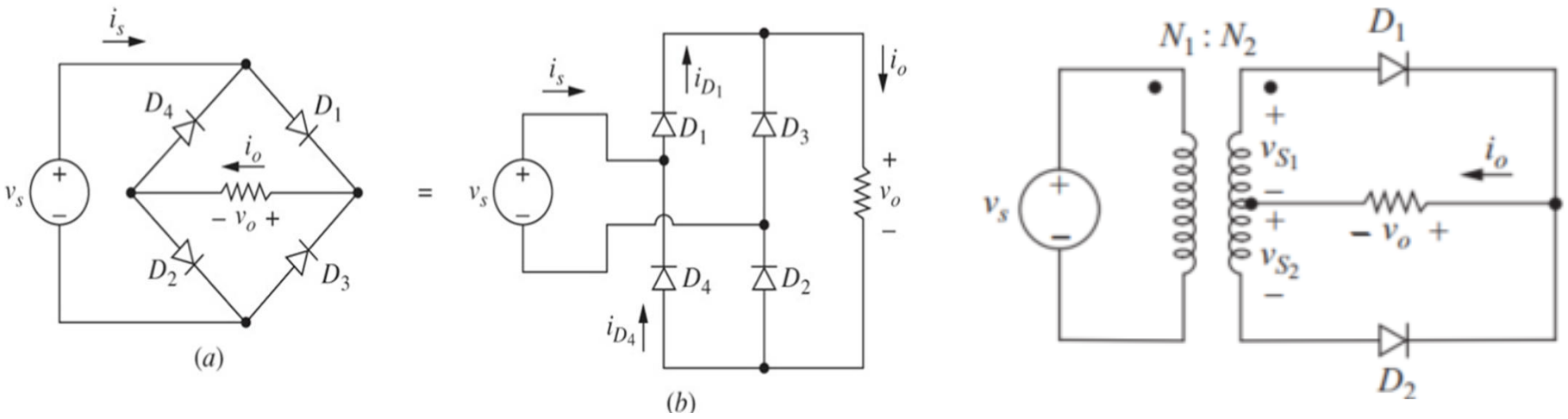
4. Full-Wave Rectifier



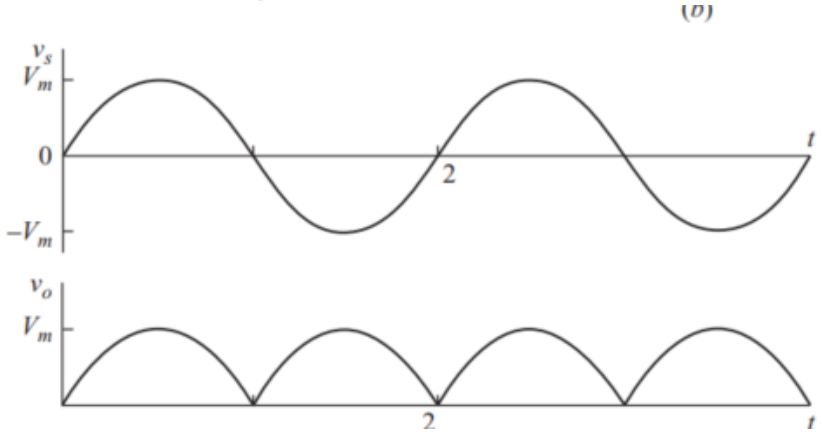
- Better than Half-wave rectifier – utilization of the entire cycle
- Less ripple than half-wave rectifier
- Topics
  - Bridge rectifier (focus) & center-tapped rectifier
  - Uncontrolled full-wave rectifier
  - Controlled full-wave rectifier
  - Single-phase only
  - Load types: R, R-L

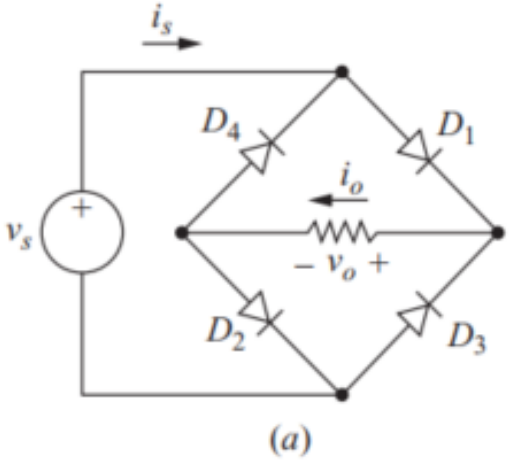
Figure 4.1

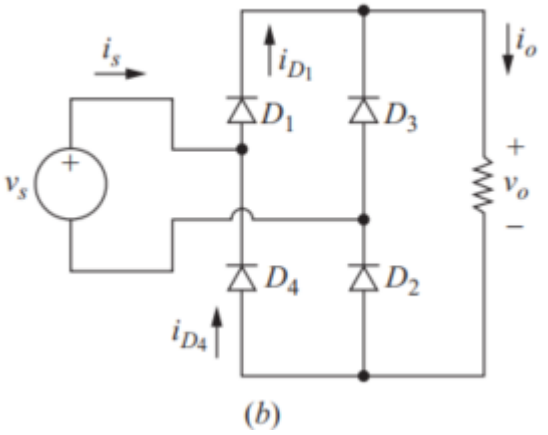
SINGLE-PHASE FULL-WAVE RECTIFIERS

BRIDGE RECTIFIER

- D1 & D2 conduct together
- D3 & D4 conduct together
- Load voltage *vo* = *vs* or –*vs*





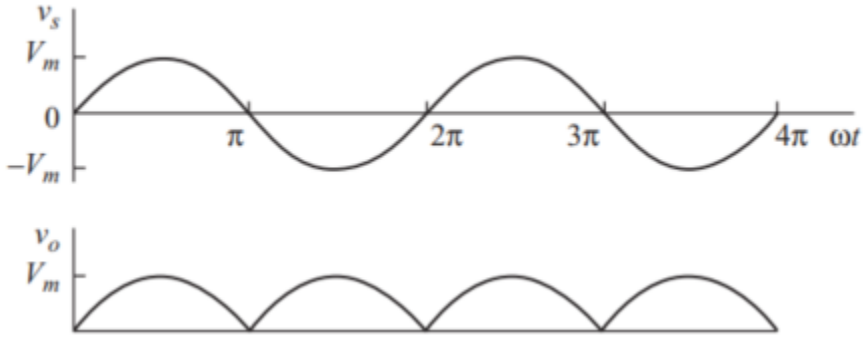


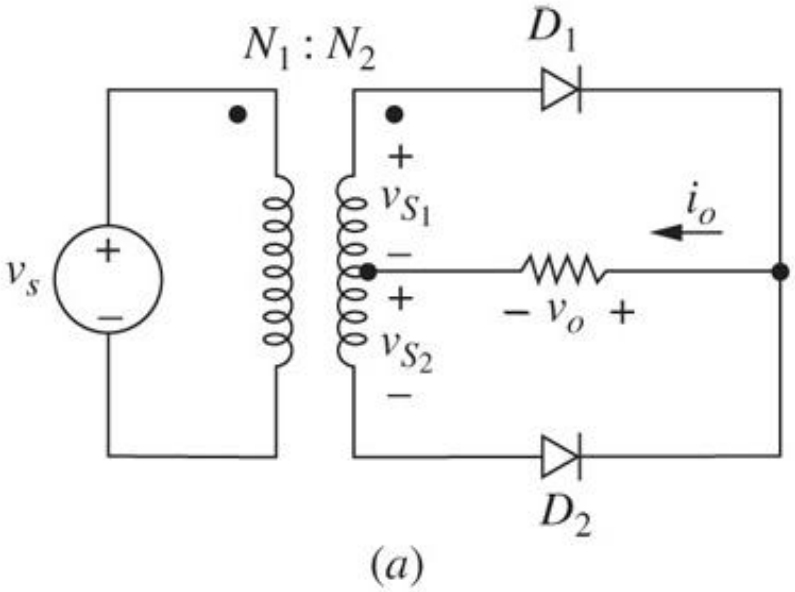
- Source current *is* = *iD1-iD4*
- *is* = *io* or –*io*
- *isRMS* = *ioRMS*
- Output voltage: fundamental freq = 2* (freq of the source)
- Fourier series: even terms

