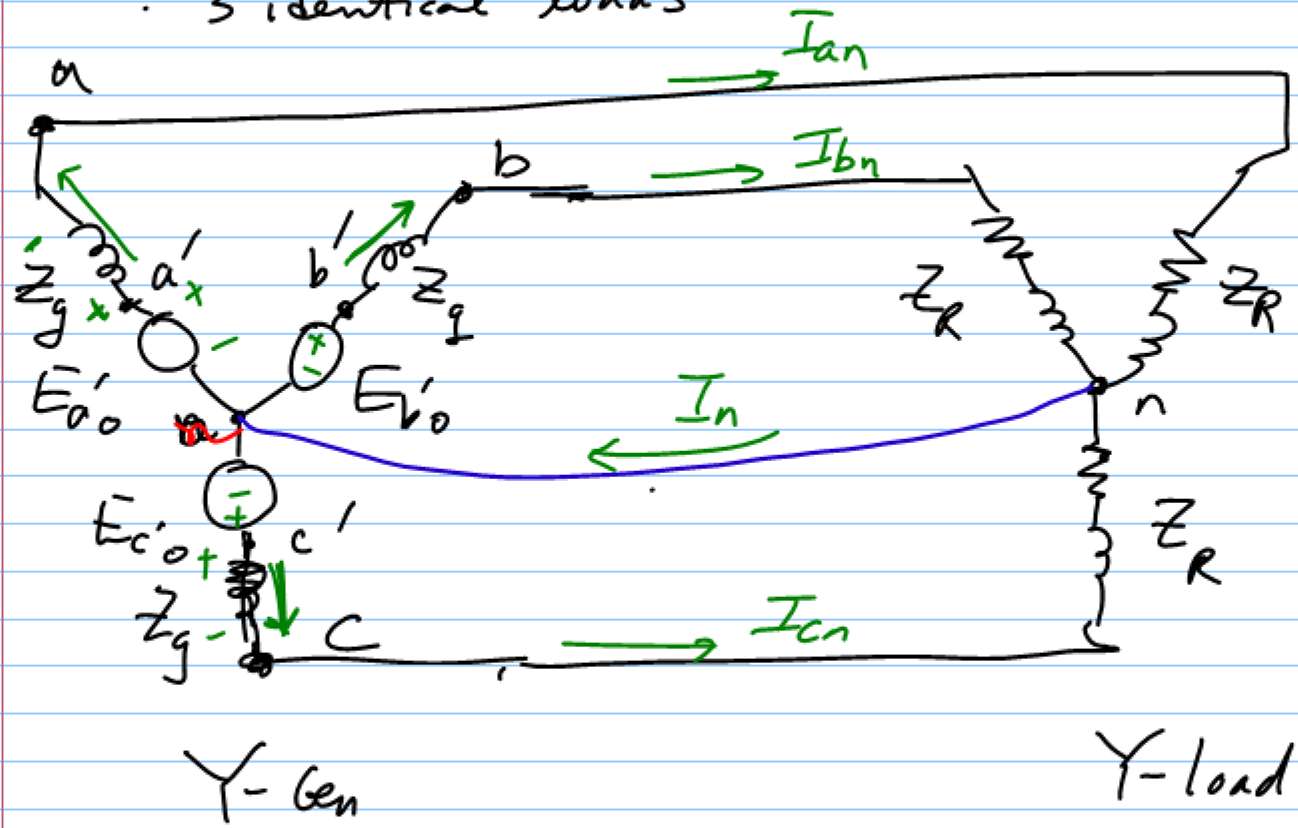
3-Phase Circuits

"Balanced 3-ϕ system"

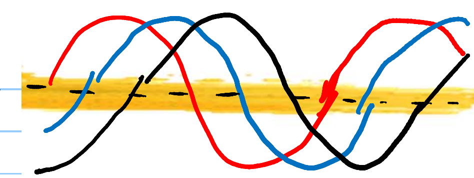
: 3ϕ generator

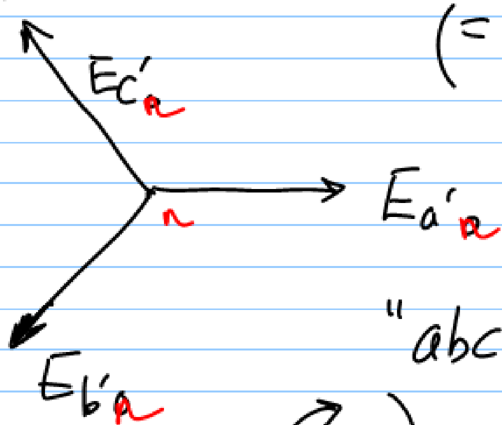
: 3 identical loads



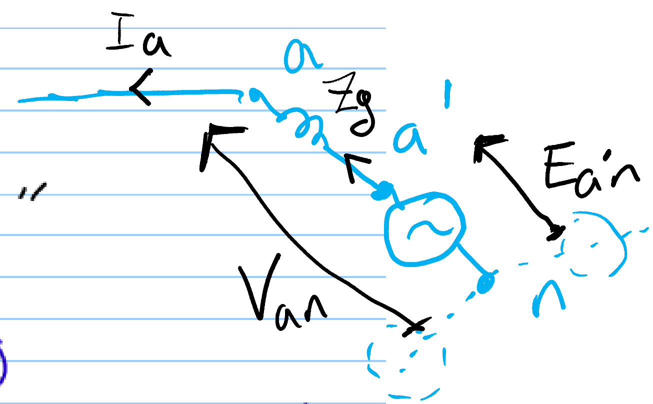
Balanced 3-Phase System

$E_{a'n} = E\angle 0^\circ$ $\quad E_{b'n} = E\angle -120^\circ$ $\left(= E\angle 240^\circ\right)$ $\quad E_{c'n} = E\angle 120^\circ$





