Chapter 3

HALF-WAVE RECTIFIERS
(The basics of Analysis)

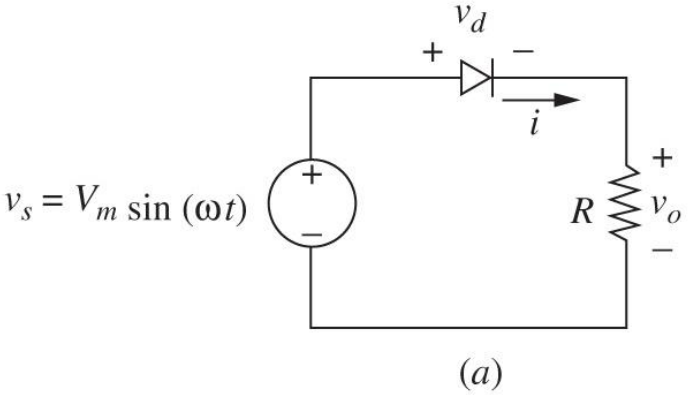
HALF-WAVE RECTIFIERS

- R LOAD
- R-L LOAD
- LTSPICE SIMULATION

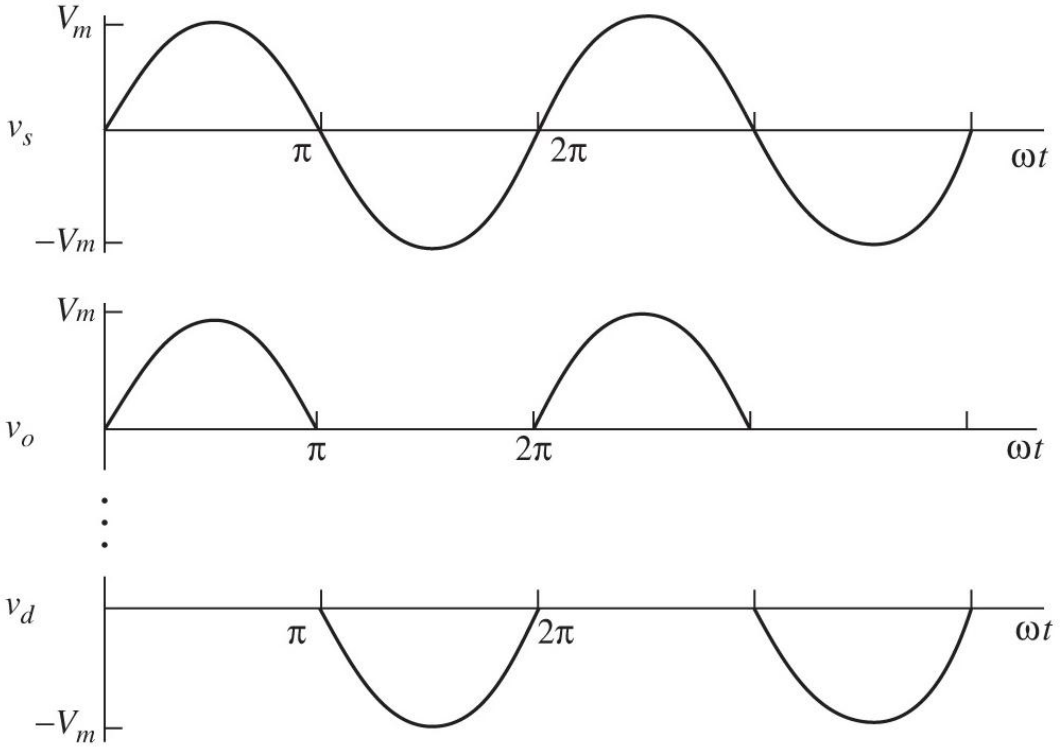
- THE FREEWHEELING DIODE
- HALF-WAVE RECTIFIER WITH A CAPACITOR FILTER
- CONTROLLED HALF-WAVE RECTIFIER
- LTSPICE SOLUTIONS FOR CONTROLLED RECTIFIERS

Figure 3.1

THE HALF-WAVE RECTIFIER – RESISTIVE LOAD



(a)



(b)

$$V_o = V_{avg} = \frac{1}{2\pi}\int_0^T V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}$$

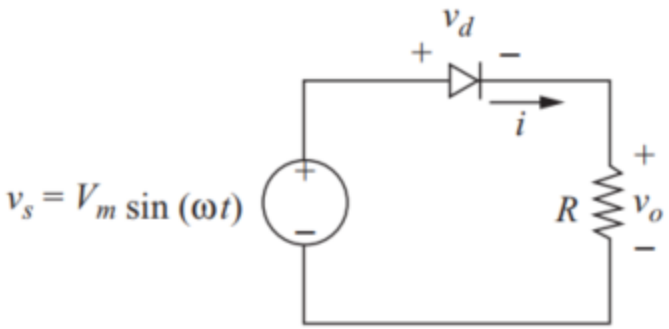
$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2}$$

$$I_{rms} = \frac{V_m}{2R}$$

- Example 3-1

For the half-wave rectifier of Fig. 3-1*a*, the source is a sinusoid of 120 V rms at a frequency of 60 Hz. The load resistor is 5 Ω. Determine (*a*) the average load current, (*b*) the average power absorbed by the load and (*c*) the power factor of the circuit.

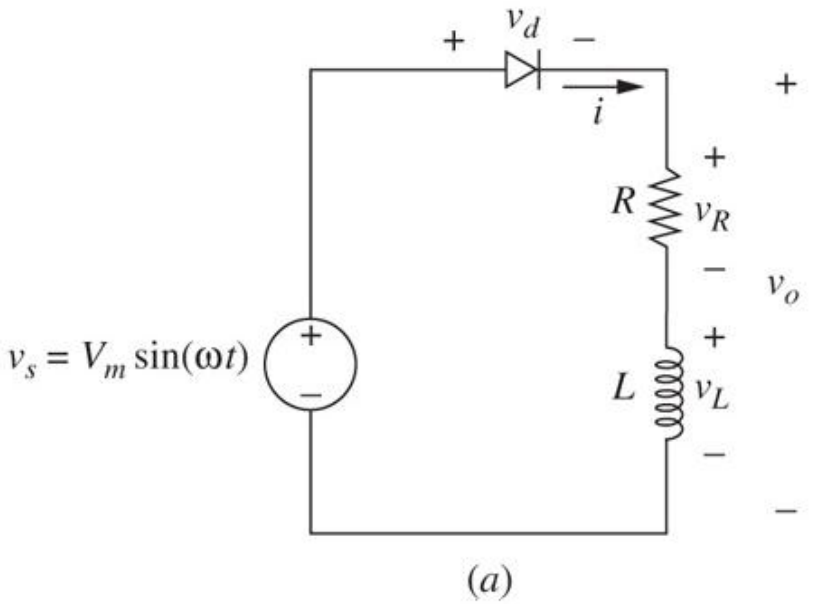


(*a*)

Figure 3.2

R-L LOAD

Positive Half-cycle: forward-biased



(a)

Kirchhoff's Voltage Law:

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt}$$

$$i(t) = i_f(t) + i_n(t)$$

$$i_f(t) = \left(\frac{V_m}{Z}\right)\sin(\omega t - \theta)$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

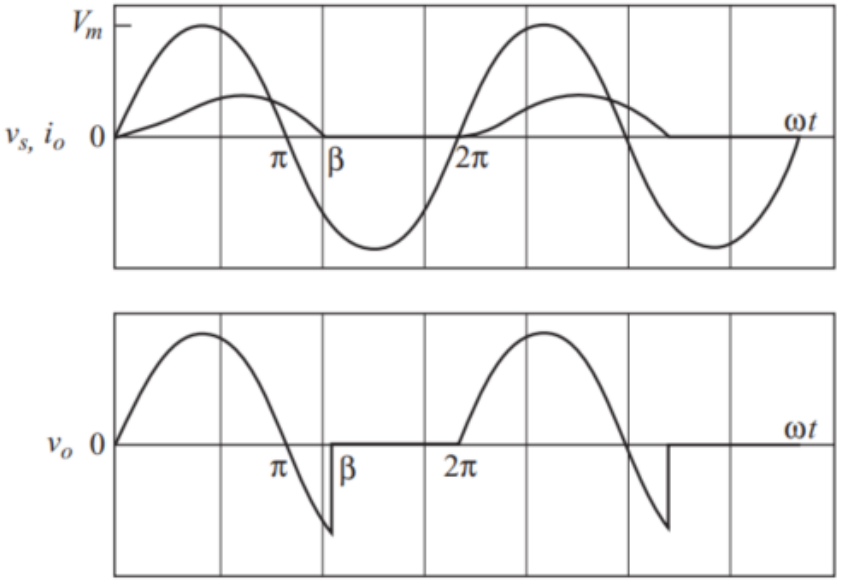
$$i_n(t) = A e^{-t/\tau}$$

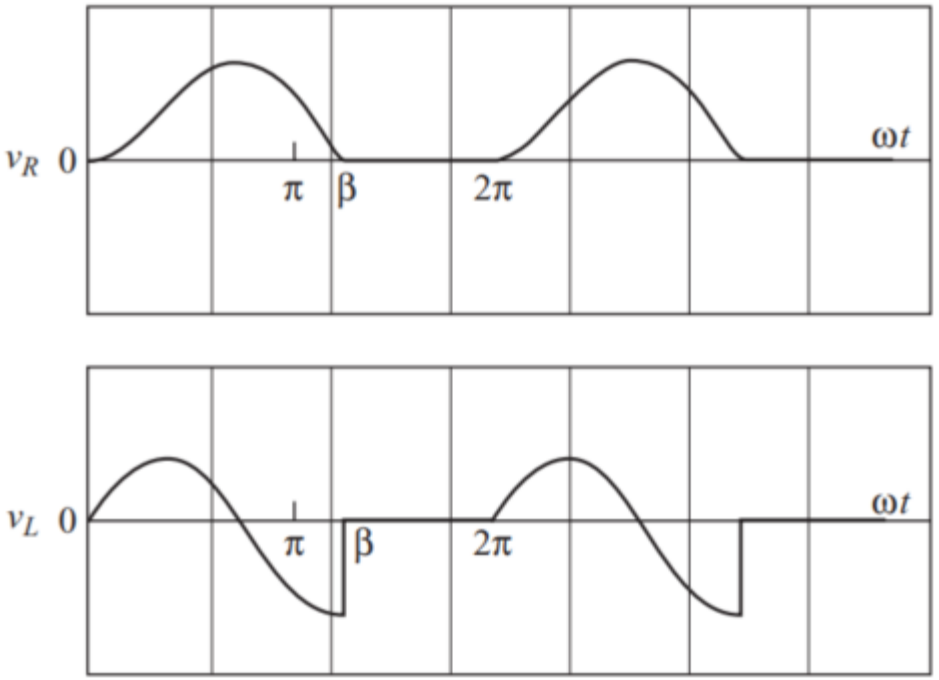
$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

Apply the initial condition to get coefficient A

In terms of angle (wt)



