\angle 0^\circ}{1\angle 36.9^\circ} + \frac{1\angle 0^\circ}{1.25\angle -53.1^\circ}$$

$$= 1\angle -36.9^\circ + 0.8\angle 53.1^\circ$$

$$= 0.8 - j0.6 + 0.48 + j0.64$$

$$= 1.28 + j0.04$$

$$= 1.28\angle 1.8^\circ$$

cf. Actual Value ?

$$\bar{I}_a = \bar{I}_{apu} \cdot I_{base}$$

$$= (1.28\angle 1.8^\circ)(72.2) = 92.4\angle 1.8^\circ$$

Changing the Base of PU Quantities

* When making computations, all impedance in a system must be expressed on the same impedance base !!

→ Necessary to convert pu Z from one base to another.

Remember,

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{\left\{\frac{(kV_{base})^2 \times 10^3}{kVA_{base}}\right\}}$$

$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P}$

$$= \frac{(Z_{actual})(kVA_{base})}{(kV_{base})^2 (10^3)}$$

Alternatively

$Z_{pu_1} \rightarrow Z_{actual} \rightarrow Z_{pu_2}$

(Base 1: V_{base1}, S_{base1}) (Base 2: V_{base2}, S_{base2})

$Z_{pu} \propto kVA_{base}$

$Z_{pu} \propto \frac{1}{(kV_{base})^2}$

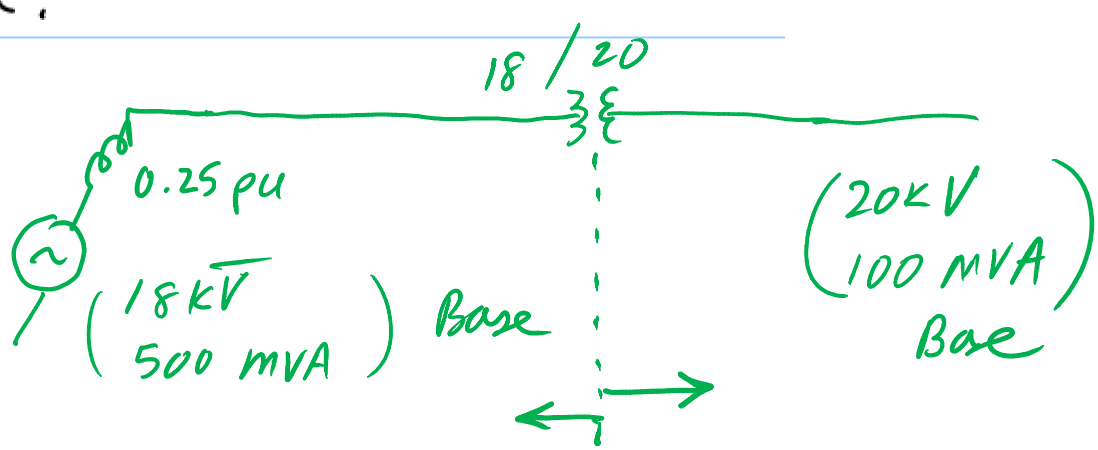
$$\rightarrow Z_{pu_{New}} = Z_{pu_{old}} \left(\frac{kV_{old}}{kV_{New}}\right)^2 \left(\frac{kVA_{New}}{kVA_{old}}\right)$$

EXAMPLE - Changing the Base of PU Quantities

The reactance a generator (X'')

$X''_{pu} = 0.25$ on generator's nameplate rating of 18 kV, 500 MVA.

Now, the base for circuit calculations is 20 kV, 100 MVA. Find X'' on the new base.



EXAMPLE - Changing the Base of PU Quantities

The reactance a generator (X'')

$X''_{pu} = 0.25$ on generator's nameplate rating of 18 kV, 500 MVA.

Now, the base for circuit calculation is 20 kV, 100 MVA. Find X'' on the new base.

Alternatively,

① Find X'' in Ω from the old base.

$$X''_{actual} = X''_{pu} \cdot \left(\frac{18^2}{500}\right) = (0.25)\left(\frac{18^2}{500}\right)$$

$$= 0.162\ (\Omega)$$

← old Base

② Convert the Ω value in to pu in the new Base

$$X''_{pu} = \frac{(0.162)\ \text{actual}}{\left(20^2/100\right)\ \text{new base}} = \frac{(0.162)(100)}{20^2}$$

$$= 0.0405\ \text{pu}$$

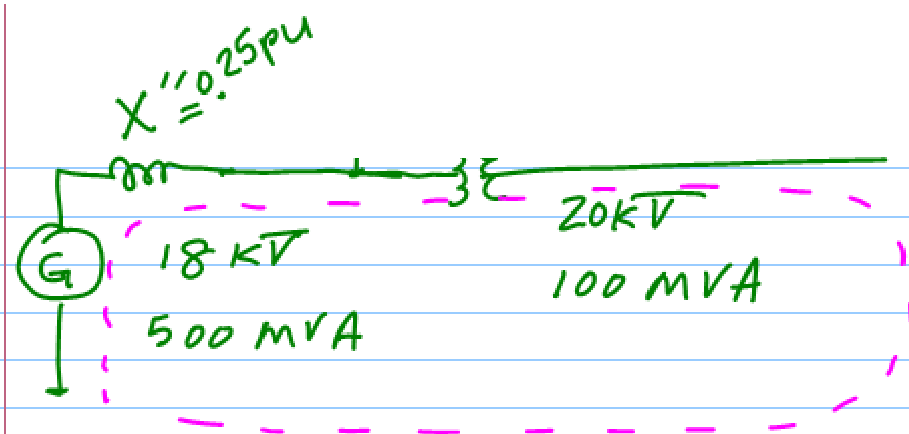
EXAMPLE - Changing the Base of PU Quantities

The reactance a generator (X'')
$X''_{pu} = 0.25$ on generator's nameplate rating of 18 kV, 500 MVA.

Now, the base for circuit calculation is 20 kV, 100 MVA. Find X'' on the new base.

- Using the formula

$$Z_{pu_{New}} = Z_{pu_{old}} \left(\frac{kV_{old}}{kV_{New}}\right)^2 \left(\frac{kVA_{New}}{kVA_{old}}\right)$$



$$X''_{pu_{New}} = (0.25)\left(\frac{18}{20}\right)^2\left(\frac{100}{500}\right)$$

$$= 0.0405 \text{ pu}$$