Figure 4.2

CENTER-TAPPED TRANSFORMER RECTIFIER

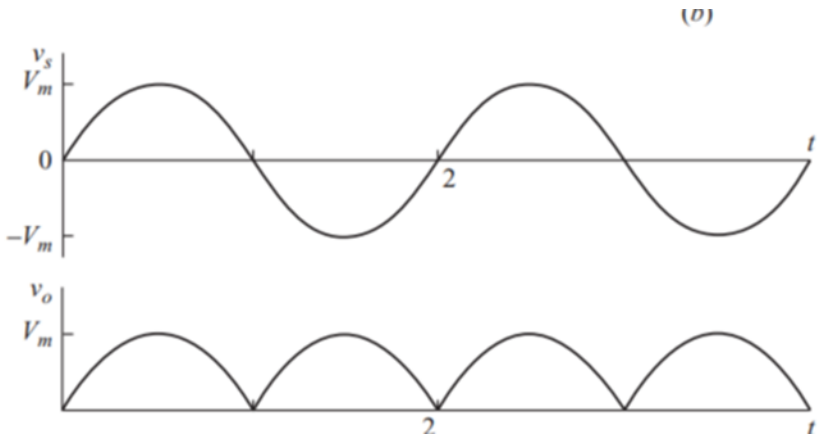
- Only one diode conducts at a time
- 1 diode voltage drop
- Load current is always positive
- vS1=vS2= vS*{(N2/N1)/2 }
- Vo = vS1 or – vS2

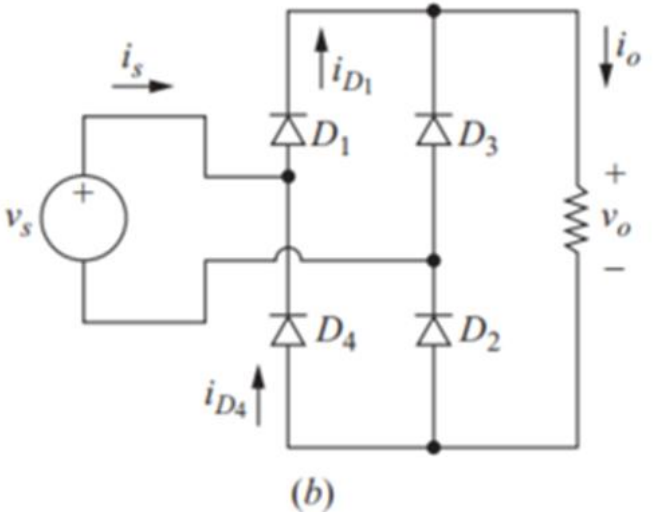




- Transformer provides electrical isolation
- Output voltage: fundamental freq
  = 2* (freq of the source)

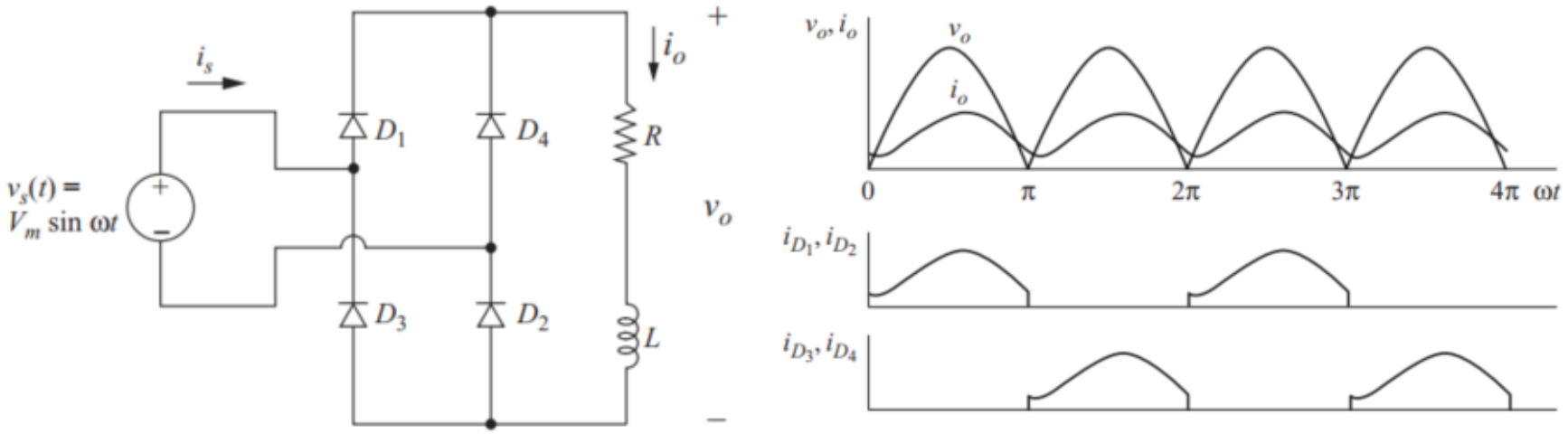
- Bridge Rectifier with R load





- Load voltage $$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{for } 0 \le \omega t \le \pi \\ -V_m \sin \omega t & \text{for } \pi \le \omega t \le 2\pi \end{cases}$$
- Average (dc) voltage: $$V_o = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t) = \frac{2V_m}{\pi}$$
- Average current: $$I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R}$$
- RMS current: $$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$$ Power absorbed by the load resistor $I_{\text{rms}}^2 R$

- Bridge Rectifier with R-L load



- Load voltage Fourier series

$$v_o(t) = V_o + \sum_{n=2,4\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

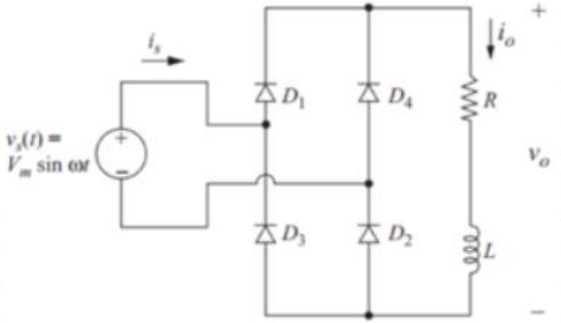
$$V_o = \frac{2V_m}{\pi} \quad \text{and} \quad V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

- Average current: $I_0 = \dfrac{V_0}{R}$

- Harmonic currents: $I_n = \dfrac{V_n}{Z_n} = \dfrac{V_n}{|R + jn\omega L|}$

• Example 4-1

The bridge rectifier circuit of Fig. 4-3*a* has an ac source with $V_m = 100$ V at 60 Hz and a series *RL* load with $R = 10\ \Omega$ and $L = 10$ mH. (*a*) Determine the average current in the load. (*b*) Estimate the peak-to-peak variation in load current based on the first ac term in the Fourier series. (*c*) Determine the power absorbed by the load and the power factor of the circuit. (*d*) Determine the average and rms currents in the diodes.