"abc- Sequence"



Gen

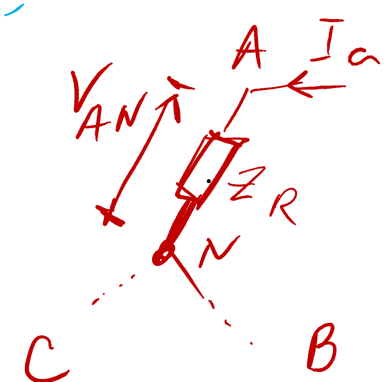
Load

$$V_{an} = E_{a'n} - I_a Z_g \qquad V_{an} = I_a \cdot Z_R \rightarrow I_a = \frac{V_{an}}{Z_R}$$

$$V_{bn} = E_{b'n} - I_b Z_g \qquad V_{bn} = I_b Z_R \rightarrow I_b = \frac{V_{bn}}{Z_R}$$

$$V_{cn} = E_{c'n} - I_c Z_g \qquad V_{cn} = I_c Z_R \rightarrow I_c = \frac{V_{cn}}{Z_R}$$

$V_{an} =$ $V_{bn} =$ $V_{cn} =$



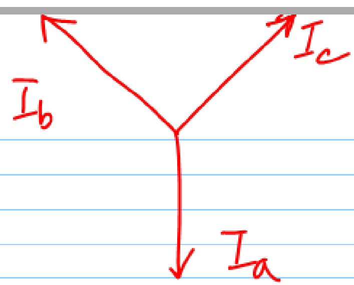
$$E_{a'A} = I_a (Z_g + Z_R) \rightarrow I_a = \frac{E_{a'a}}{Z_g + Z_R} = \frac{V_{an}}{Z_R}$$

$$E_{b'A} = I_b (Z_g + Z_R) \rightarrow I_b = \frac{E_{b'a}}{Z_g + Z_R} = \frac{V_{bn}}{Z_R}$$

$$E_{c'A} = I_c (Z_g + Z_R) \rightarrow I_c = \frac{E_{c'a}}{Z_g + Z_R} = \frac{V_{cn}}{Z_R}$$

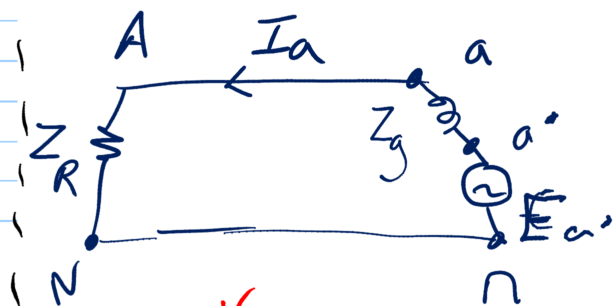
Since $E_{a'A}$, $E_{b'A}$, and $E_{c'A}$ are balanced,

I_a, I_b, & I_c are also balanced.



and $I_a + I_b + I_c = 0$

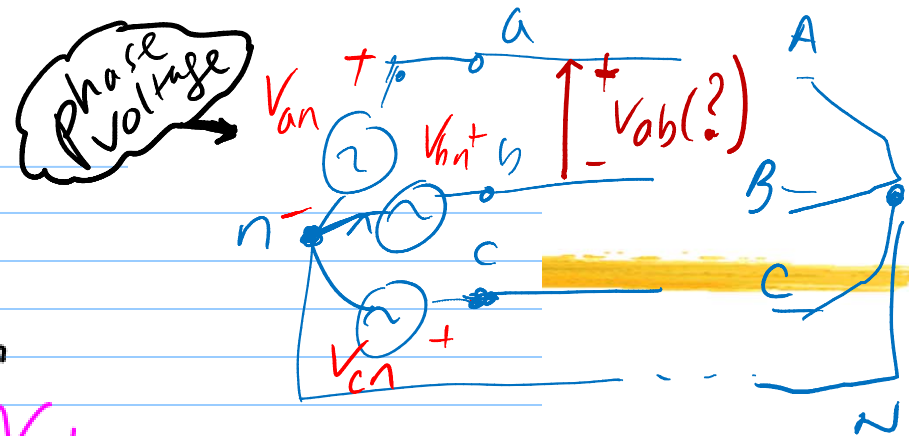
$\rightarrow I_n = 0$ No current flow back to generator.



$I_a = \frac{V_{AN}}{Z_R}$ @ Load

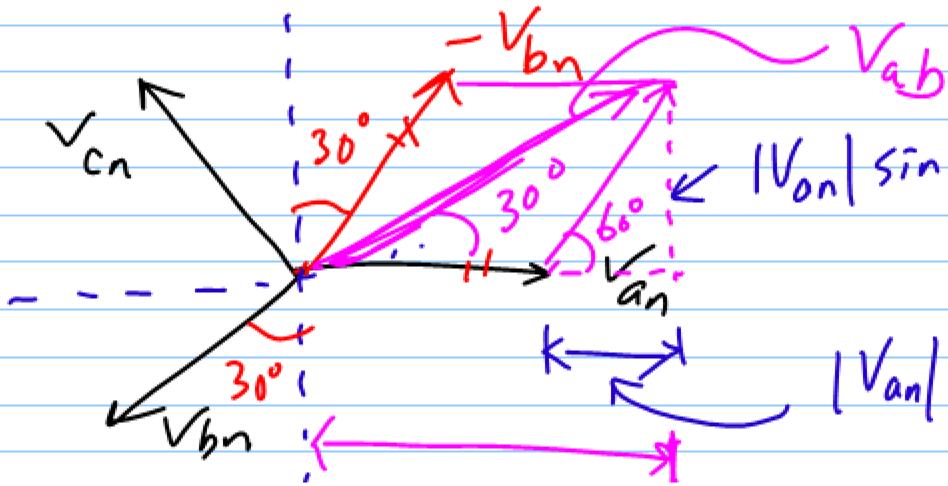
$I_a = \frac{E_{a'n}}{Z_g + Z_R}$ @ network