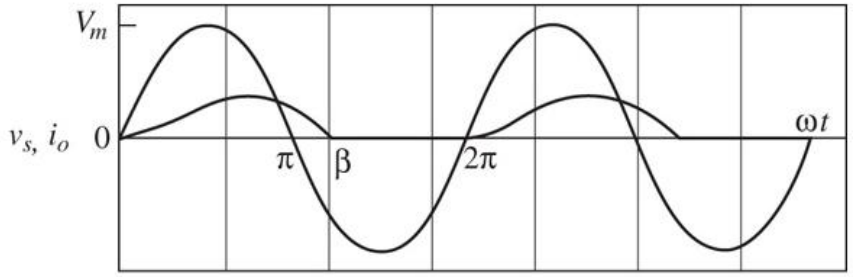


- Diode remains forward-biased longer than π (up to β [extinction angle])
  - Current does not stop abruptly in inductor(L)
- The output voltage is negative from π to β

The current goes to zero at angle β

$$i(\beta) = \frac{V_m}{Z}\left[\sin(\beta-\theta)+\sin(\theta)e^{-\beta/\omega\tau}\right]=0$$

$$\sin(\beta-\theta)+\sin(\theta)e^{-\beta/\omega\tau}=0$$



Solution for β? → Transcendental equation → 'solve" in Smath

$R := 100 \qquad L := 0.1$

$\tau := \frac{L}{R} = 0.001$

$\varphi := \text{atan}\left(\frac{w \cdot L}{R}\right) = 0.3605$

$g(\beta) := \sin(\beta-\varphi)+\sin(\varphi)\cdot\exp\left(-\frac{\beta\cdot R}{w\cdot L}\right)$

$\text{solve}(g(\beta),\beta,0,5)=\begin{bmatrix}0\\3.5021\end{bmatrix}$

$g2(\beta) := \sin(\beta-\varphi)+\sin(\varphi)\cdot\exp\left(-\frac{\beta}{w\cdot\tau}\right)$

$\text{beta} := \text{solve}(g2(\beta),\beta,0,2\cdot\pi)$

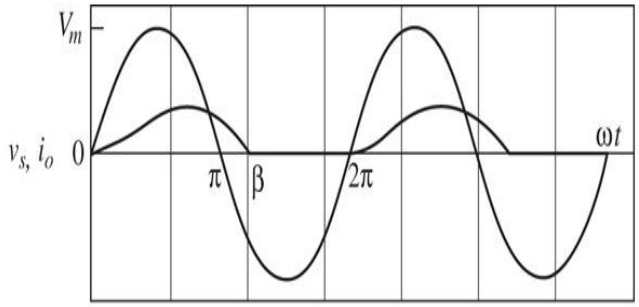
$\text{beta}=\begin{bmatrix}0\\3.5021404\end{bmatrix}$ rad

$\text{extAngle} := \text{beta}_{|2}\cdot\frac{180}{\pi}=200.6578643$ deg

Current in half-wave rectifier circuit:

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}$$

where $Z = \sqrt{R^2 + (\omega L)^2}$, $\theta = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$, and $= \dfrac{L}{R}$



RMS current:

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d(\omega t)}$$

Average current:

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\,d(\omega t)$$

Example 3-2

For the half-wave rectifier of Fig. 3-2*a*, $R = 100\ \Omega$, $L = 0.1$ H, $\omega = 377$ rad/s, and $V_m = 100$ V. Determine (*a*) an expression for the current in this circuit, (*b*) the average current, (*c*) the rms current, (*d*) the power absorbed by the *RL* load, and (*e*) the power factor.

$f := 60$ $\quad Vm := 100$

$w := 2 \cdot \pi \cdot f = 376.9911184$ $\quad$ RL circuit

$R := 100$ $\quad L := 0.1$

$v(wt) := Vm \cdot \sin(wt)$

$\tau := \frac{L}{R} = 0.001$ $\quad Z := \sqrt{R^2 + (w \cdot L)^2} = 106.8701223$

$\theta := \operatorname{atan}\left(\frac{w \cdot L}{R}\right) = 0.3605$

$g(x) := \sin(x - \theta) + \sin(\theta) \cdot \exp\left(-\frac{x}{w \cdot \tau}\right)$

$beta := \operatorname{solve}(g(x), x, 0, 2 \cdot \pi)$

$beta = \begin{bmatrix} 0 \\ 3.5021404 \end{bmatrix}$ rad

$\beta := beta_{|2} = 3.5021404$ rad $\quad \beta \cdot \frac{180}{\pi} = 200.6578643$ deg

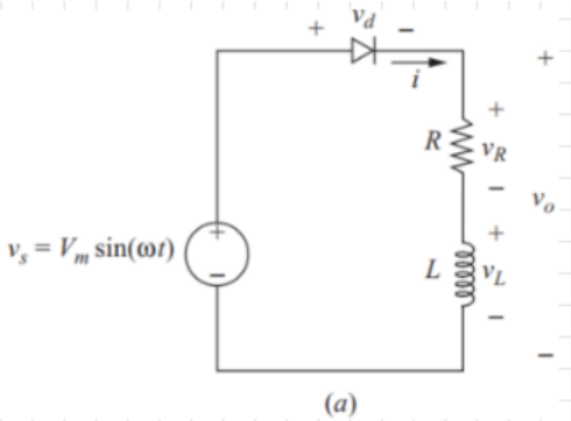
Equation for current $\quad i(wt) := \frac{Vm}{Z} \cdot \left(\sin(wt - \theta) + \sin(\theta) \cdot e^{-\frac{wt}{w \cdot \tau}}\right)$

Average current: $\quad Io := \frac{1}{2 \cdot \pi} \cdot \int_0^{\beta} i(wt)\, d\,wt = 0.3080768$

RMS current: $\quad Irms := \sqrt{\frac{1}{2 \cdot \pi} \cdot \int_0^{\beta} i(wt)^2\, d\,wt} = 0.4735832$

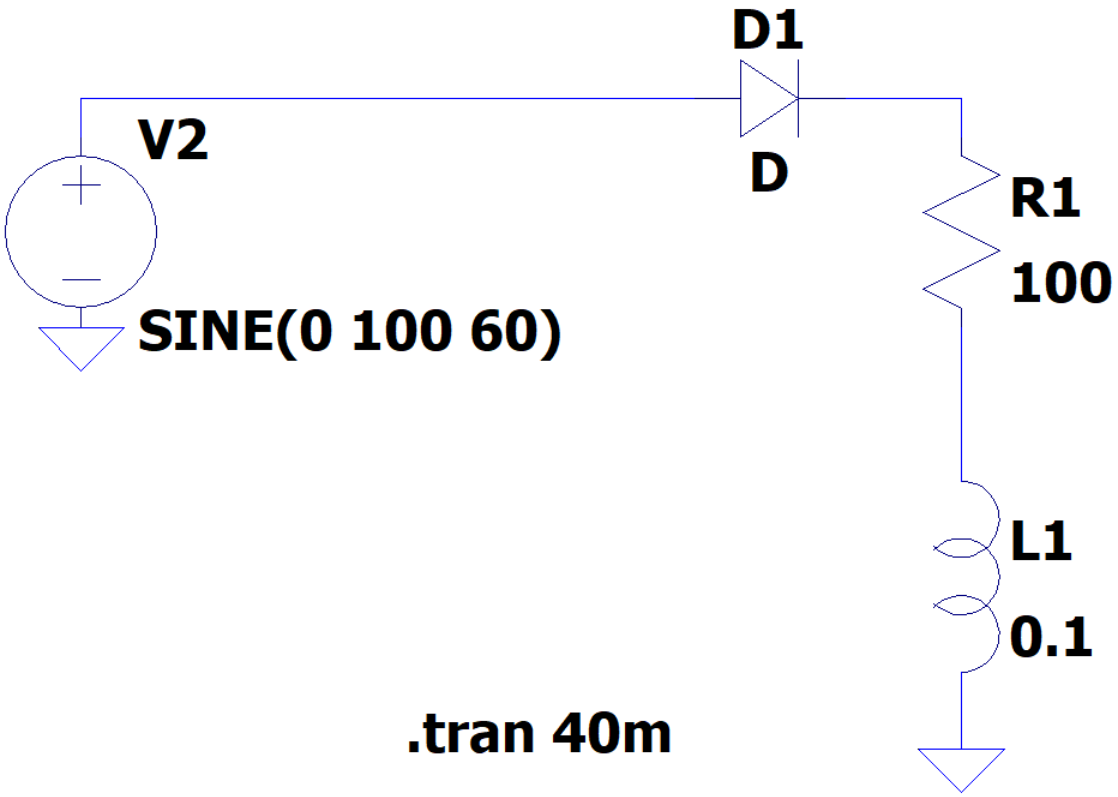
Power Absrobed by the resistor: $\quad P := Irms^2 \cdot R = 22.4281057$ $\quad P := \frac{1}{2 \cdot \pi} \cdot \int_0^{\beta} v(wt) \cdot i(wt)\, d\,wt = 22.4281056$

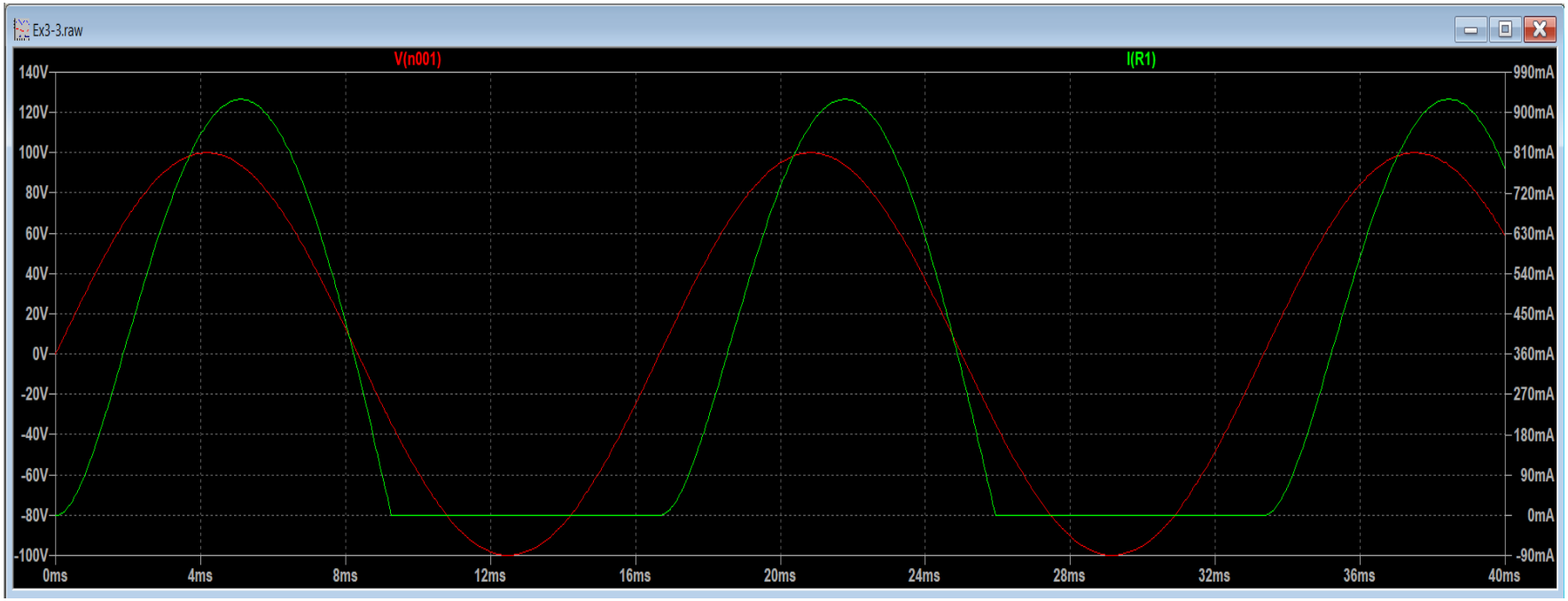
power factor: $\quad Vrms := \frac{Vm}{\sqrt{2}} = 70.7106781$ $\quad pf := \frac{P}{Vrms \cdot Irms} = 0.6697478$