$Vm := 100 \quad f := 60 \quad R := 10$

$L := 10 \cdot 10^{-3}$

RL circuit

$v(wt) := Vm \cdot \sin(wt)$

$w := 2 \cdot \pi \cdot f = 376.9911184$

Average voltage $V0 := \frac{2 \cdot Vm}{\pi} = 63.6619772$

$V(n) := \frac{2 \cdot Vm}{\pi} \cdot \left(\frac{1}{n-1} - \frac{1}{n+1} \right)$ n even

$Z(n) := \left| R + i \cdot (n \cdot w \cdot L) \right|$

Average current: $I0 := \frac{V0}{R} = 6.3661977$

First ac term in Fourier Series

$V2 := V(2) = 42.4413182 \quad V4 := V(4) = 8.4882636$

$I2 := \frac{V(2)}{Z(2)} = 3.3888168 \quad I4 := \frac{V(4)}{Z(4)} = 0.4691183$

$\Delta io := 2 \cdot I2 = 6.7776337$

$$\Delta I_o \approx 2I_1 \qquad (3\text{-}36)$$

RMS current:

$$Irms := \sqrt{I0^2 + \left(\frac{I2}{\sqrt{2}}\right)^2 + \left(\frac{I4}{\sqrt{2}}\right)^2} = 6.8103267$$

Power Absrobed by the resistor:

$P := Irms^2 \cdot R = 463.8054922$

$Vrms := \frac{Vm}{\sqrt{2}} = 70.7106781$

power factor:

$pf := \frac{P}{Vrms \cdot Irms} = 0.9631256$

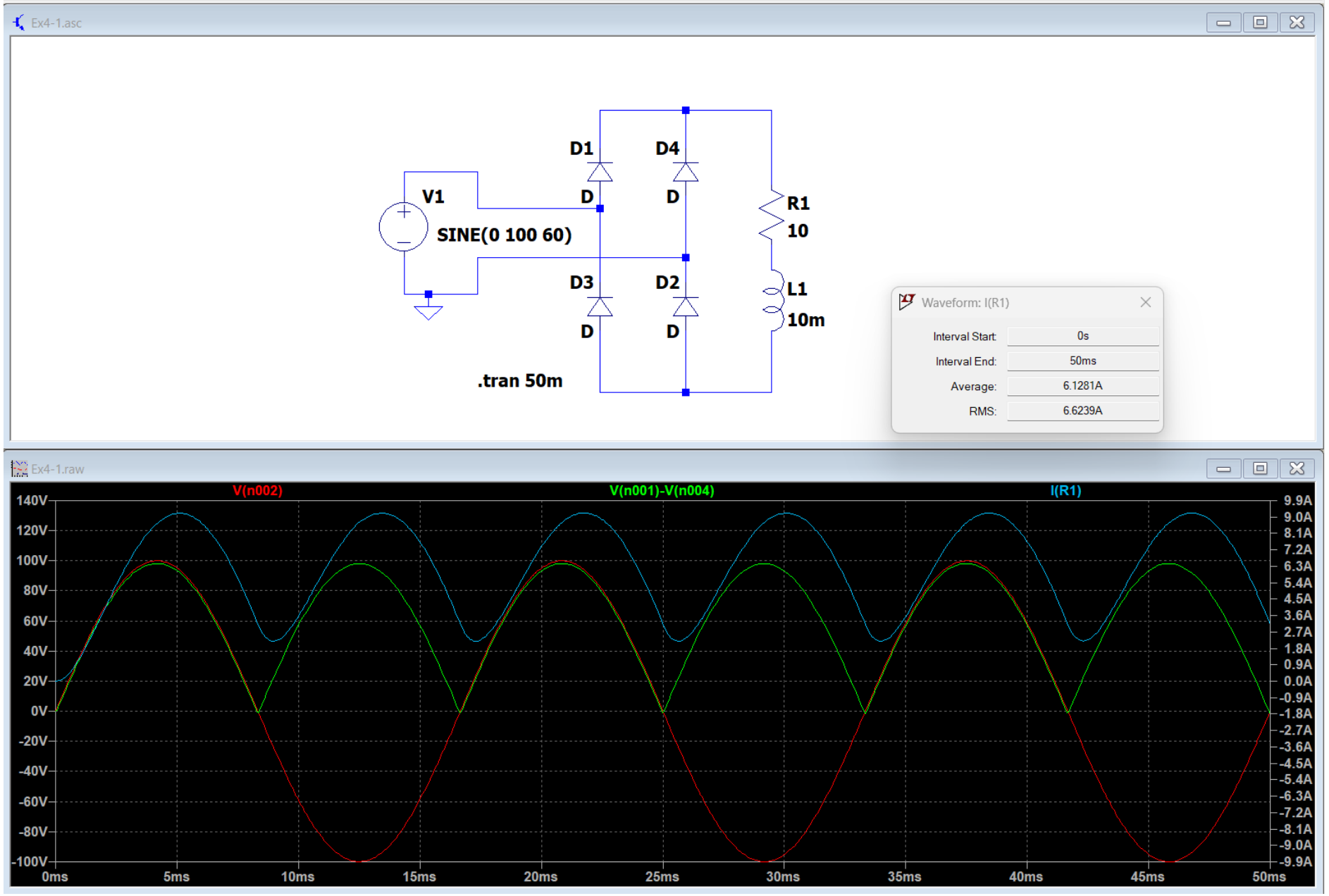
Diode current

$IDavg := \frac{I0}{2} = 3.1830989$

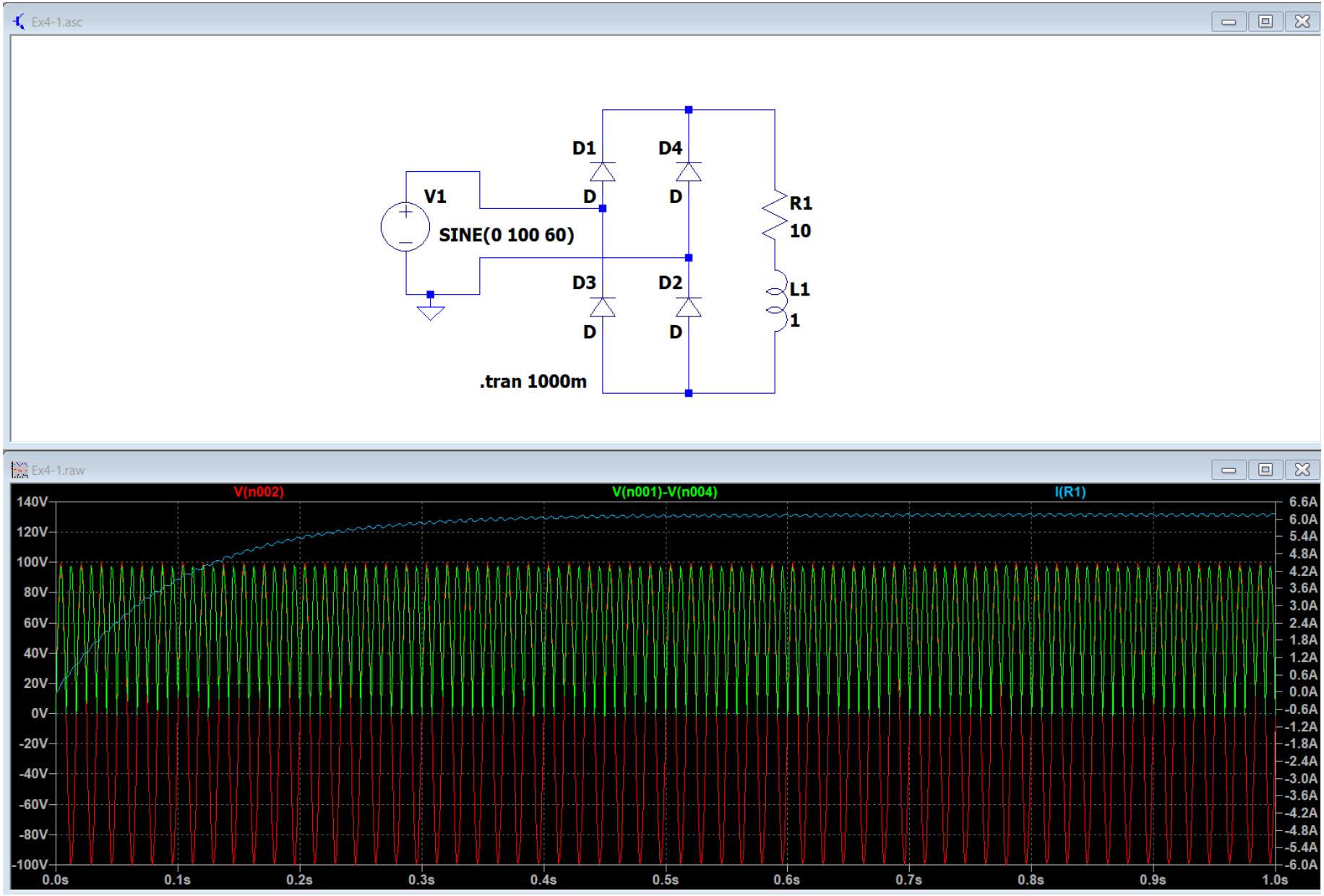
$IDrms := \frac{Irms}{\sqrt{2}} = 4.8156282$