Line-to-Line Voltage



Line-to-Line Voltage

$$\bar{V}_{ab} = \bar{V}_{an} + \bar{V}_{nb} = \bar{V}_{an} - \bar{V}_{bn}$$



$$|V_{an}| \sin 60° = \frac{\sqrt{3}}{2} |V_{an}|$$

$$|V_{an}| \cos 60° = \frac{|V_{an}|}{2}$$

$$|V_{ab}| = \sqrt{\left(\frac{\sqrt{3}}{2}|V_{an}|\right)^2 + \left(\frac{3}{2}|V_{an}|\right)^2} = \sqrt{\frac{12}{4}|V_{an}|^2} = \sqrt{3}\,|V_{an}|$$

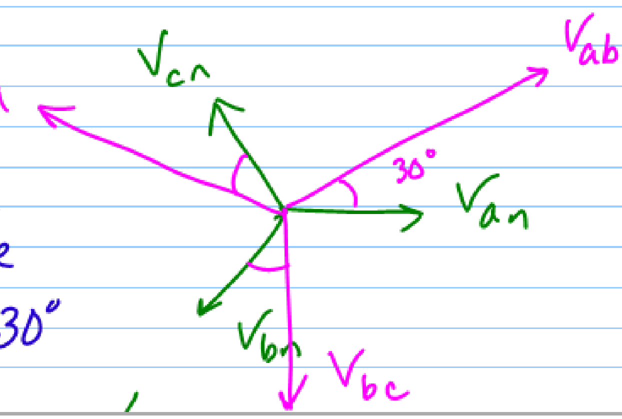
3

$$|V_{ab}| = \sqrt{\left(\frac{\sqrt{3}}{2}|V_{an}|\right)^2 + \left(\frac{3}{2}|V_{an}|\right)^2} = \sqrt{\frac{12}{4}|V_{an}|^2} = \sqrt{3}\,|V_{an}|$$

3

$$\Rightarrow \underline{\bar{V}_{ab} = \sqrt{3}\,\bar{V}_{an} \angle 30^\circ}$$

$$\Rightarrow \bar{V}_{bc} = \sqrt{3}\,\bar{V}_{bn} \angle 30^\circ \qquad \Rightarrow \bar{V}_{ca} = \sqrt{3}\,\bar{V}_{cn} \angle 30^\circ$$



Line Voltage leads phase Voltge by 30°

Also,

$$V_{ab} + V_{bc} + V_{ca} = 0$$

⌘Algebraic Calculation

$V := 100$

$Van := V \cdot \exp(i \cdot 0) = 100$

$Vbn := V \cdot \exp\left[i \cdot (-120) \cdot \frac{\pi}{180}\right] = -50 - 86.6025404 \cdot i$

$Vcn := V \cdot \exp\left[i \cdot (120) \cdot \frac{\pi}{180}\right] = -50 + 86.6025404 \cdot i$

$Vab := Van - Vbn = 150 + 86.6025404 \cdot i$

$|Vab| = 173.2050808$

$\theta ab := \operatorname{atan}\left(\frac{Im(Vab)}{Re(Vab)}\right) \cdot \frac{180}{\pi} = 30$

$Vca := Vcn - Van = -150 + 86.6025404 \cdot i$

$|Vca| = 173.2050808$

$\theta ca := \operatorname{atan}\left(\frac{Im(Vca)}{Re(Vca)}\right) \cdot \frac{180}{\pi} = -30$

$\theta ca := \theta ca + 180 = 150$

$|V_{ab}| = \sqrt{\left(\frac{\sqrt{3}}{2}|V_{an}|\right)^2 + \left(\frac{3}{2}|V_{an}|\right)^2} = \sqrt{\frac{12}{4}|V_{an}|^2} = \sqrt{3}|V_{an}|$

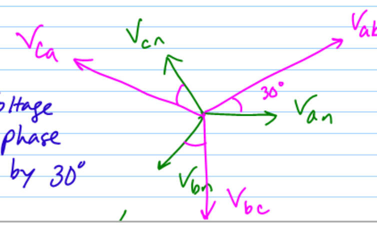
3

$\Rightarrow \underline{\overline{V}_{ab} = \sqrt{3}\,\overline{V}_{an} \angle 30°}$

$\Rightarrow \overline{V}_{bc} = \sqrt{3}\,\overline{V}_{bn} \angle 30°$

$\Rightarrow \overline{V}_{ca} = \sqrt{3}\,\overline{V}_{cn} \angle 30°$

Line Voltage leads phase Voltge by 30°



Also,

$V_{ab} + V_{bc} + V_{ca} = 0$

$V := 100$

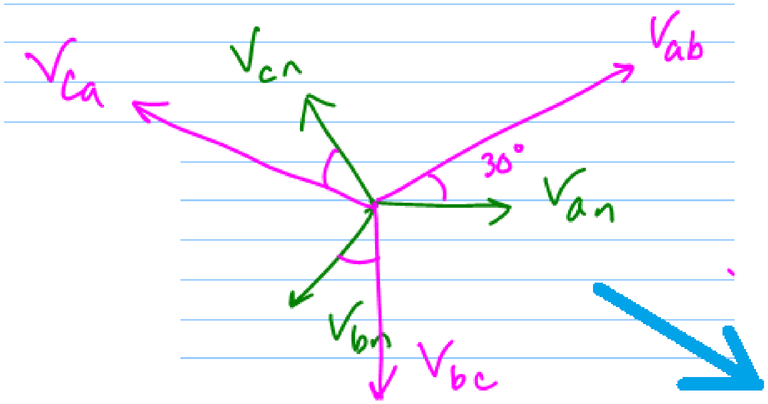
$\theta a := 0 \quad \theta b := -120 \cdot \frac{\pi}{180} \quad \theta c := 120 \cdot \frac{\pi}{180}$