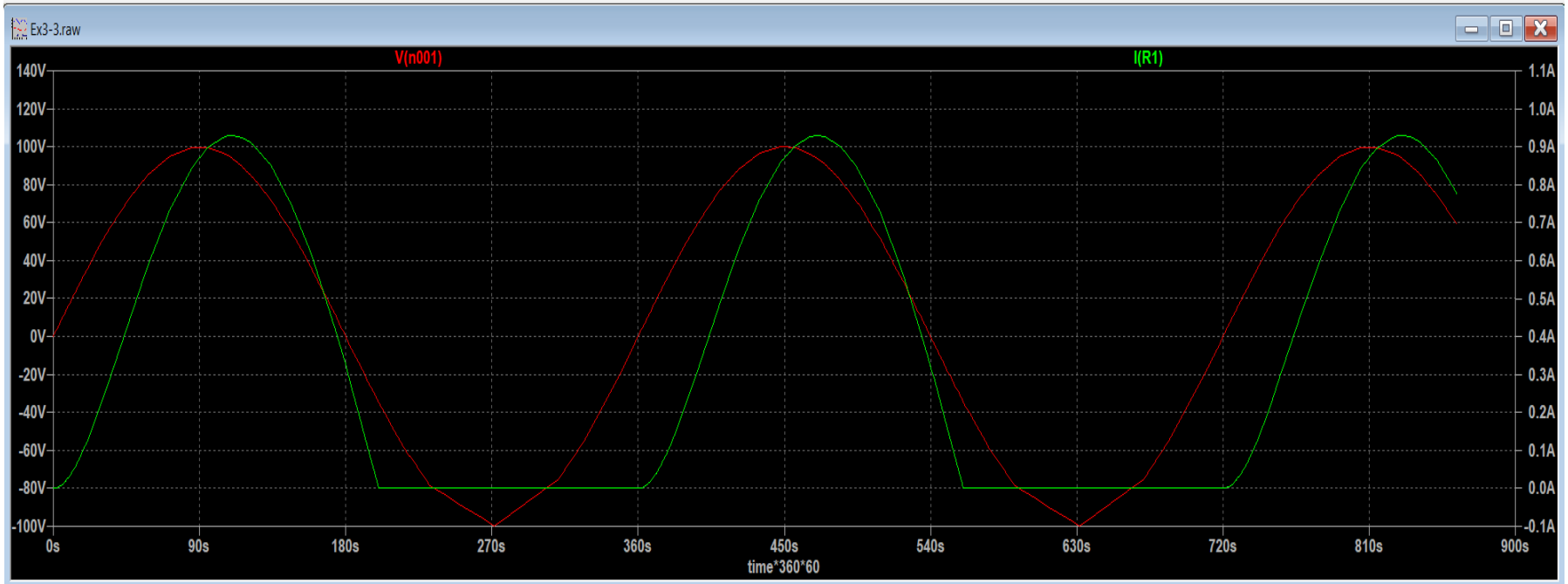


(*a*)

LTSpice simulation of Example 3-2

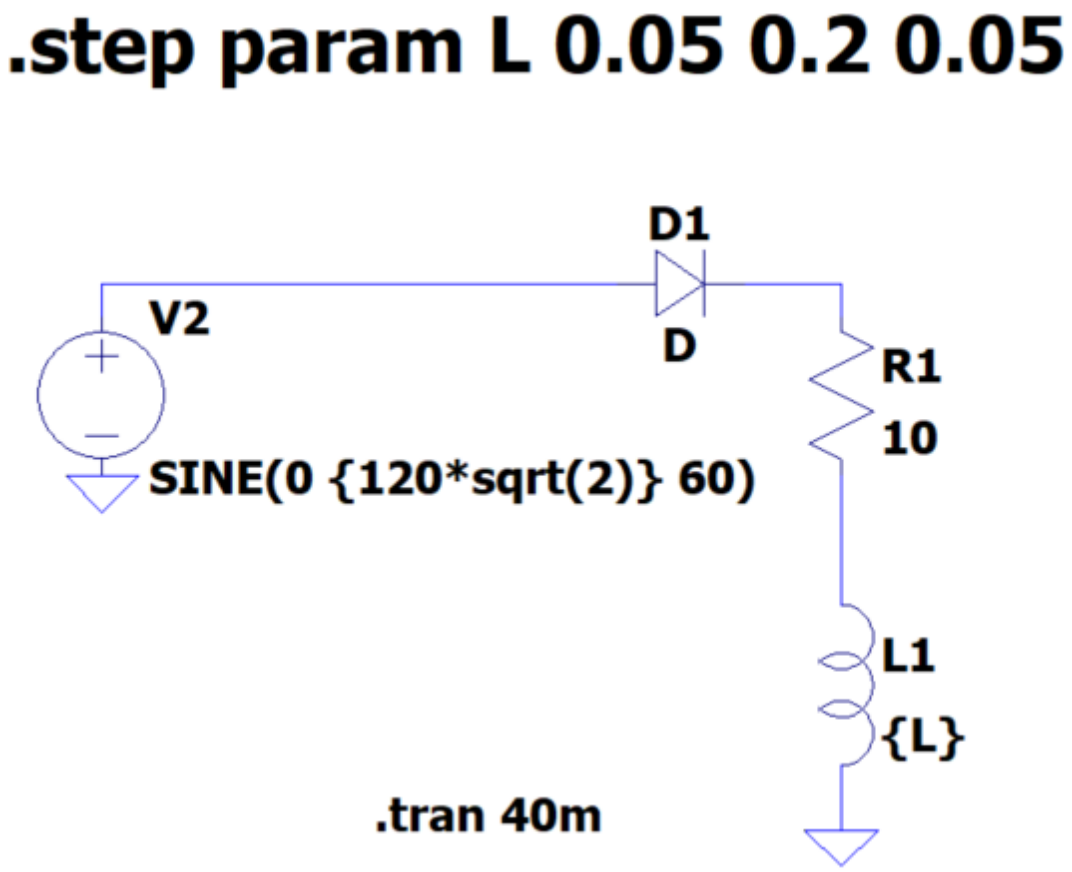



Ex3-3.raw
V(n001)
I(R1)
140V
120V
100V
80V
60V
40V
20V
0V
-20V
-40V
-60V
-80V
-100V
990mA
900mA
810mA
720mA
630mA
540mA
450mA
360mA
270mA
180mA
90mA
0mA
-90mA
0ms
4ms
8ms
12ms
16ms
20ms
24ms
28ms
32ms
36ms
40ms


Ex3-3.raw
V(n001)
I(R1)
140V
120V
100V
80V
60V
40V
20V
0V
-20V
-40V
-60V
-80V
-100V
1.1A
1.0A
0.9A
0.8A
0.7A
0.6A
0.5A
0.4A
0.3A
0.2A
0.1A
0.0A
-0.1A
0s
90s
180s
270s
360s
450s
540s
630s
720s
810s
900s
time*360*60

Example 3-4

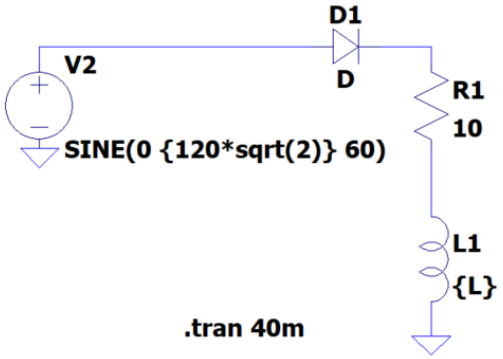
Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.

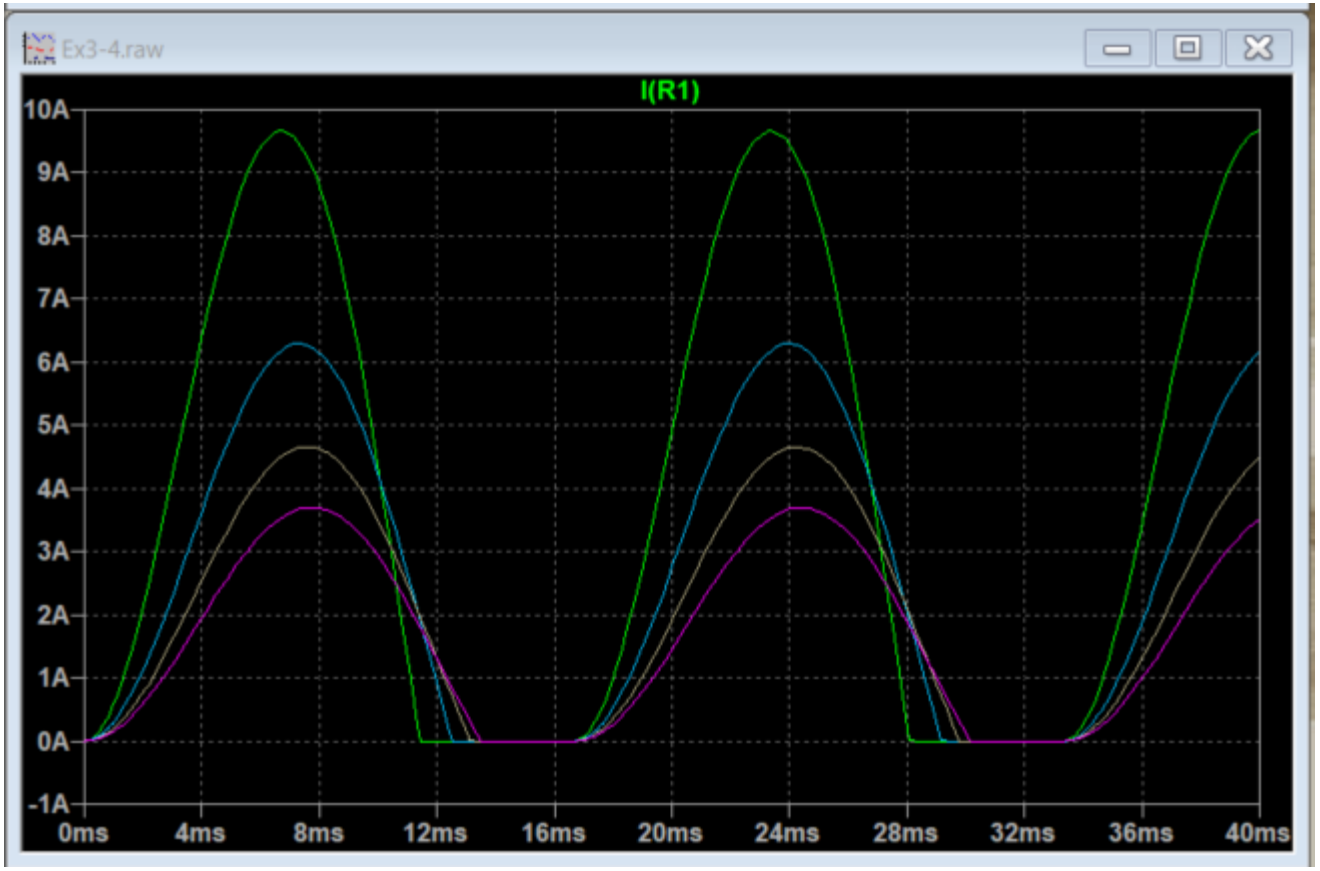


Example 3-4

Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.