ID flows only the half of the time, T/2

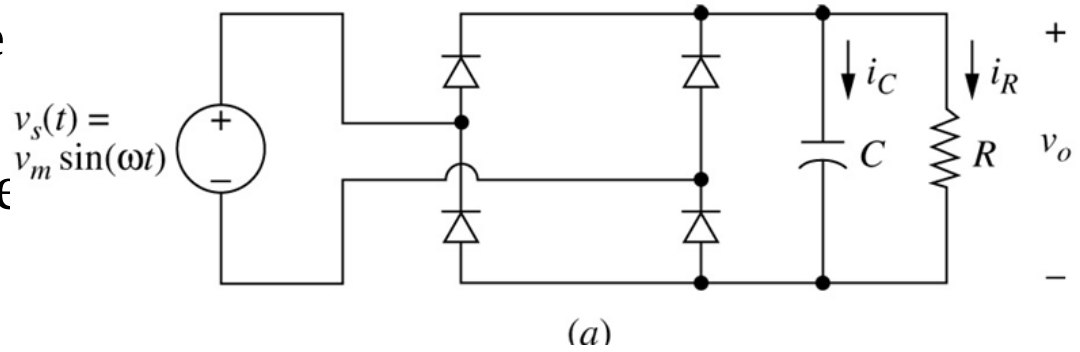
- **Example 1 simulation**



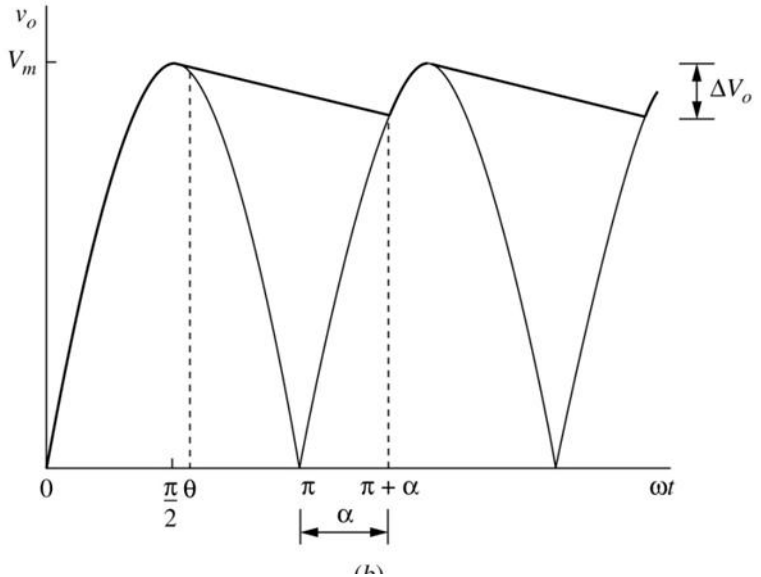
- **Large L (L = 1 H)**


Ex4-1.asc
D1
D4
V1
D
D
R1
10
SINE(0 100 60)
D3
D2
L1
1
D
D
.tran 1000m
Ex4-1.raw
V(n002)
V(n001)-V(n004)
I(R1)
140V
120V
100V
80V
60V
40V
20V
0V
-20V
-40V
-60V
-80V
-100V
6.6A
6.0A
5.4A
4.8A
4.2A
3.6A
3.0A
2.4A
1.8A
1.2A
0.6A
0.0A
-0.6A
-1.2A
-1.8A
-2.4A
-3.0A
-3.6A
-4.2A
-4.8A
-5.4A
-6.0A
0.0s
0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s
1.0s

- **Capacitor Filter for Full-Wave Rectifier**
  - Similar to Capacitor filte half-wave rectifier
  - In full-wave, capacitor discharge is smaller.
  - Ripple is one-half of half-wave rectifier
  - Peak voltage is slightly lower: 2 diode drops (instead of 1)
  - Output Voltage:

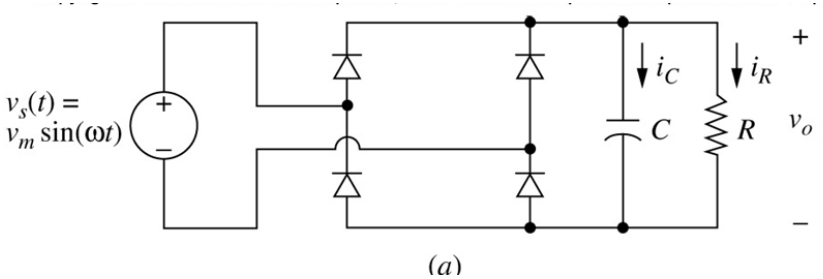


(a)

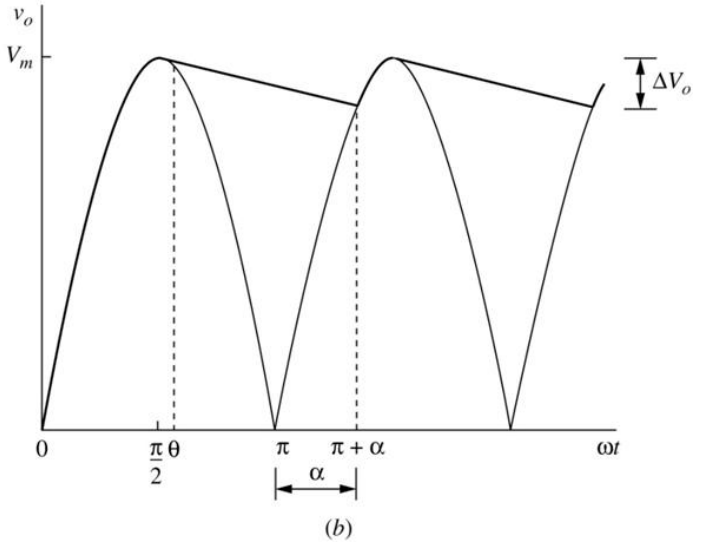


$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin \theta) e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases}$$

- **Capacitor Filter for Full-Wave Rectifier**
  - The angle θ is where the ON diodes become reverse biased (ref. p.89)