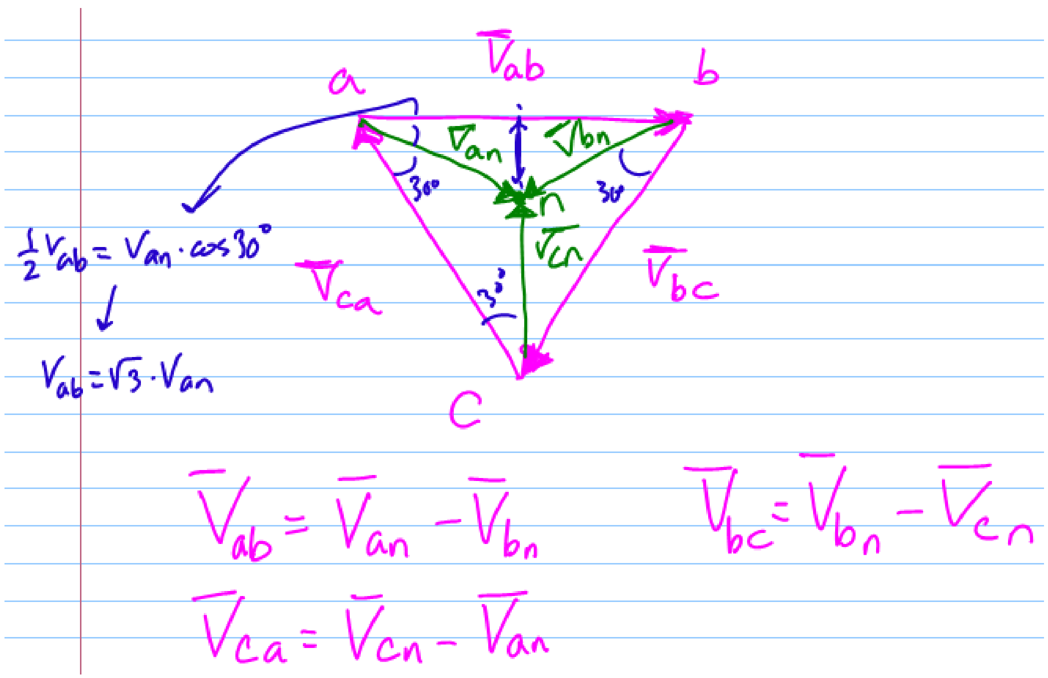
$van := V \cdot (\cos(\theta a) + i \cdot \sin(\theta a)) = 100$

$vbn := V \cdot (\cos(\theta b) + i \cdot \sin(\theta b)) = -50 - 86.6025404 \cdot i$

$vcn := V \cdot (\cos(\theta c) + i \cdot \sin(\theta c)) = -50 + 86.6025404 \cdot i$

Line-To-Line Voltage (V_{ab}) vs. Phase Voltage (V_{an})





Example 2.2

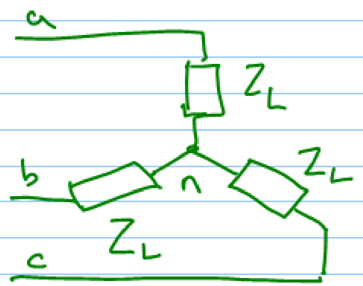
In a balanced 3-ϕ circuit, $\overline{V_{ab}} = 173.2\angle 0^\circ$ V. Determine all the voltages and the currents in a Y-connected load having $Z_L = 10\angle 20^\circ\ \Omega$.

- Can you draw a circuit diagram for this example question?

Example 2.2

In a balanced 3-ϕ circuit, $\bar{V}_{ab} = 173.2\angle 0^\circ$ V. Determine all the voltages and the currents in a Y-connected load having $Z_L = 10\angle 20^\circ\ \Omega$.

Solution



$\bar{V}_{ab} = 173.2\angle 0^\circ$

→ $\bar{V}_{bc} = 173.2\angle -120^\circ$

$\bar{V}_{ca} = 173.2\angle 120^\circ$

From $\bar{V}_{ab} = \sqrt{3}\,\bar{V}_{an}\angle 30^\circ$

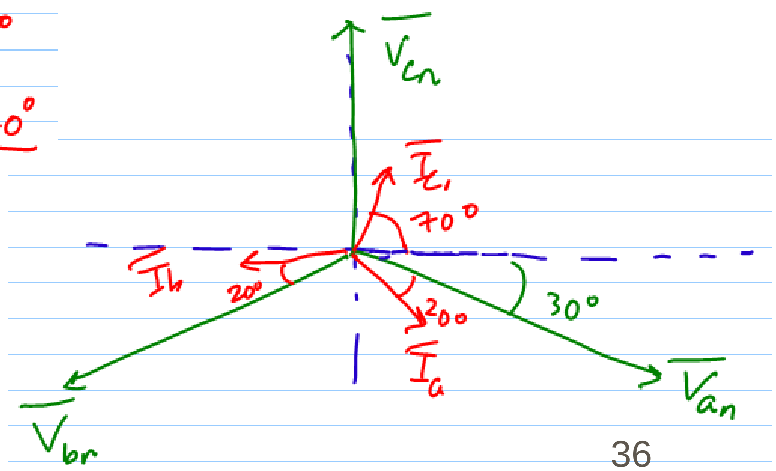
$$\bar{V}_{an} = \frac{1}{\sqrt{3}}\bar{V}_{ab}\angle -30^\circ = 100\angle -30^\circ$$