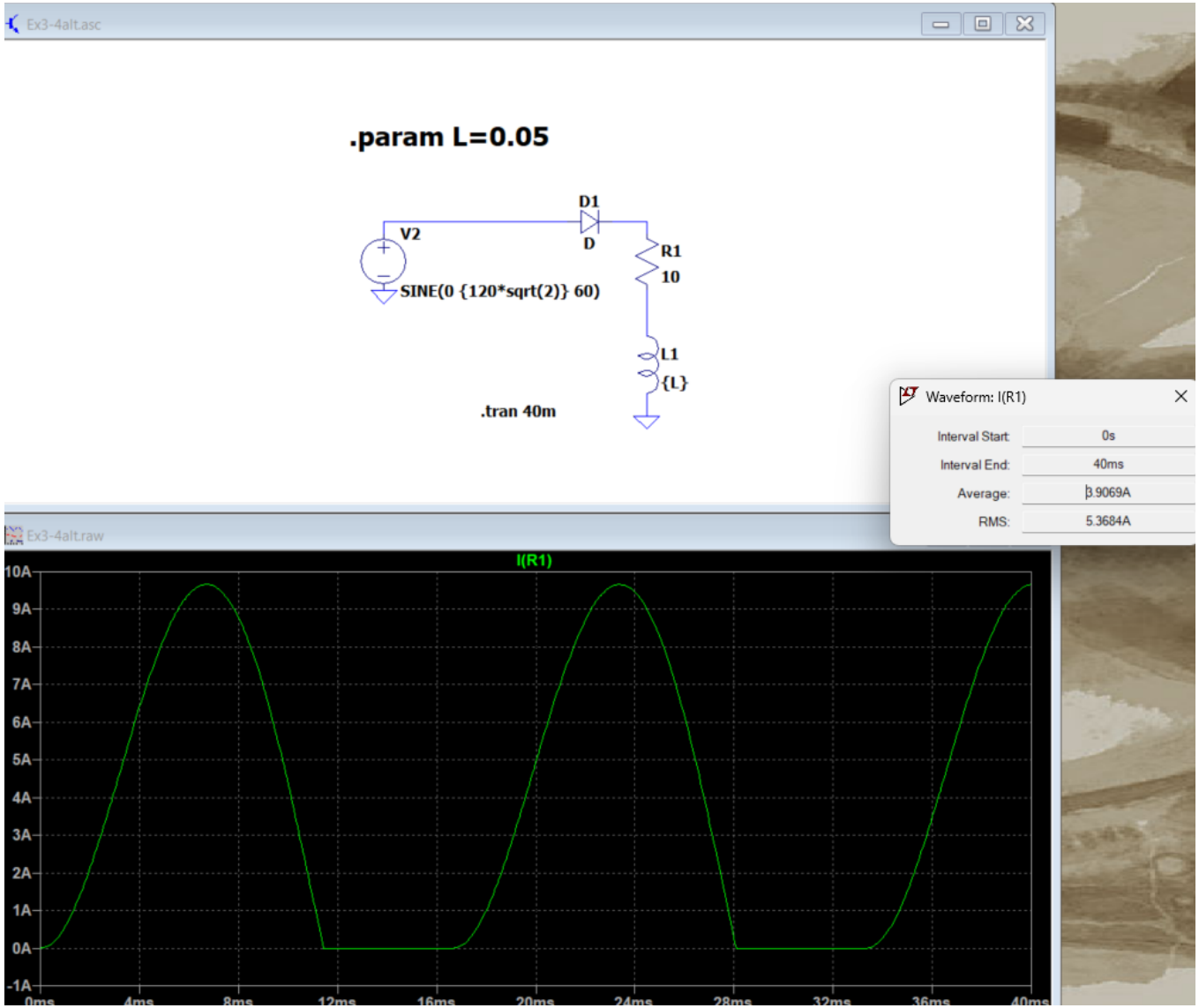
.step param L 0.05 0.2 0.05





Example 3-4 (without parametric sweep)

Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.



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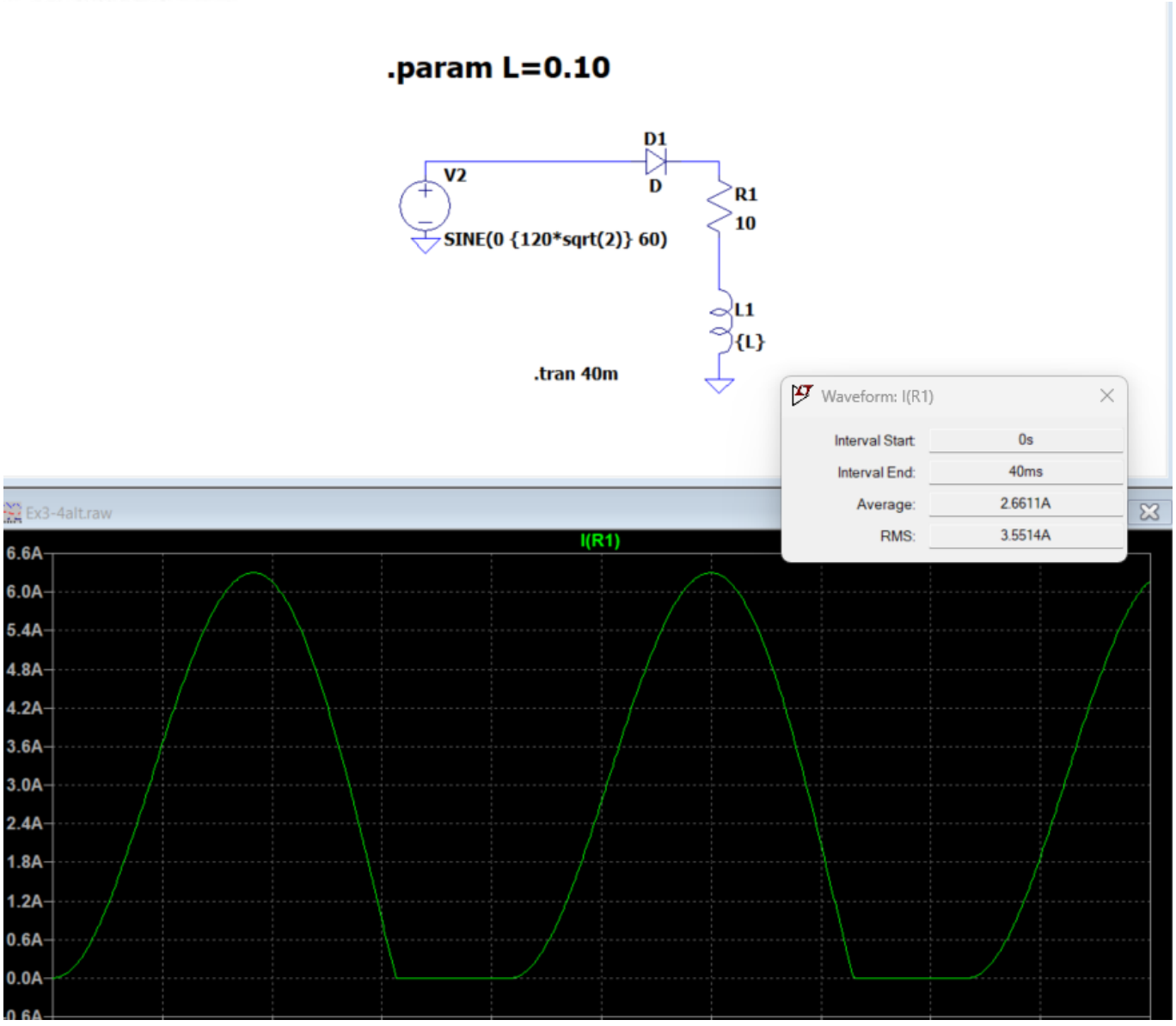
Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.



Example 3-4 (without parametric sweep)

Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.

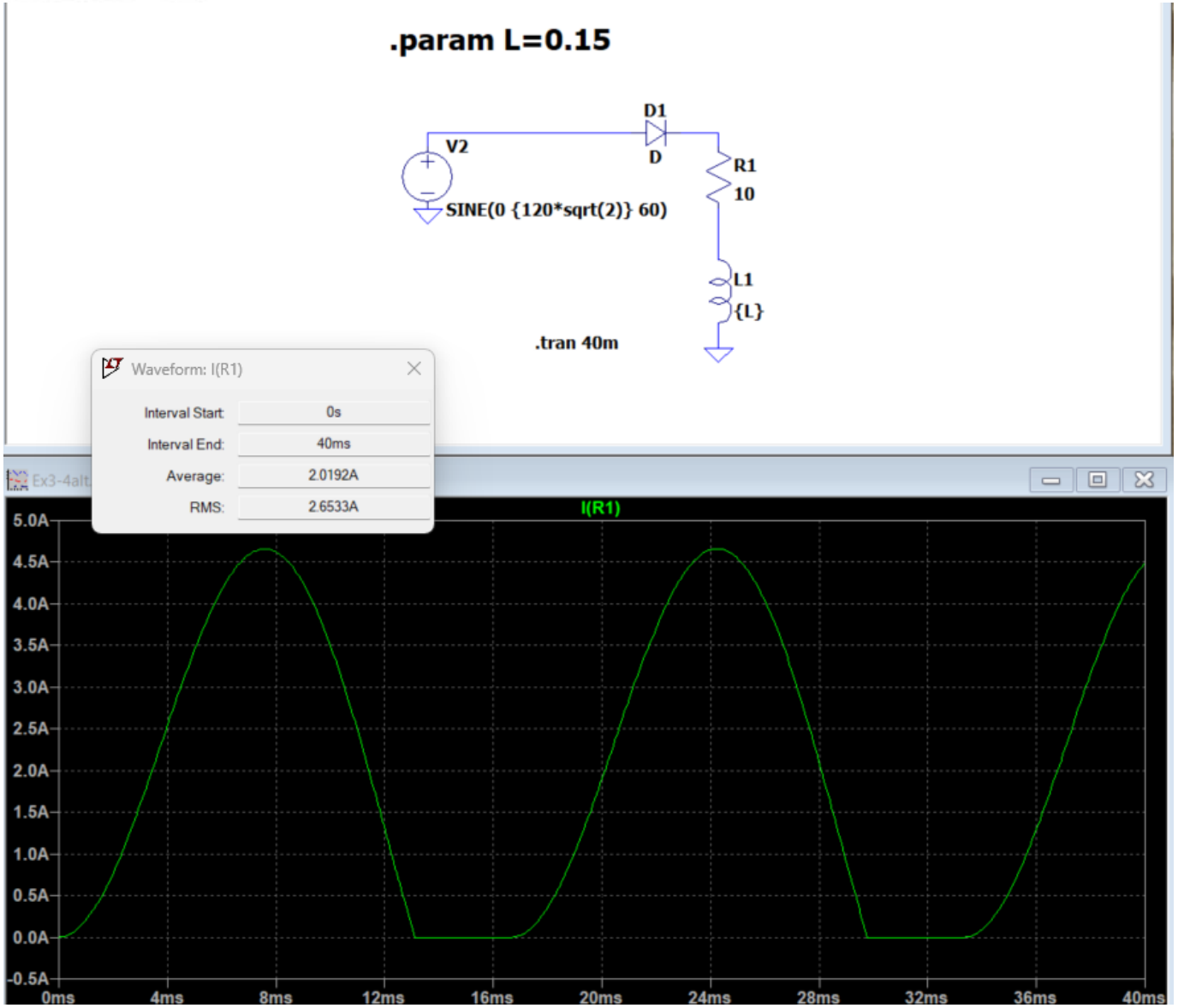
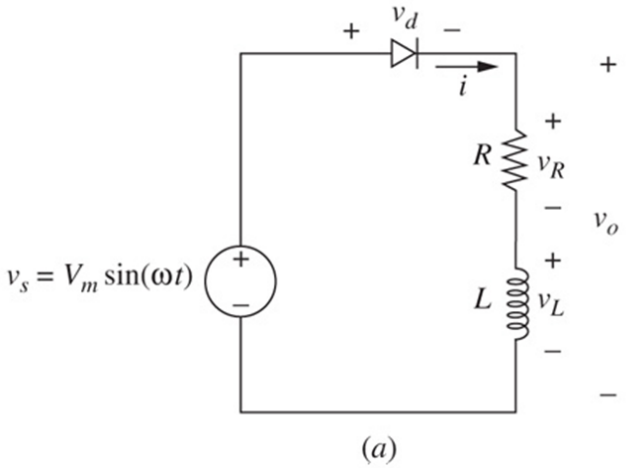