$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$



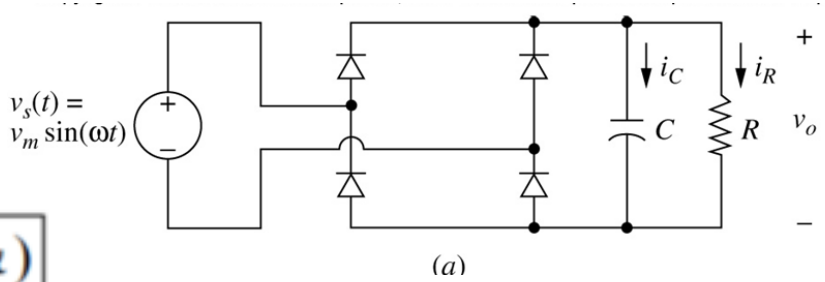
  - Max voltage = Vm
  - Min voltage is determined when the other pair of diode turns ON at wt = π + α

$$(V_m \sin\theta)e^{-(\pi+\alpha-\theta)/\omega RC} = -V_m \sin(\pi+\alpha)$$

$$(\sin\theta)e^{-(\pi+\alpha-\theta)/\omega RC} - \sin\alpha = 0$$

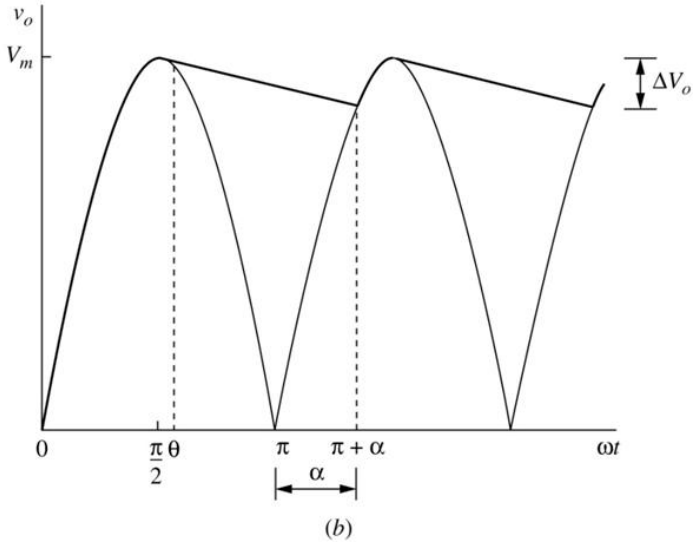
  - '**Solve**" in Smath for α

- **Capacitor Filter for Full-Wave Rectifier**
- Ripple voltage = Vmax-Vmin



(a)

$$\Delta V_o = V_m - |V_m \sin(\pi + \alpha)| = V_m(1 - \sin\alpha)$$



(b)

- If the time constant R*C is large,

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

$$\theta \approx \pi/2 \qquad \alpha \approx \pi/2$$

- Then Vmin:

$$v_o(\pi + \alpha) = V_m e^{-(\pi + \pi/2 - \pi/2)/\omega RC} = V_m e^{-\pi/\omega RC}$$

- Ripple voltage (approximation)

$$\Delta V_o \approx V_m(1 - e^{-\pi/\omega RC})$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

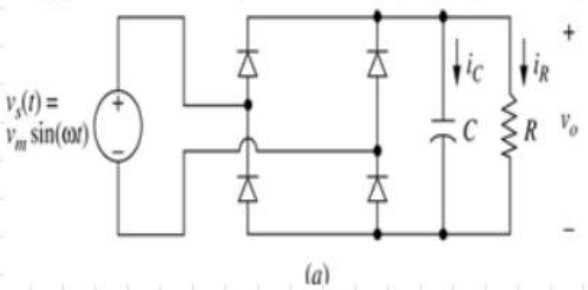
$$e^{-\pi/\omega RC} \approx 1 - \frac{\pi}{\omega RC}$$

$$\Delta V_o \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}$$

- **Example 4-4**

Full-Wave Rectifier with Capacitance Filter

The full-wave rectifier of Fig. 4-6*a* has a 120 V source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu F$. (*a*) Determine the peak-to-peak voltage variation of the output. (*b*) Determine the value of capacitance that would reduce the output voltage ripple to 1percent of the dc value.



(a)

$Vsrms := 120 \qquad f := 60 \qquad R := 500 \qquad C := 100 \cdot 10^{-6}$

$w := 2 \cdot \pi \cdot f = 376.9911184 \qquad Vm := \sqrt{2} \cdot Vsrms = 169.7056275$

$\theta := -\operatorname{atan}(w \cdot R \cdot C) + \pi = 1.6237983$ rad

α calculation

$$g(x) := \sin(\theta) \cdot \exp\left(\frac{-\pi - x + \theta}{w \cdot R \cdot C}\right) - \sin(x)$$

$$alpha := \operatorname{solve}\left(g(x), x, 0, \frac{\pi}{2}\right) = 1.0573939$$

$\alpha := alpha = 1.0573939$ rad

Ripple voltage (Vmax- Vmin)

$$\Delta V := Vm - \left|Vm \cdot \sin(\pi + \alpha)\right| = 21.8787129$$

For the ripple with 1 percent: $\Delta Vnew := \frac{Vm}{100} \cdot 1 = 1.6970563$

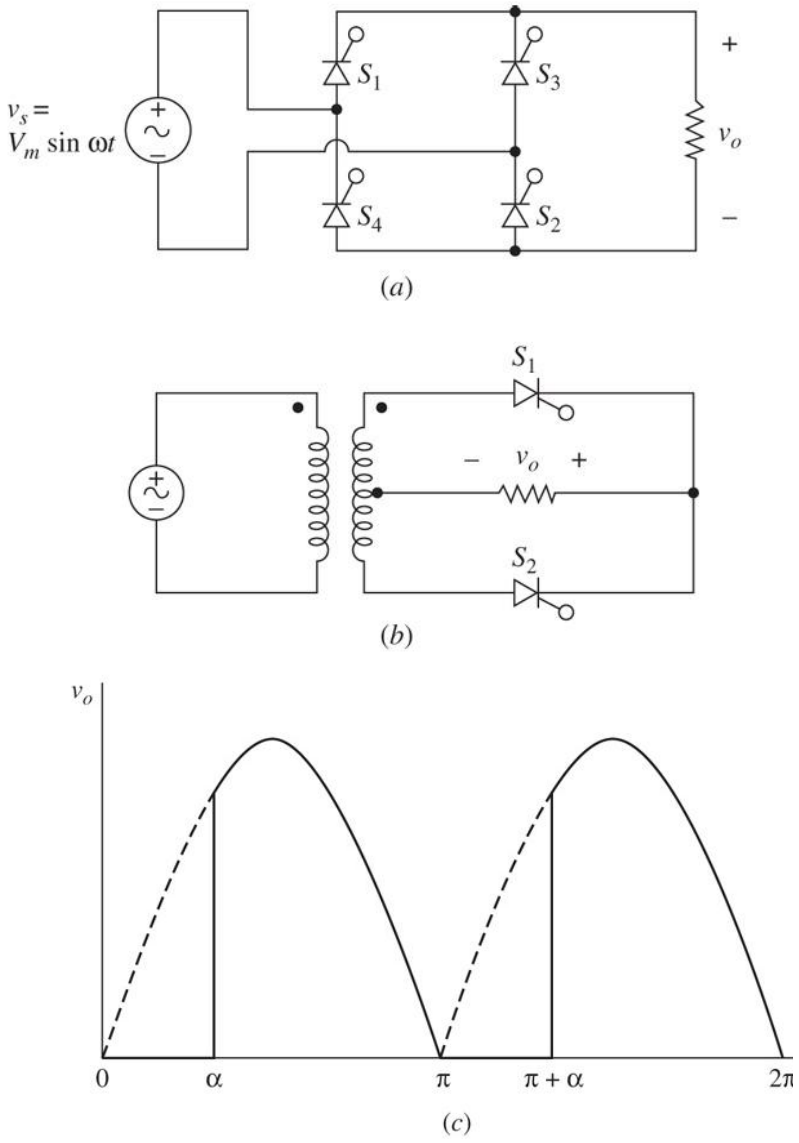
Capacitance requirement

$$C := \frac{Vm}{2 \cdot f \cdot R \cdot \Delta Vnew} = 0.0016667 \quad F$$

$C \cdot 10^{6} = 1666.6666667$ uF

Figure 4.10

Controlled Full-Wave Rectifiers



- Output voltage adjustment
- Diode → SCR (Thyristor)
- Delay angle **α** by gate pulse
- Bridge rectifier
- Center-tapped rectifier
- **R Load case**
  - Average voltage