$$\bar{V}_{bn} = 100\angle -150^\circ$$

$$\bar{V}_{cn} = 100\angle 90^\circ$$

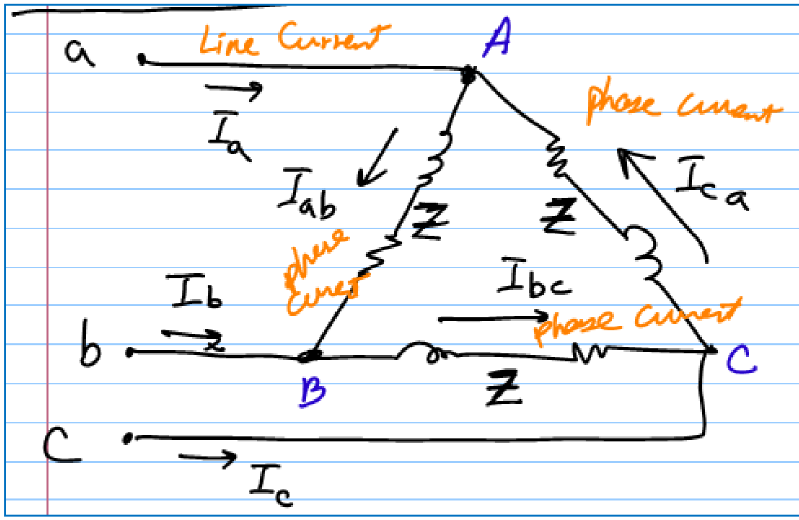
$$\bar{I}_a = \frac{V_{an}}{Z_L} = \frac{100\angle -30^\circ}{10\angle 20} = 10\angle -50^\circ$$

$\bar{I}_b = 10\angle -170^\circ$, $\bar{I}_c = 10\angle 70^\circ$

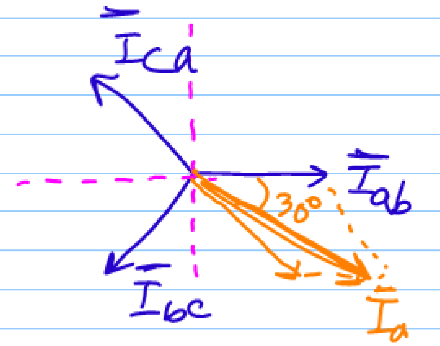


Delta (Δ) Load

I_a ?



Balanced Load



$$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = \bar{I}_{ab} - \bar{I}_{ab}\angle 120°$$

$$= \bar{I}_{ab}\left(1 - \angle 120°\right)$$

$$= \bar{I}_{ab}\left(1 - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right)$$

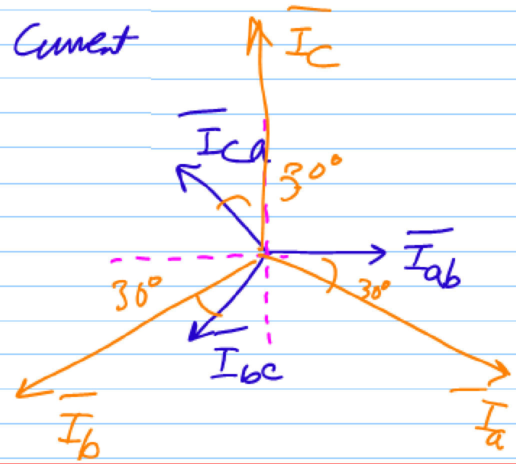
$$= \bar{I}_{ab}\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}\,\bar{I}_{ab}\angle -30°$$

$$\bar{I}_b = \sqrt{3}\,\bar{I}_{bc}\angle -30°$$

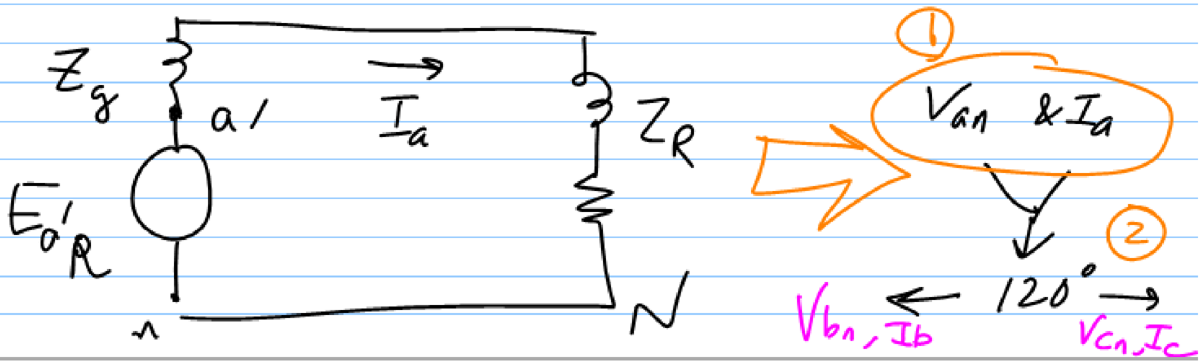
$$\bar{I}_c = \sqrt{3}\,\bar{I}_{ca}\angle -30°$$