Figure 3.7

THE FREEWHEELING DIODE

Half-wave rectifier (WITHOUT freewheeling diode)



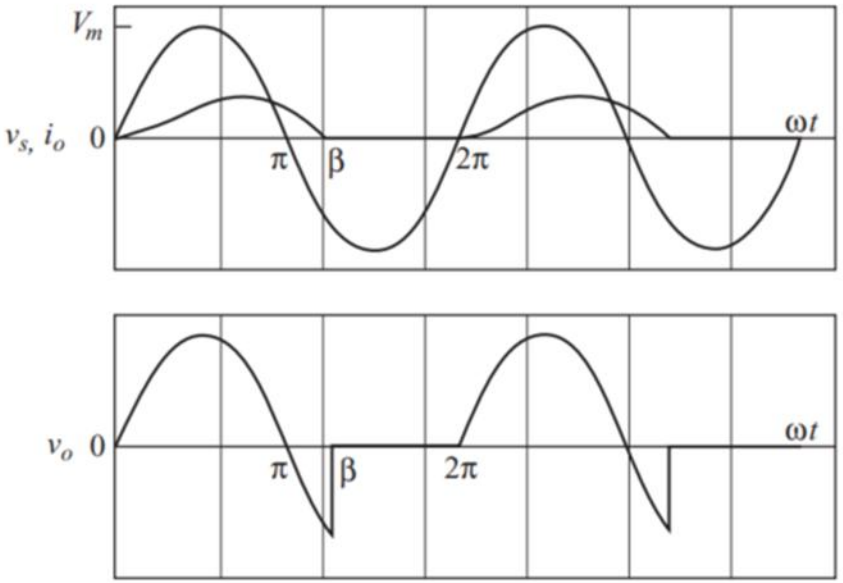
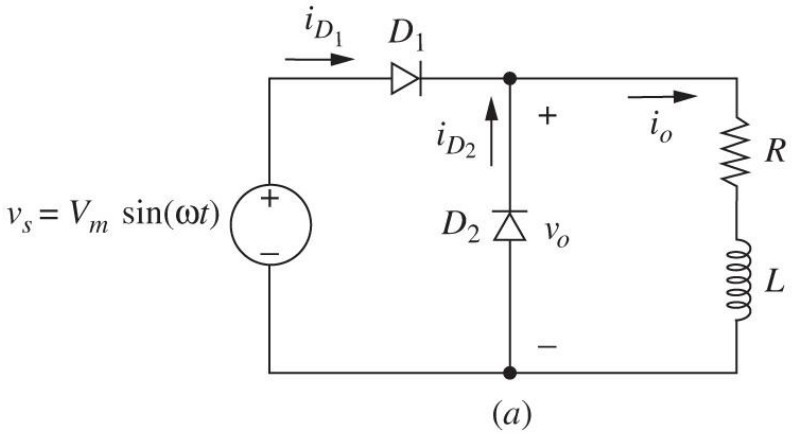
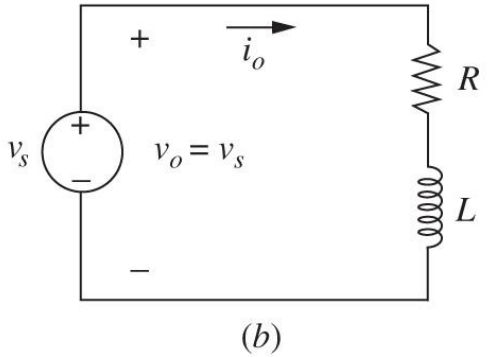


Figure 3.7

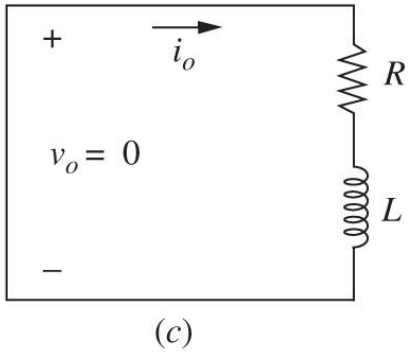
THE FREEWHEELING DIODE

- Vs>0: D1 ON & D2 OFF
- Vs<0: D1 OFF & D2 ON





D1 ON & D2 OFF



D1 OFF & D2 ON

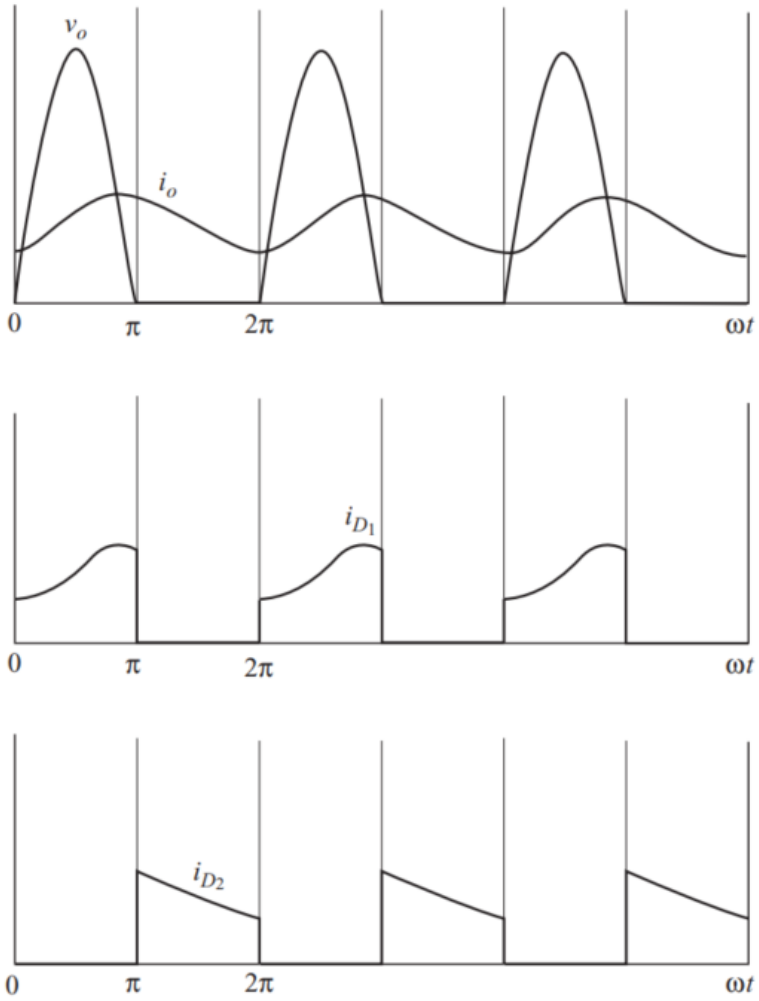


Figure 3-9 Steady-state load voltage and current waveforms with freewheeling diode.