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}(1+\cos\alpha)$$

  - Average output current

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

  - RMS current

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)}$$

$$= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

- Example 4-6 (R Load case)

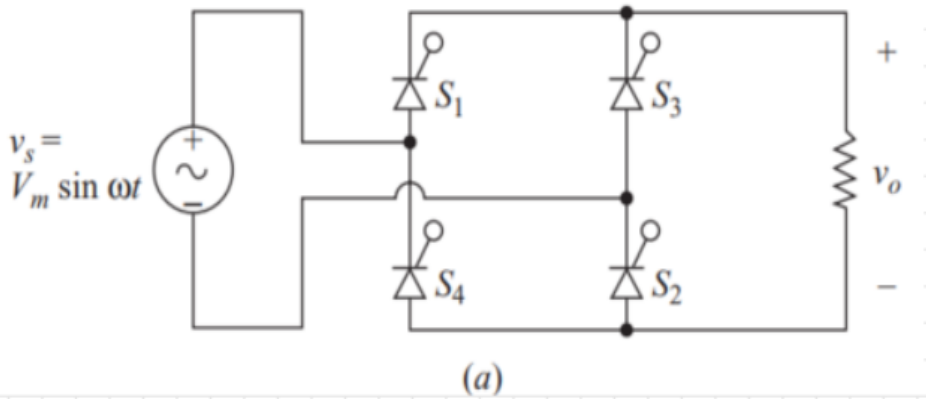
Controlled Full-Wave Rectifier with Resistive Load

The full-wave controlled bridge rectifier of Fig. 4-10*a* has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

$Vsrms := 120 \quad R := 20 \quad f := 60$

$w := 2 \cdot \pi \cdot f = 376.9911184 \quad \alpha := 40 \cdot \frac{\pi}{180} = 0.6981317$ rad

$Vm := \sqrt{2} \cdot Vsrms = 169.7056275$



(*a*)

Average output voltage

$$Vo := \frac{Vm}{\pi} \cdot (1 + \cos(\alpha)) = 95.3999176$$

Average load current

$$Io := \frac{Vo}{R} = 4.7699959$$

RMS current

$$Irms := \frac{Vm}{R} \cdot \sqrt{\frac{1}{2} - \frac{\alpha}{2 \cdot \pi} + \frac{\sin(2 \cdot \alpha)}{4 \cdot \pi}} = 5.8002183$$

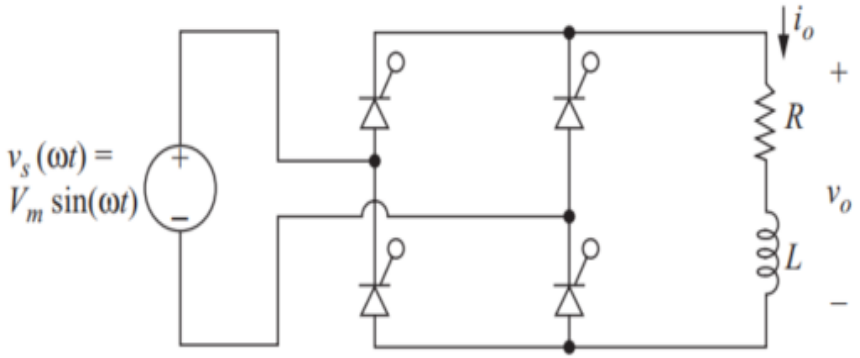
Power absrobed by the load $PR := Irms^2 \cdot R = 672.8506558$

Apparanet Power (volt-ampere) at the source $S := Vsrms \cdot Irms = 696.0262008$

power factor $pf := \frac{PR}{S} = 0.9667031$

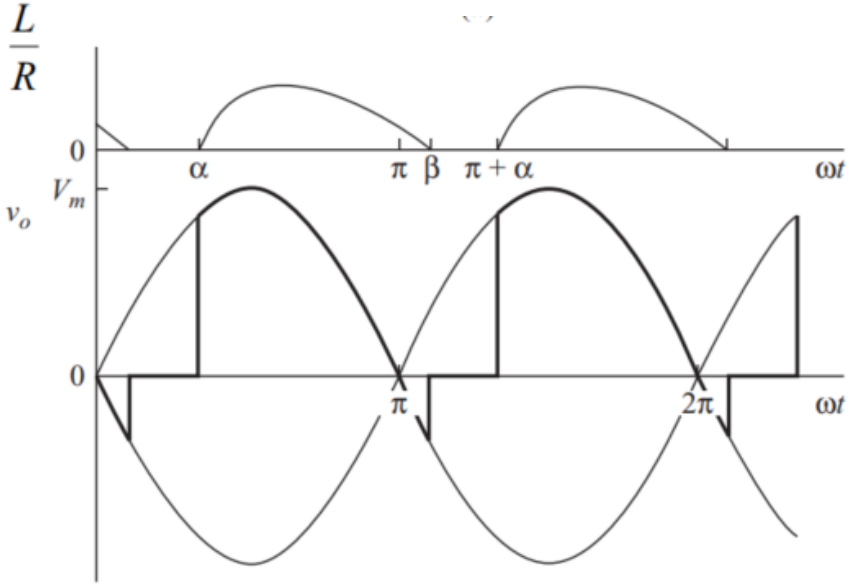
- R-L load case of controlled full-wave rectifier

- **1) DCM** analysis
- Similar to controlled ½ wave rectifier
- Load current:



$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$



$\beta < \alpha + \pi \rightarrow$ discontinuous current

- Same as in ½ wave controlled rectifier (except that the period is π instead of 2*π)

- Example 4-7

Compare with Example 3-11 of Controlled half-Wave rectifier

Controlled Full-Wave Rectifier, Discontinuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, $R = 10\ \Omega$, $L = 20$ mH, and $\alpha = 60°$. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

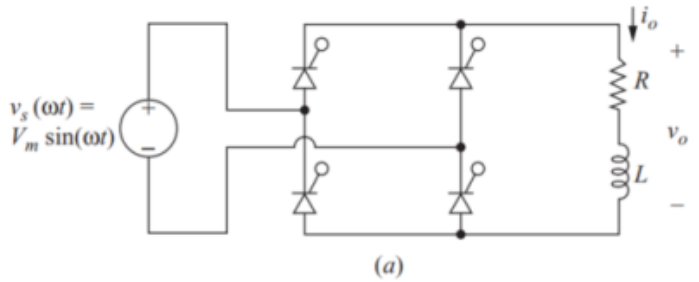
$f := 60$ $\quad Vsrms := 120$ $\quad R := 10$ $\quad L := 0.02$

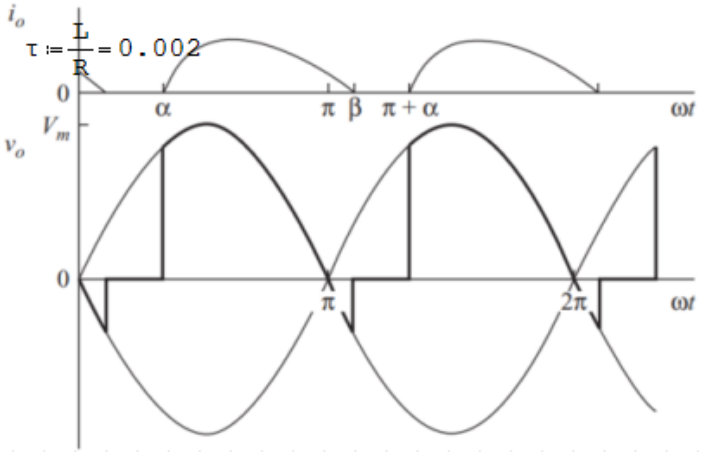
$w := 2 \cdot \pi \cdot f = 376.9911184$