Phase current leads line current by 30°



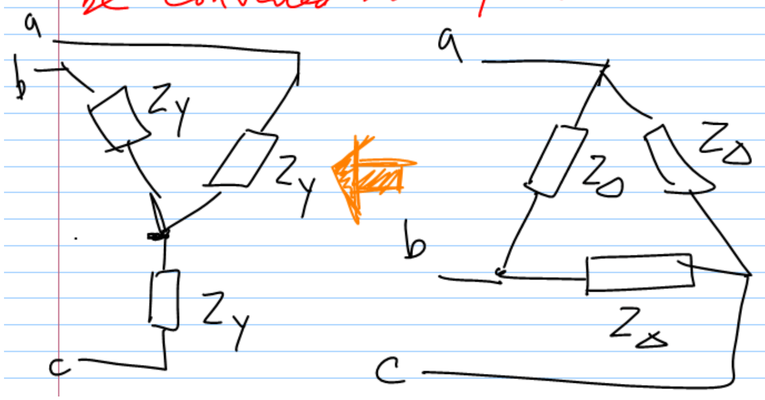
Per-phase analysis

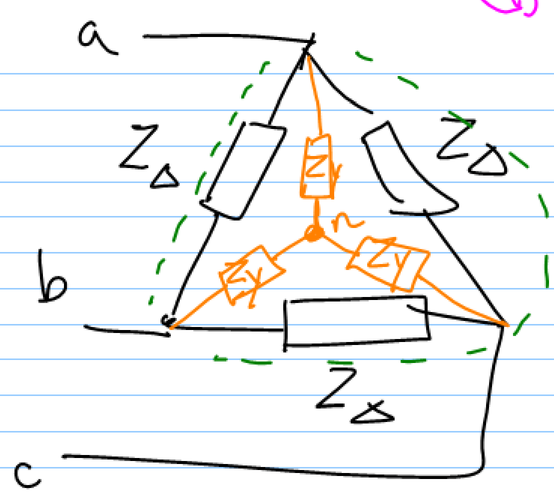
Since, in Balanced System, Voltage and currents in all 3 phases are the same in Magnitude and the only differen is the phase angle which we know (120° apart) we can do 3-ϕ circuit w/ single-phase equivalent circuit. → "per-phase analysis"



Per-phase analysis with Delta (Δ) load → Convert to Y-load

CAVEAT: If the load is in Δ connection, it must be converted to Y-connected Load.





① $Z_{ab} = Z_\Delta // 2Z_\Delta$

$$= \frac{2 \cdot Z_\Delta^2}{3 Z_\Delta} = \frac{2 \cdot Z_\Delta}{3}$$

② $Z_{ab} = Z_Y + Z_Y = 2Z_Y$

$$\frac{\cancel{2}}{3} Z_\Delta = \cancel{2} Z_Y$$

$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$

Load in Δ-Load

Equivalent Load in Y-Load converted from Δ-Load

Example 2.3

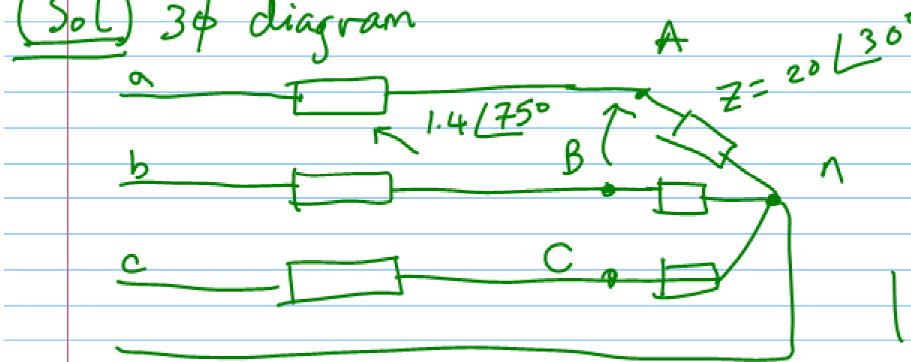
<u>Example 2.3 (P. 28).</u> The terminal Voltage of a Y-connected load cosisting of 3 equal impedances of $20\angle 30^\circ\ \Omega$ is 4.4 kV line-to-line. The impedance of each of the 3 lines connecting the load to a bus at a substation is $Z_L = 1.4\angle 75^\circ\ \Omega$. Find the line-to-line Voltage at the substation bus.

⌘Draw a circuit diagram for this example question.

Example 2.3

Example 2.3 (P. 28). The terminal Voltage of a Y-connected load cosisting of 3 equal impedances of $20\angle 30^\circ\ \Omega$ is 4.4 kV line-to-line. The impedance of each of the 3 lines connecting the load to a bus at a substation is $Z_L = 1.4\angle 75^\circ\ \Omega$. Find the line-to-line Voltage at the substation bus.

(Sol) 3ϕ diagram

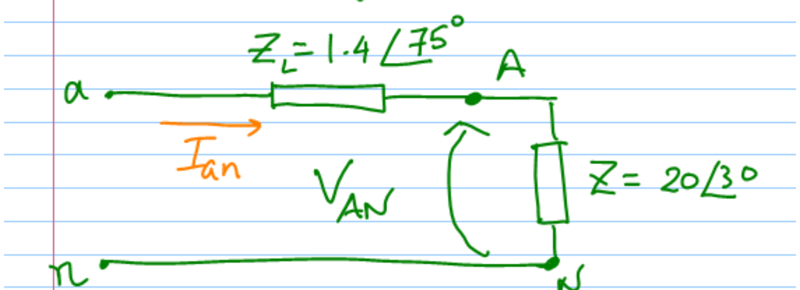


$$|V_{AB}| = 4.4\,kV$$

$$\Downarrow$$

$$|V_{AN}| = \frac{4.4}{\sqrt{3}}\,kV$$

→ Single-phase equivalent circuit



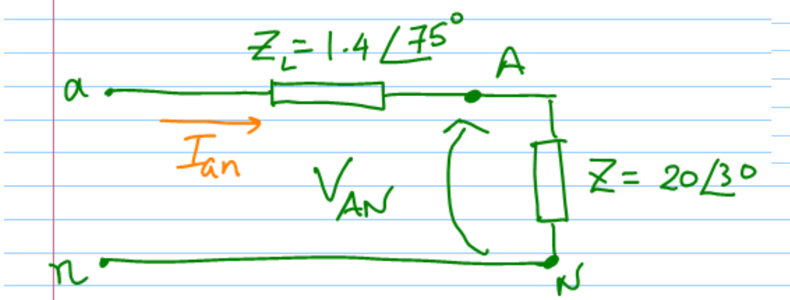
$$V_{AN} = \frac{4400}{\sqrt{3}}\angle 0^\circ \longleftarrow \text{We choose phase a Voltage as reference.}$$

$$= 2540\angle 0^\circ$$

$$I_{an} = \frac{V_{AN}}{Z} = \frac{2540\angle 0^\circ}{20\angle 30^\circ} = 127\angle -30^\circ$$