Figure 3.8

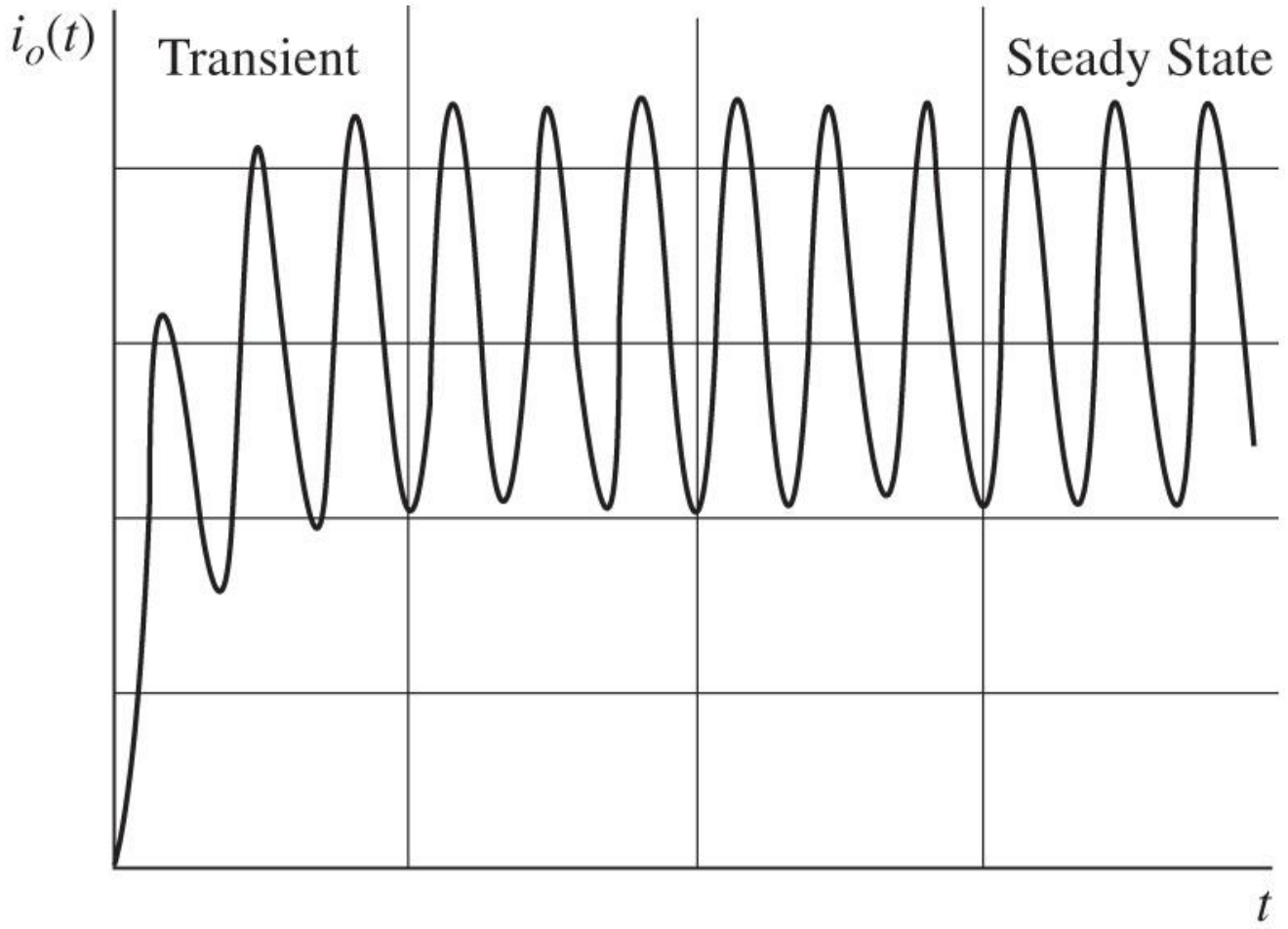
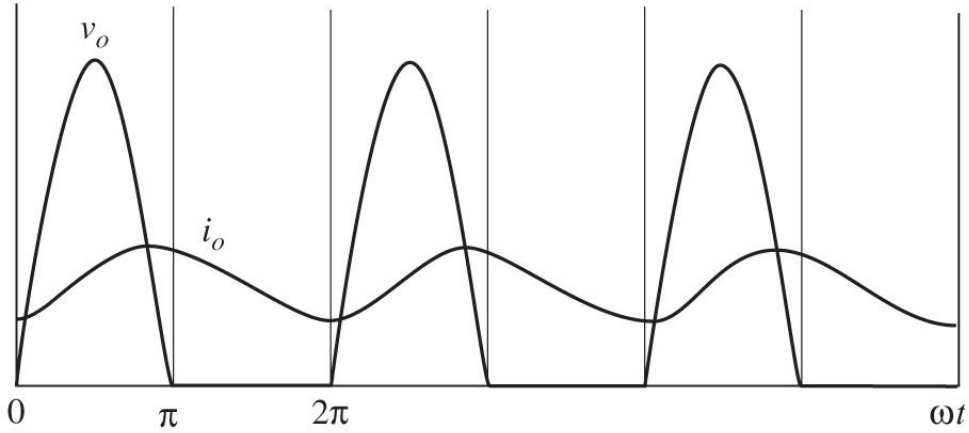


Figure 3.9

Fourier Series of the output voltage



$$v_o(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t)$$

ac current amplitude (by phasor analysis)

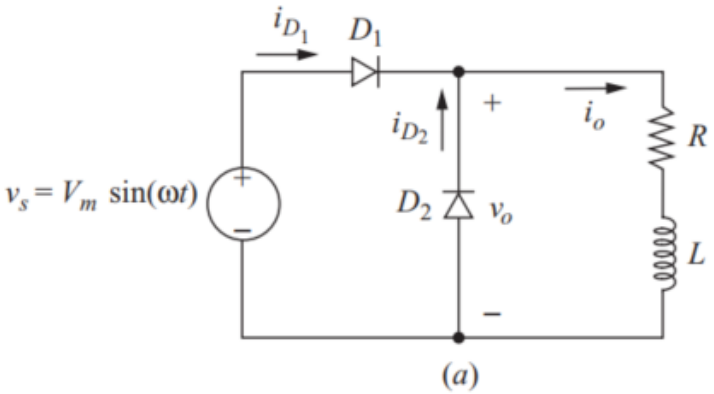
$$I_n = \frac{V_n}{Z_n}$$

where $Z_n = |R + jn\omega_0 L|$

- Example 3-7

Half-Wave Rectifier with Freewheeling Diode

Determine the average load voltage and current, and determine the power absorbed by the resistor in the circuit of Fig. 3-7*a*, where $R = 2\ \Omega$ and $L = 25$ mH, V_m is 100 V, and the frequency is 60 Hz.



(a)

$f := 60$ $Vm := 100$ $R := 2$ $L := 25 \cdot 10^{-3}$

$w := 2 \cdot \pi \cdot f = 376.9911184$ $\tau := \frac{L}{R} = 0.0125$

$$v_o(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t)$$

$V0 := \frac{Vm}{\pi} = 31.8309886$

$I0 := \frac{V0}{R} = 15.9154943$

Find current from Fourier Series (Amplitudes)

$V1 := \frac{Vm}{2} = 50$

$V(n) := \frac{2 \cdot Vm}{(n^2 - 1)\cdot \pi}$

$V2 := V(2) = 21.2206591$

$V4 := V(4) = 4.2441318$

$V6 := V(6) = 1.8189136$

Impedance Z:

$Z(n) := \sqrt{R^2 + (n \cdot w \cdot L)^2}$

$Z0 := Z(0) = 2$

$Z1 := Z(1) = 9.6346479$

$Z2 := Z(2) = 18.9553623$

$Z4 := Z(4) = 37.7521262$

$Z6 := Z(6) = 56.5840245$

- Example 3-7

Current

$$I(n) := \frac{V(n)}{Z(n)}$$

$$I0 := \frac{V0}{Z0} = 15.9154943 \qquad I1 := \frac{V1}{Z1} = 5.1896033$$

$$I2 := I(2) = 1.1195069$$

$$I4 := I(4) = 0.112421 \qquad I6 := I(6) = 0.0321454$$

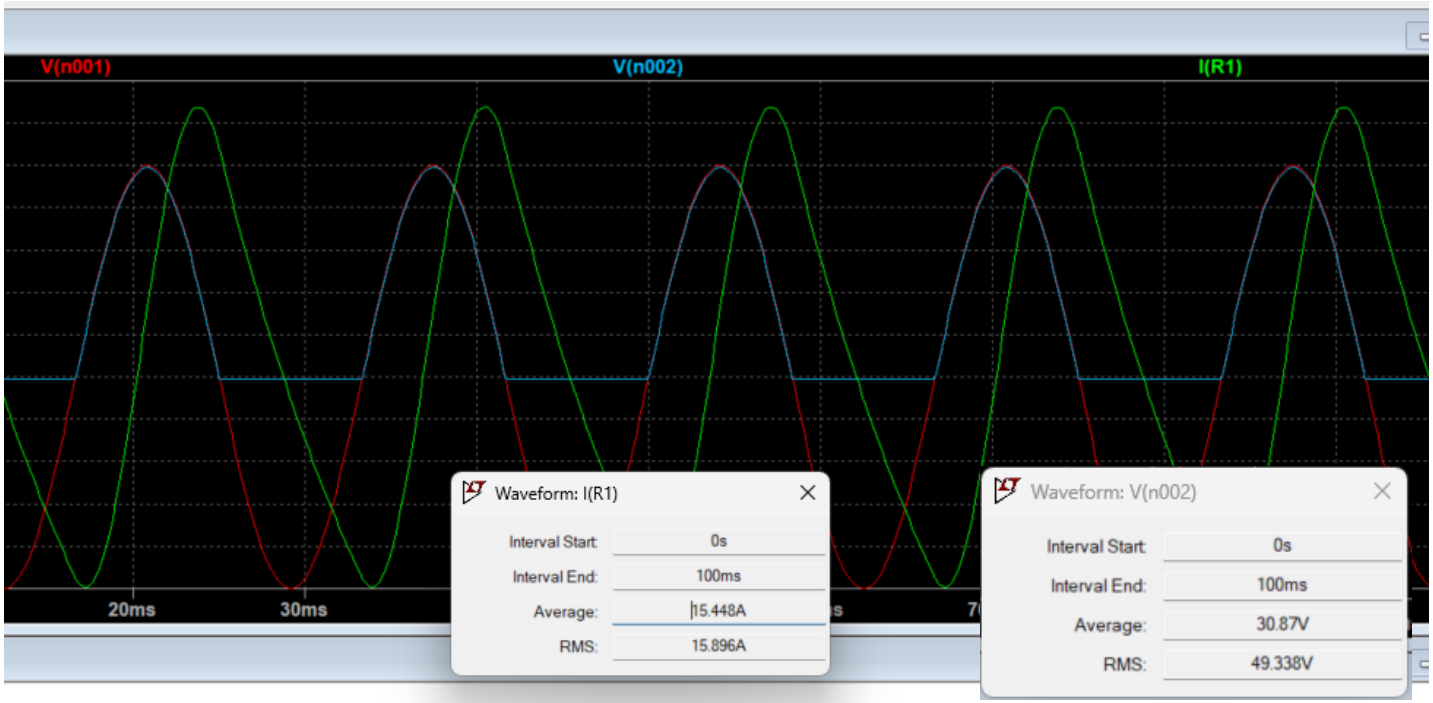
RMS current

$$Irms := \sqrt{I0^2 + \left(\frac{I1}{\sqrt{2}}\right)^2 + \left(\frac{I2}{\sqrt{2}}\right)^2 + \left(\frac{I4}{\sqrt{2}}\right)^2 + \left(\frac{I6}{\sqrt{2}}\right)^2} = 16.3524443$$

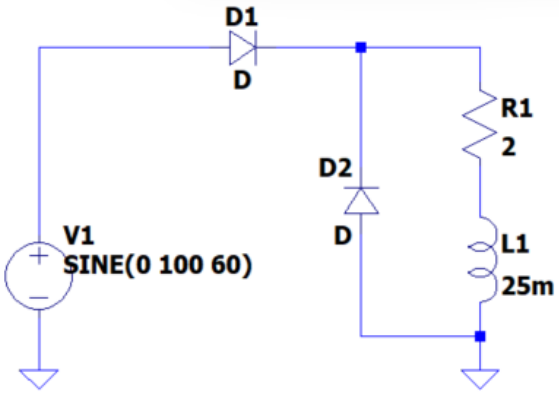
Power absorbed by the resistor

$$P := Irms^2 \cdot R = 534.8048677$$

- Example 3-7 in LTSpice



.four 60Hz 10 I(R1)