$Vm := Vsrms \cdot \sqrt{2} = 169.7056275$ $\quad \theta := \operatorname{atan}\left(\frac{w \cdot L}{R}\right) = 0.6460449$

$Z := \sqrt{R^2 + (w \cdot L)^2} = 12.5239339$

Delay angle $\quad \alpha := 60 \cdot \frac{\pi}{180} = 1.0471976$ rad





From

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

$$i(wt) := \frac{Vm}{Z} \cdot \sin(wt - \theta) - \left(\frac{Vm}{Z} \cdot \sin(\alpha - \theta)\right) \cdot e^{\frac{(\alpha - wt)}{w \cdot \tau}}$$

$\frac{Vm}{Z} = 13.5505048$

$\sin(\alpha - \theta) = 0.3904798$

$\frac{Vm}{Z} \cdot \sin(\alpha - \theta) = 5.291198$

$\exp\left(\frac{\alpha}{w \cdot \tau}\right) = 4.0103916$

Extinction angle

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha-\beta)/\omega\tau}\right]$$

$$g(x) := \frac{Vm}{Z}\cdot\left(\sin(x-\theta) - \sin(\alpha-\theta)\cdot e^{\frac{(\alpha-x)}{w\cdot\tau}}\right)$$

$$beta := solve(g(x), x, \pi, 2\cdot\pi)$$

$$\beta := beta = 3.7771872$$

$$\beta\cdot\frac{180}{\pi} = 216.416885$$

Average current

$$Io := \frac{1}{\pi}\cdot\int_{\alpha}^{\beta} i(wt)\,d(wt) = 7.0479582$$

RMS current

$$Irms := \sqrt{\frac{1}{\pi}\cdot\int_{\alpha}^{\beta} i(wt)^2\,d(wt)} = 8.3494234$$

Load power

$$P := Irms^2\cdot R = 697.1287057$$

Source apparent Power

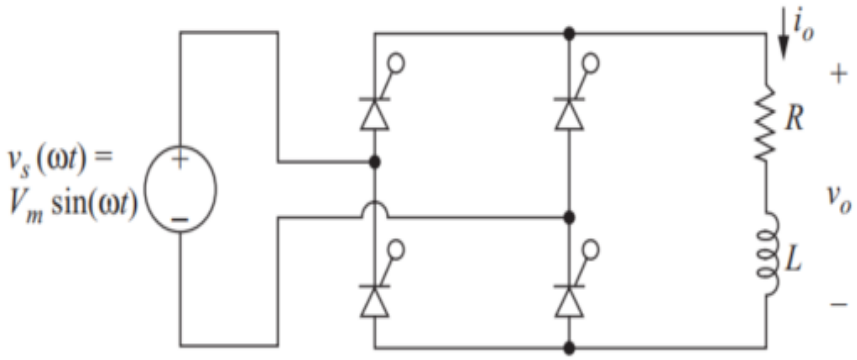
$$S := Vsrms\cdot Irms = 1001.9308041$$

power factor

$$pf := \frac{P}{S} = 0.6957853$$

- R-L load case of controlled full-wave rectifier
  - **2) CCM** analysis
  - Load current still positive at wt = π + α

$$i(\pi + \alpha) \geq 0$$



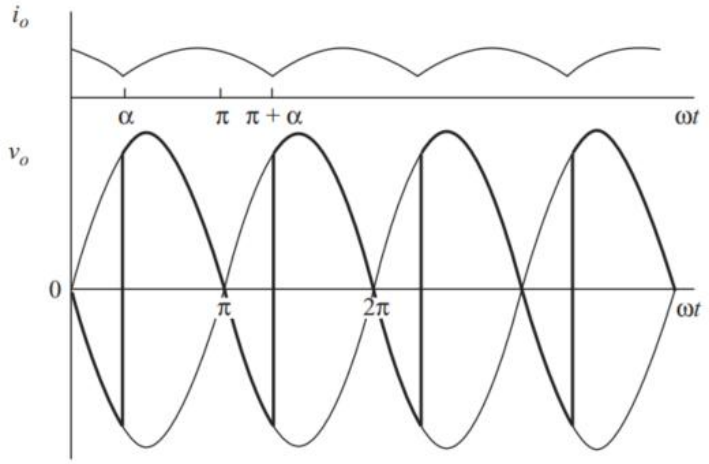
  - Applying to the current eqn

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta)\, e^{-(\pi+\alpha-\alpha)/\omega\tau} \geq 0$$

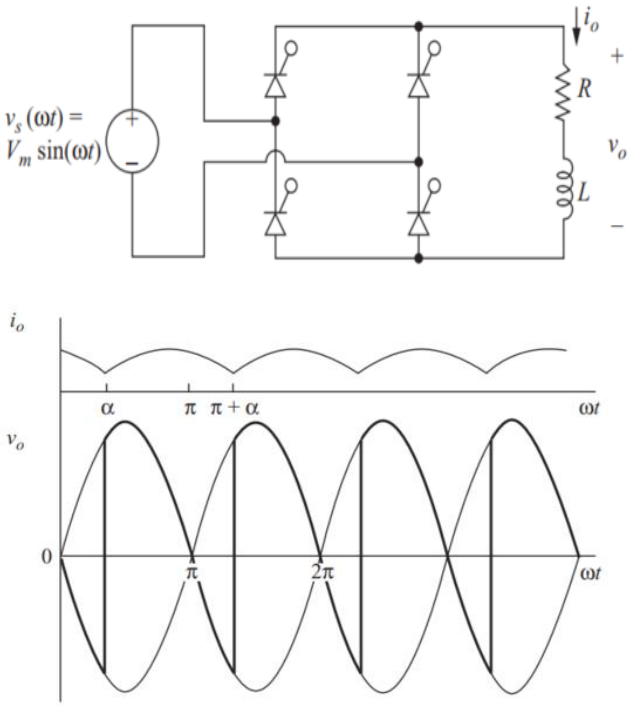
  - Therefore, CCM condition

$$\alpha \leq \theta \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\boxed{\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{for continuous current}}$$



- R-L load case of controlled full-wave rectifier
  - **2) CCM** analysis – continued
  - Analysis of output voltage and current → Fourier Analysis
  - Voltage waveform:



$$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

  - Average (dc) voltage:

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha$$

  - Amplitudes of harmonic terms

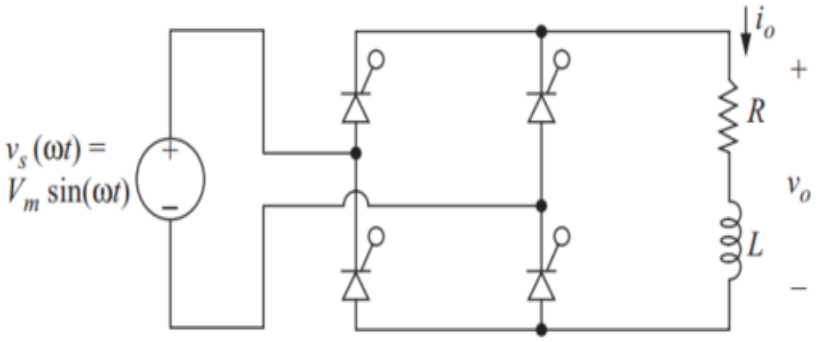
$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