Example 2.3 - Continued

→ Single-phase equivalent circuit



$V_{AN} = \frac{4400}{\sqrt{3}}\angle 0^\circ$ ← we choose phase a voltage as reference,

$= 2540\angle 0^\circ$

$I_{an} = \frac{V_{AN}}{Z} = \frac{2540\angle 0^\circ}{20\angle 30^\circ} = 127\angle -30^\circ$

→ $V_{an} = V_{AN} + I_{an} \cdot Z_L = 2540\angle 0^\circ + (127\angle -30^\circ)(1.4\angle 75^\circ)$

$= 2666 + j\,125.7$

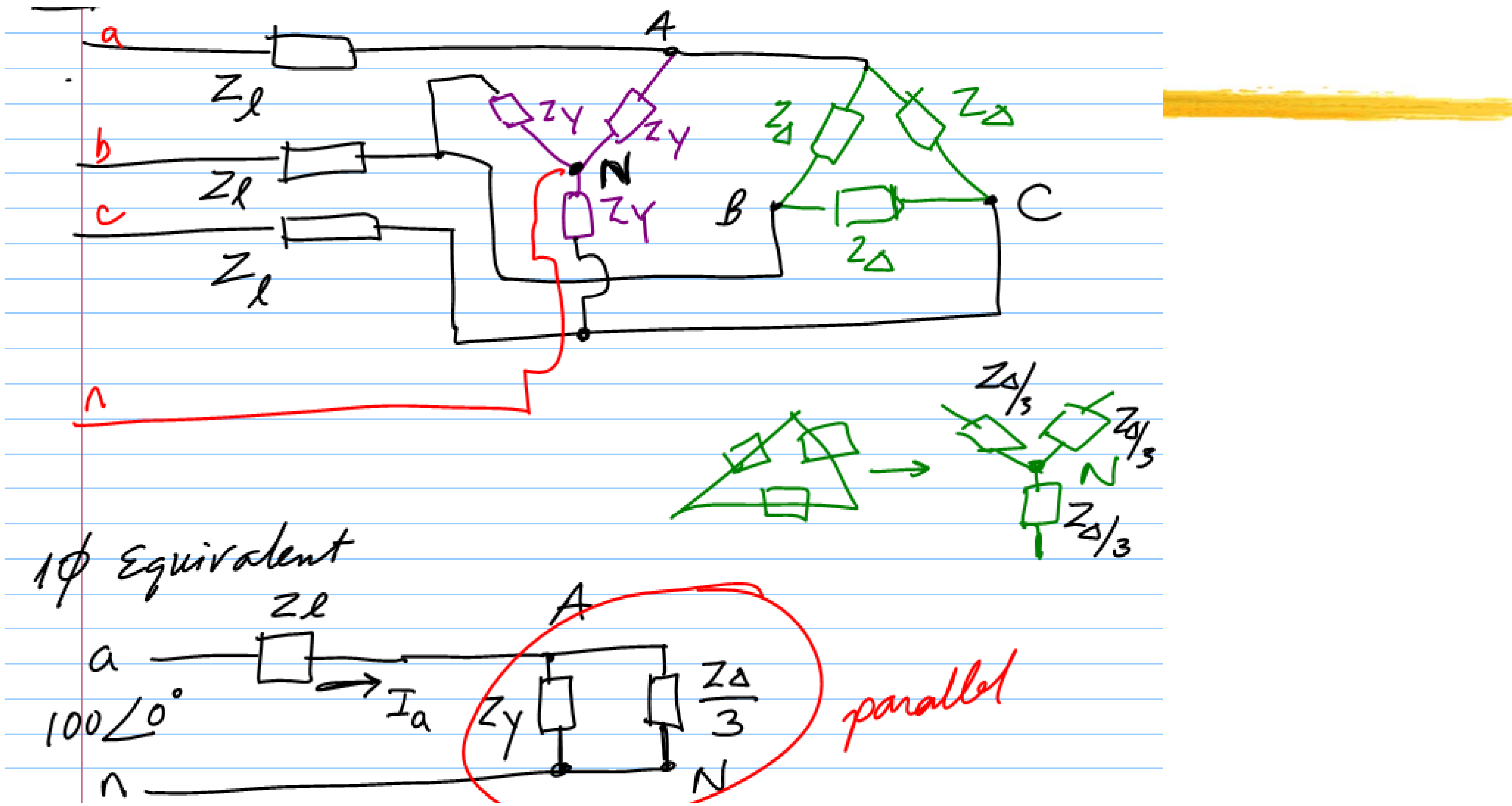
$= 2670\ \angle 2.70^\circ$

→ $V_{ab} = \sqrt{3}\, V_{an}\angle 30^\circ = \sqrt{3}\,(2670)\angle 32.70^\circ$