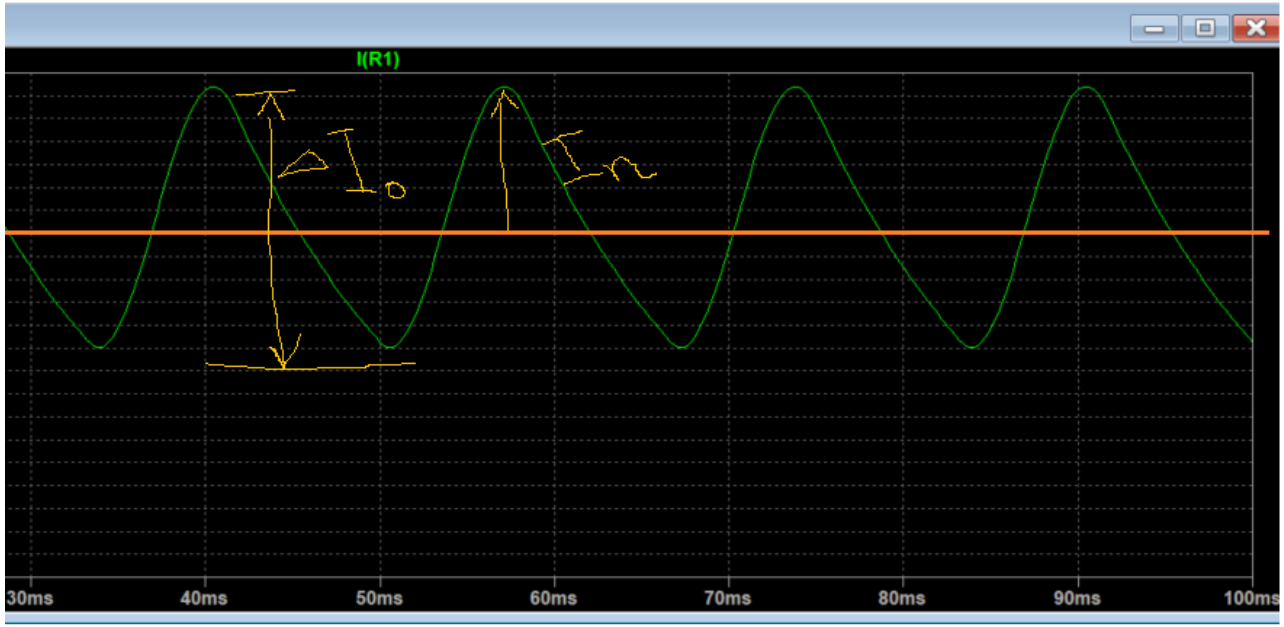
.tran 0 200m 100m

- Example 3-7 in LTSpice

SPICE Error Log: C:\bE2025\TF25\PowerElectronics\LT\Ex3-7.log

```
Circuit: * C:\bE2025\TF25\PowerElectronics\LT\Ex3-7.asc

.OP point found by inspection.
N-Period=1
Fourier components of I(r1)
DC component:15.4483

Harmonic     Frequency     Fourier     Normalized    Phase       Normalized
 Number        [Hz]       Component     Component   [degree]     Phase [deg]
    1       6.000e+01    5.178e+00    1.000e+00     -78.01°        0.00°
    2       1.200e+02    1.111e+00    2.146e-01    -173.93°      -95.93°
    3       1.800e+02    6.171e-04    1.192e-04     -87.10°       -9.09°
    4       2.400e+02    1.093e-01    2.110e-02    -176.85°      -98.84°
    5       3.000e+02    5.991e-04    1.157e-04     -93.30°      -15.30°
    6       3.600e+02    3.020e-02    5.832e-03    -177.55°      -99.54°
    7       4.200e+02    5.850e-04    1.130e-04     -97.37°      -19.37°
    8       4.800e+02    1.204e-02    2.324e-03    -177.46°      -99.45°
    9       5.400e+02    5.674e-04    1.096e-04    -100.87°      -22.86°
   10       6.000e+02    5.811e-03    1.122e-03    -176.83°      -98.82°
Total Harmonic Distortion: 21.568037%(21.568226%)



Date: Thu Aug 28 11:59:42 2025
Total elapsed time: 0.046 seconds.

tnom = 27
temp = 27
method = modified trap
```

- How can we reduce the Load Current harmonics?



$I_{AVG} = I_o$

$$I(n) := \frac{V(n)}{Z(n)}$$

$$Z(n) := \sqrt{R^2 + (n \cdot w \cdot L)^2}$$

$I0 := \frac{V0}{Z0} = 15.9154943$ $\quad I1 := \frac{V1}{Z1} = 5.1896033$

$I2 := I(2) = 1.1195069$ $\quad I4 := I(4) = 0.112421$ $\quad I6 := I(6) = 0.0321454$

- Increasing Zn (which means increasing L since R is the load) would reduce Harmonic Currents
- If L/R → ∞, then

$$i_o(t) \approx I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \qquad \frac{L}{R} \to \infty$$

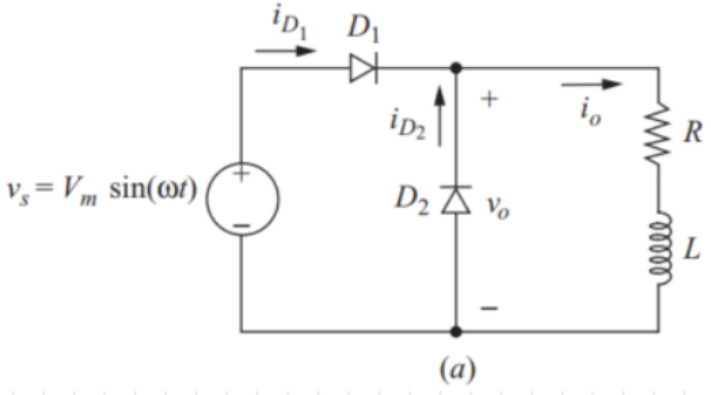
Then current ripple is twice the first harmonic term, I1

$$\Delta I_o \approx 2I_1$$

- Example 3-8

Half-Wave Rectifier with Freewheeling Diode: $L/R \rightarrow \infty$

For the half-wave rectifier with a freewheeling diode and *RL* load as shown in Fig. 3-7*a*, the source is 240 V rms at 60 Hz and $R = 8\ \Omega$. (*a*) Assume *L* is infinitely large. Determine the power absorbed by the load and the power factor as seen by the source. Sketch v_o, i_{D_1}, and i_{D_2}. (*b*) Determine the average current in each diode. (*c*) For a finite inductance, determine *L* such that the peak-to-peak current is no more than 10 percent of the average current.



(*a*)

Vrms := 240

L := 1

f := 60

Vm := Vrms · √2

R := 8

w := 2 · π · f = 376.9911184

τ := L/R = 0.125

$$v_o(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t)$$

V0 := Vm/π = 108.0379579

I0 := V0/R = 13.5047447

Find current from Fourier Series (Amplitudes)

$$V(n) := \frac{2 \cdot Vm}{(n^2 - 1)\cdot \pi}$$

V1 := Vm/2 = 169.7056275

V2 := V(2) = 72.0253053

V4 := V(4) = 14.4050611

V6 := V(6) = 6.1735976

Impedance Z:

$$Z(n) := \sqrt{R^2 + (n \cdot w \cdot L)^2}$$

Z0 := Z(0) = 8

Z1 := Z(1) = 377.0759915

Z2 := Z(2) = 754.024677

Z4 := Z(4) = 1507.9856942

Z6 := Z(6) = 2261.9608576

Load Current

$$I(n) := \frac{V(n)}{Z(n)}$$

$I0 := \frac{V0}{Z0} = 13.5047447$ $I1 := \frac{V1}{Z1} = 0.4500568$

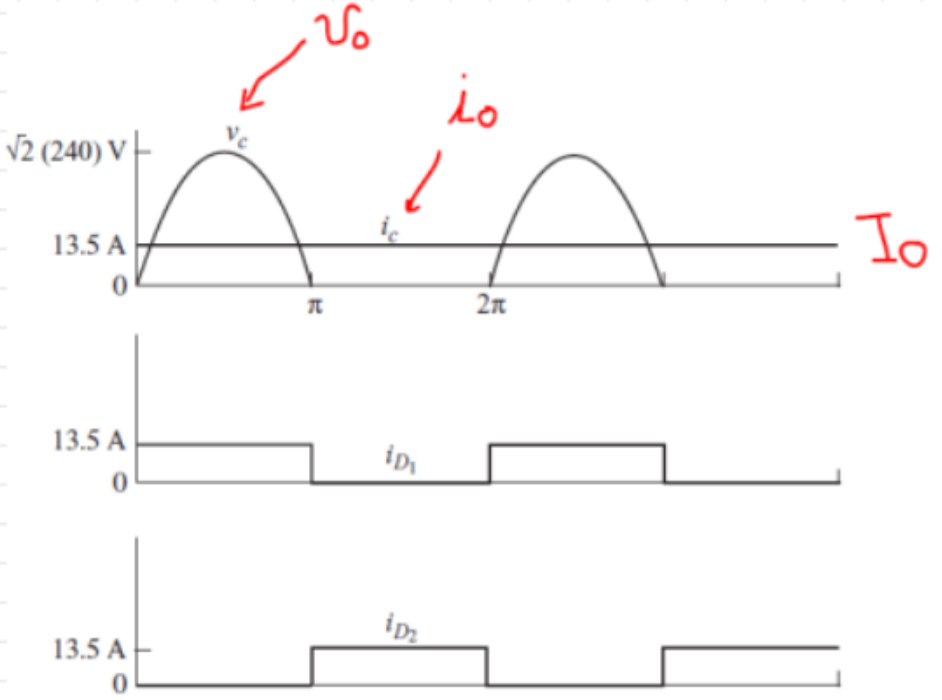
$I2 := I(2) = 0.0955212$ $I4 := I(4) = 0.0095525$ $I6 := I(6) = 0.0027293$

Load RMS current

$$Irms := \sqrt{I0^2 + \left(\frac{I1}{\sqrt{2}}\right)^2 + \left(\frac{I2}{\sqrt{2}}\right)^2 + \left(\frac{I4}{\sqrt{2}}\right)^2 + \left(\frac{I6}{\sqrt{2}}\right)^2} = 13.5086645$$

Power absorbed by the resistor

$P := Irms^2 \cdot R = 1459.872141$



Source RMS current

$$ISrms := \sqrt{\frac{1}{2 \cdot \pi} \cdot \int_0^{\pi} Irms^2 \, d\theta} = 9.5520683$$

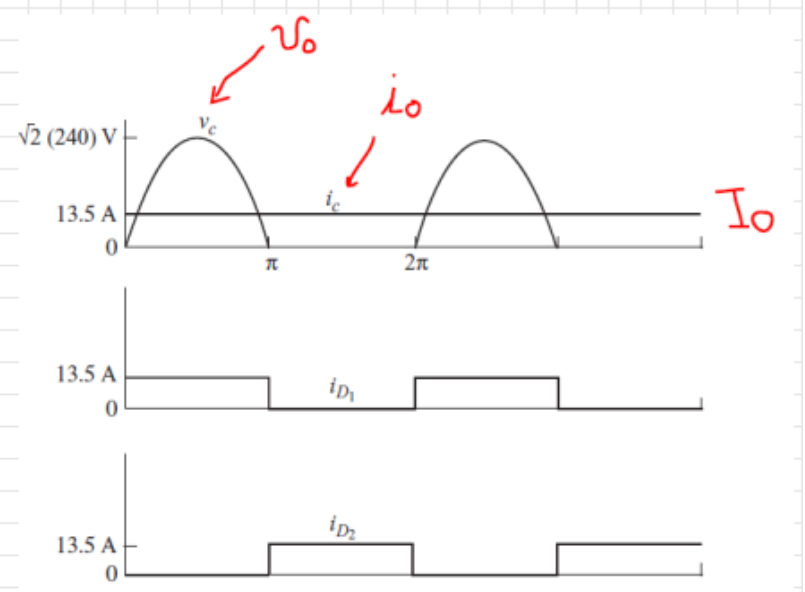
power factor

$$pf := \frac{P}{Vrms \cdot ISrms} = 0.6368046$$

Diode current (each diode turns ON half the time):