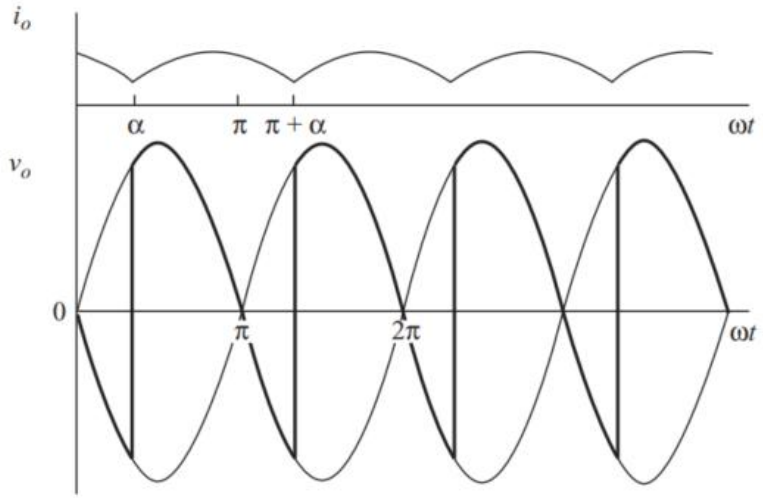
$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

- R-L load case of controlled full-wave rectifier
  - **2) CCM** analysis – continued
  - Analysis of output voltage and current → Fourier Analysis



  - Average current: $I_o = \frac{V_o}{R}$



  - Amplitude of harmonic current:

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega_0 L|}$$

  - RMS current:

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

- **Example 4-8**

Controlled Full-Wave Rectifier with *RL* Load, Continuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, an *RL* load where $R = 10\ \Omega$ and $L = 100$ mH. The delay angle $\alpha = 60°$ (same as Example 4-7 except L is larger). (*a*) Verify that the load current is continuous. (*b*) Determine the dc (average) component of the current. (*c*) Determine the power absorbed by the load.

$f := 60$ $Vsrms := 120$ $R := 10$ $L := 0.1$

$\tau := \frac{L}{R} = 0.01$

$w := 2 \cdot \pi \cdot f = 376.9911184$

$Vm := Vsrms \cdot \sqrt{2} = 169.7056275$ $\theta := \operatorname{atan}\left(\frac{w \cdot L}{R}\right) = 1.3115093$

Delay angle $\alpha := 60 \cdot \frac{\pi}{180} = 1.0471976$ rad

CCM check: $\alpha - \theta = -0.2643118$ $\alpha < \theta$; therefoe CCM

Average (dc) voltage

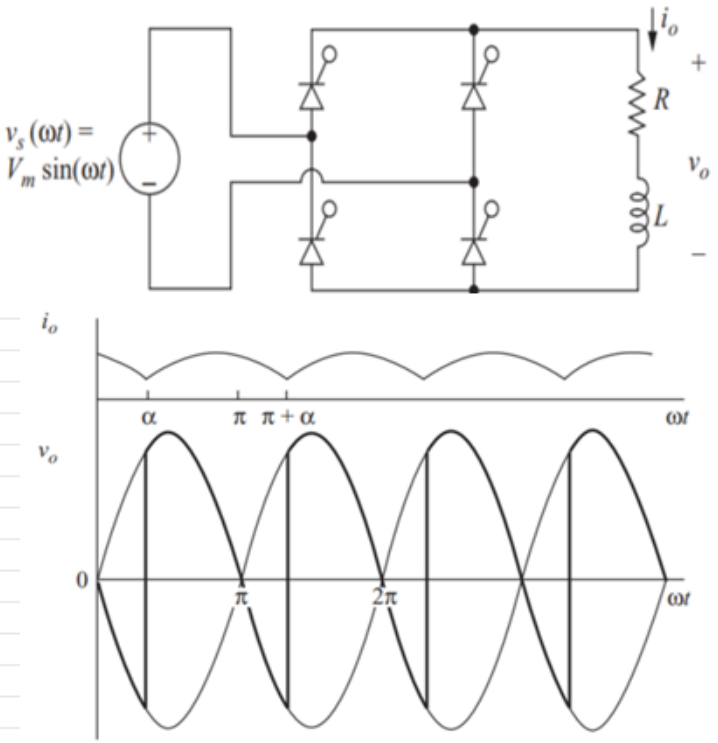
$Vo := \frac{2 \cdot Vm}{\pi} \cdot \cos(\alpha) = 54.018979$

Amplitude of harmonics

$Z(n) := |R + i \cdot n \cdot w \cdot L|$

$a(n) := \frac{2 \cdot Vm}{\pi} \cdot \left(\frac{\cos((n+1) \cdot \alpha)}{n+1} - \frac{\cos((n-1) \cdot \alpha)}{n-1} \right)$ $b(n) := \frac{2 \cdot Vm}{\pi} \cdot \left(\frac{\sin((n+1) \cdot \alpha)}{n+1} - \frac{\sin((n-1) \cdot \alpha)}{n-1} \right)$

$V(n) := \sqrt{a(n)^2 + b(n)^2}$ $I(n) := \frac{V(n)}{Z(n)}$



$$Io := \frac{Vo}{R} = 5.4018979$$ Average current

Amplitudes of harmonic currents

$$I2 := I(2) = 1.7071794$$

$$I4 := I(4) = 0.3336101$$

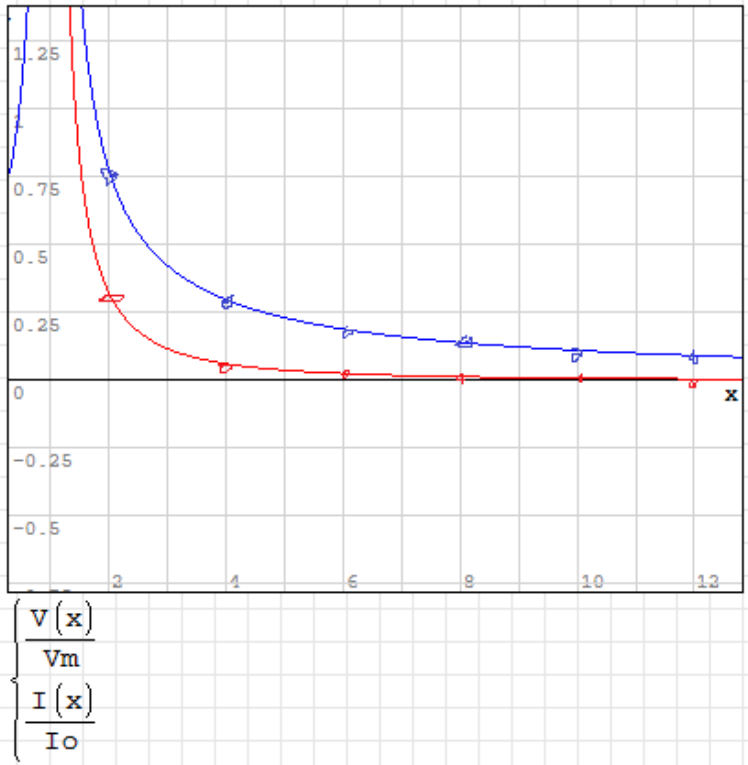
$$I6 := I(6) = 0.1423361$$

RMS current

$$Irms := \sqrt{Io^2 + \left(\frac{I2}{\sqrt{2}}\right)^2 + \left(\frac{I4}{\sqrt{2}}\right)^2 + \left(\frac{I6}{\sqrt{2}}\right)^2} = 5.5410747$$

Power Absorbed by the resistor

$$PR := Irms^2 \cdot R = 307.0350924$$



- **Example 4-9 (simulation of Example 4-8) (using vSW instead of SCR)**