$= 4620\angle 32.70^\circ$

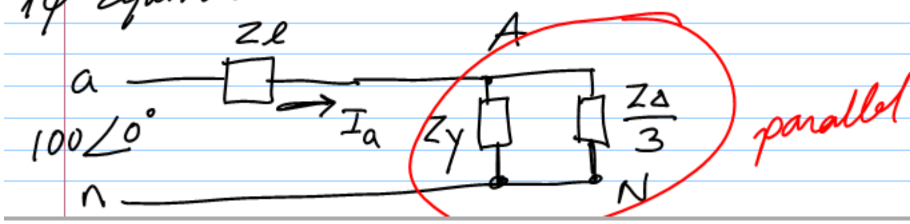
$\therefore |V_{ab}| = 4.62\ kV$

Y-load and Δ-load in parallel



Y-load and Δ-load in parallel - Example

1φ Equivalent



$$Z_\ell = j5, \quad Z_y = 10 + j20, \quad Z_c = -j5$$

$$Z_y \,//\, Z_c \rightarrow \frac{(10+j20)(-j5)}{(10+j20)+(-j5)} = \frac{100-j50}{10+j15}$$

$$= \frac{20-j10}{2+j3} = \frac{(20-j10)(2-j3)}{(2+j3)(2-j3)}$$

$$= \frac{40-j60-j20-30}{13} = \frac{10-j80}{13}$$

$$= 0.7692 - j\,6.1538$$

Zy = 10 + i·20 Zd = -i·5

$$Zp = \frac{Zy \cdot Zd}{Zy + Zd} = 0.7692 - 6.1538 \cdot i$$

$$Z_p + Z_\ell = 0.7692 - j\,6.1538 + j5$$

$$= 0.7692 - j\,1.1538$$

$$= \sqrt{0.7692^2 + 1.1538^2}\ \angle \tan^{-1}\frac{-1.1538}{0.7692}$$

$$= 1.3868 \angle -56.31^\circ$$

V = 100 + i·0

Z = Zp + i·5 = 0.7692 − 1.1538·i

|Z| = 1.3868

$$\arg(Z)\cdot\frac{180}{\pi} = -56.3099$$

$$I = \frac{V}{Z} = 40 + 60 \cdot i$$

|I| = 72.111

$$\arg(I)\cdot\frac{180}{\pi} = 56.3099$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{100 \angle 0^\circ}{1.3868 \angle -56.31^\circ}$$

$$= 72.11 \angle -56.31^\circ$$

Power in balanced 3-phase circuits

$$P_{3\phi} = P_a + P_b + P_c$$

$$= 3 P_a \quad \text{(in Balanced System)}$$

V_p: phase voltage magnitude to neutral

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

I_p: phase current magnitude for Y-Load

$$I_p = |I_{an}| = |I_{bn}| = |I_{cn}|$$

$$\Rightarrow P_{3\phi} = 3 V_p I_p \cos\theta_p$$

θ_p: phase angle between $\bar{V}_p$ & $\bar{I}_p$

: angle of the impedance $\longrightarrow \theta_Z$

Power in balanced 3-phase circuits

V_L: Line-to-Line Voltage magnitude

$V_p = \frac{V_L}{\sqrt{3}}$ and $I_p = I_L$ (Y-Load case) or 1ϕ equ

I_L: current over a Line

$I_L = I_p\sqrt{3}$ Δ Load case

$$\Rightarrow P_{3\phi} = 3 V_p I_p \cos\theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\theta_p = \sqrt{3} V_L I_L \cos\theta_p$$

$$\& \quad Q_{3\phi} = 3 V_p I_p \sin\theta_p = \sqrt{3} V_L I_L \sin\theta_p$$

Further,

$$S_{3\phi} = P_{3\phi} + j Q_{3\phi}$$

$$|S_{3\phi}| = \sqrt{P_{3\phi}^2 + Q_{3\phi}^2}$$

$$= \sqrt{3} V_L I_L$$

$$= 3 V_p I_p$$

3-phase instantaneous power $p_3(t)$

$$v_a = V_m \cos \omega t \qquad i_a = I_m \cos(\omega t - \theta)$$

$$v_b = V_m \cos(\omega t - 120^\circ) \qquad i_b = I_m \cos(\omega t - \theta - 120^\circ)$$

$$v_c = V_m \cos(\omega t + 120^\circ) \qquad i_c = I_m \cos(\omega t - \theta + 120^\circ)$$

$$\begin{aligned} p_{3\phi} &= p_a(t) + p_b(t) + p_c(t) \\ &= V_m I_m \cos \omega t \cdot \cos(\underline{\omega t} - \theta) \\ &\quad + V_m I_m \cos(\underline{\omega t - 120^\circ}) \cdot \cos(\underline{\omega t - 120^\circ} - \theta) \\ &\quad + V_m I_m \cos(\underline{\omega t + 120^\circ}) \cdot \cos(\underline{\omega t + 120^\circ} - \theta) \end{aligned}$$

3-phase instantaneous power $p_3(t)$

From, 1ϕ instantaneous power $p(t)$

Phase a:

$$p_a(t) = \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos 2\omega t + \frac{V_m I_m}{2}\sin\theta \cdot \sin 2\omega t$$

Phase b:

$$p_b(t) = \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos\left(2\omega t - 240^\circ\right)$$

$$\frac{V_m I_m}{2}\sin\theta \cdot \sin\left(2\omega t - 240^\circ\right)$$

Phase c:

$$p_c(t) = \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos\left(2\omega t + 240^\circ\right)$$

$$\frac{V_m I_m}{2}\sin\theta \cdot \sin\left(2\omega t + 240^\circ\right)$$

3-phase instantaneous power $p_3(t)$

$$p_3(t) = p_a(t) + p_b(t) + p_c(t)$$

$$= 3\left(\frac{V_m I_m}{2}\cos\theta\right) + \frac{V_m I_m}{2}\cos\theta\,\underbrace{\left\{\cos 2\omega t + \cos(2\omega t - 240^\circ) + \cos(2\omega t + 240^\circ)\right\}}_{0}$$

$$+ \frac{V_m I_m}{2}\sin\theta\,\underbrace{\left\{\sin 2\omega t + \sin(2\omega t - 240) + \sin(2\omega t + 240^\circ)\right\}}_{0}$$

$$= 3\,\frac{V_m I_m}{2}\cos\theta = 3|V||I|\cos\theta$$

Constant !!

$$p_3(t) = \text{Constant}$$