$$Id := \frac{I0}{2} = 6.7523724$$

- Example 3-8



Diode current (each diode turns ON half the time):

$$Id := \frac{I0}{2} = 6.7523724$$

Value of required inductance L with peak-to-peak current < less than 10% of I0

+

With already selected L = 1

$$V1 = 169.7056275$$

$$I1 = 0.4500568$$

$$Z1 := \frac{V1}{I1} = 377.0759915$$

Ignoring R value

$$L := \frac{Z1}{w} = 1.0002251 \quad H$$

$$\Delta I0 := 2 \cdot I1 = 0.9001137$$

$$\Delta I_o \approx 2I_1$$

Looking for L values (without given L)

$$req\Delta I0 := 0.1 \cdot (I0) = 1.3504745$$

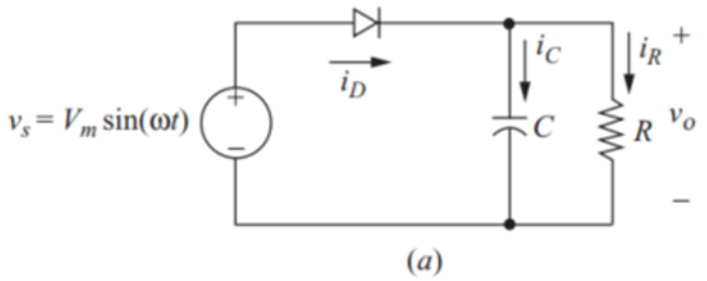
$$reqI1 := \frac{req\Delta I0}{2} = 0.6752372$$

$$newZ1 := \frac{V1}{reqI1} = 251.3274123$$

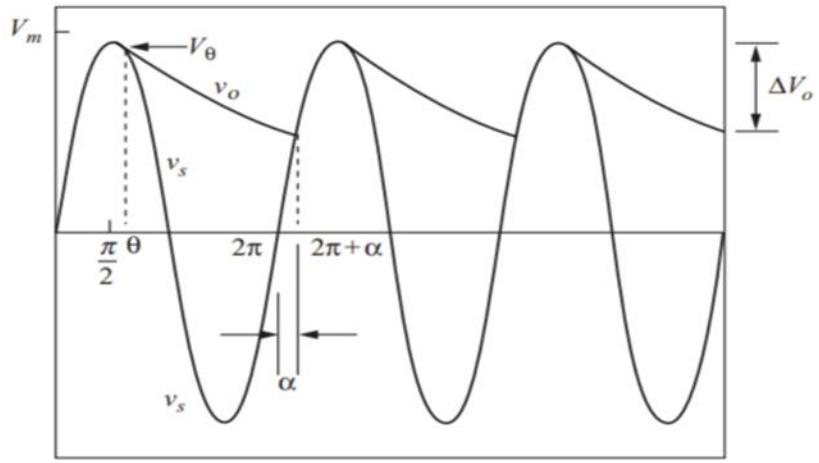
Ignoring R value

$$L := \frac{newZ1}{w} = 0.6666667 \quad H$$

Half-wave rectifier with CAPACITOR FILTER



(a)



(b)

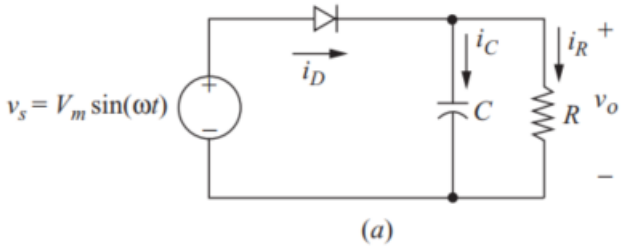
- Capacitor in parallel for reducing the ripple in the output voltage
- Filtering role of high frequency
- Capacitor is charged to Vm at wt=π/2.
- The capacitor discharges in to the load resistor
- At some point, source voltage is less than the output voltage → diode reverse-biased → output current decays exponentially

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases}$$

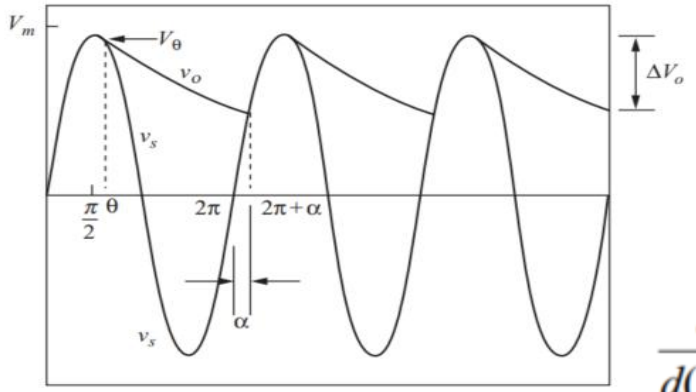
$$V_\theta = V_m \sin \theta$$

Figure 3.11

Half-wave rectifier with CAPACITOR FILTER



(a)



(b)

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases}$$

$$V_\theta = V_m \sin \theta$$

- The exact moment of diode-OFF
  - iC = iC
  - d/dt (vs) = d/dt (vo)
  - ON OFF diode

$$\frac{d}{d(\omega t)}[V_m \sin(\omega t)] = V_m \cos(\omega t)$$

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta\, e^{-(\omega t - \theta)/\omega RC}\right) = V_m \sin\theta\left(-\frac{1}{\omega RC}\right)e^{-(\omega t - \theta)/\omega RC}$$

$$V_m \cos\theta = \left(\frac{V_m \sin\theta}{-\omega RC}\right)e^{-(\theta-\theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\frac{V_m \cos\theta}{V_m \sin\theta} = \frac{1}{-\omega RC}$$

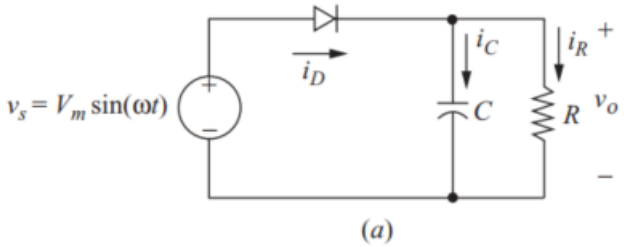
$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

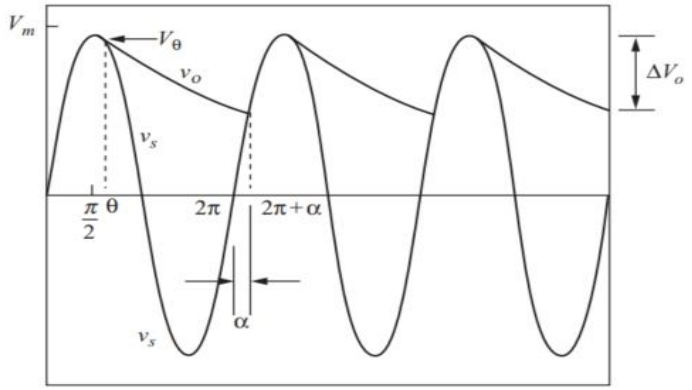
$$\theta \approx \frac{\pi}{2} \quad \text{and} \quad V_m \sin\theta \approx V_m$$

Figure 3.11

Half-wave rectifier with CAPACITOR FILTER



(a)



(b)

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases}$$

$$V_\theta = V_m \sin \theta$$

- The next period
- When source voltage is the same as the output voltage, Diode is forward-biased.
- The moment of diode ON is wt=2π + α

$$V_m \sin(2\pi + \alpha) = (V_m \sin \theta) e^{-(2\pi + \alpha - \theta)/\omega RC}$$

- Calculation of α: Smath 'solve'

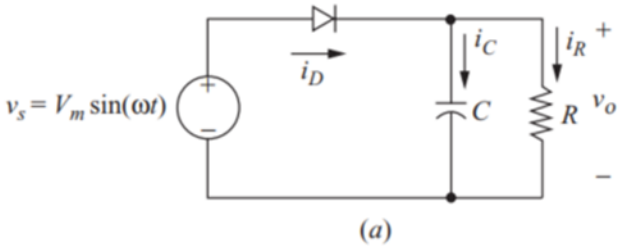
$$\sin \alpha - (\sin \theta) e^{-(2\pi + \alpha - \theta)/\omega RC} = 0$$

- Current in the resistor $i_R = v_o/R$
- Current in the capacitor

$$i_C(t) = C \frac{dv_o(t)}{dt} \qquad i_C(\omega t) = \omega C \frac{dv_o(\omega t)}{d(\omega t)}$$

$$i_C(\omega t) = \begin{cases} -\left(\frac{V_m \sin \theta}{R}\right) e^{-(\omega t - \theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases}$$

Half-wave rectifier with CAPACITOR FILTER



- **Source current = Diode current**

$$i_S = i_D = i_R + i_C$$

- **Peak capacitor current** occurs when the diode turns on at wt = 2π+α due to the abrupt change (charging)

$$I_{C,\text{peak}} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha$$

- **Resistor current** at wt = 2π+α

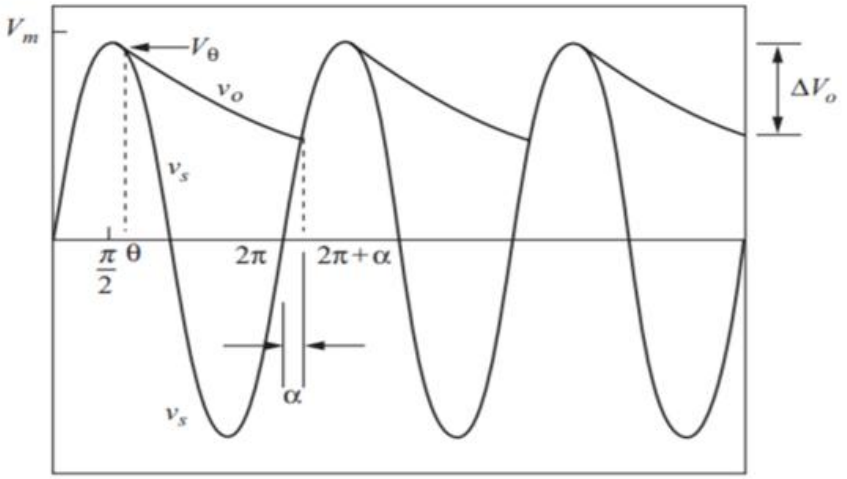
$$i_R(2\pi + \alpha) = \frac{V_m \sin(2\pi + \alpha)}{R} = \frac{V_m \sin\alpha}{R}$$

- **Peak diode current**:

$$I_{D,\text{ peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)$$

Effectiveness of capacitor filter

- Is determined by the smoothness of the output voltage
- Is the time constant τ=RC is longer than the sine wave period, output would be nearly constant dc
- Peak-to-peak ripple voltage
  - ΔV = Vmax-Vmin
  - Vmax = Vm
  - Vmin=v(2*π+α)=Vm*sin(α)



$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases}$$

$$\Delta V_o = V_m - V_m \sin \alpha = V_m(1 - \sin \alpha)$$

- Assume, $\theta \approx \pi/2$ and $\alpha = \pi/2$
- Vmin = $v_o(2\pi + \alpha) = V_m e^{-(2\pi + \pi/2 - \pi/2)\omega RC} = V_m e^{-2\pi/\omega RC}$
- Ripple voltage:

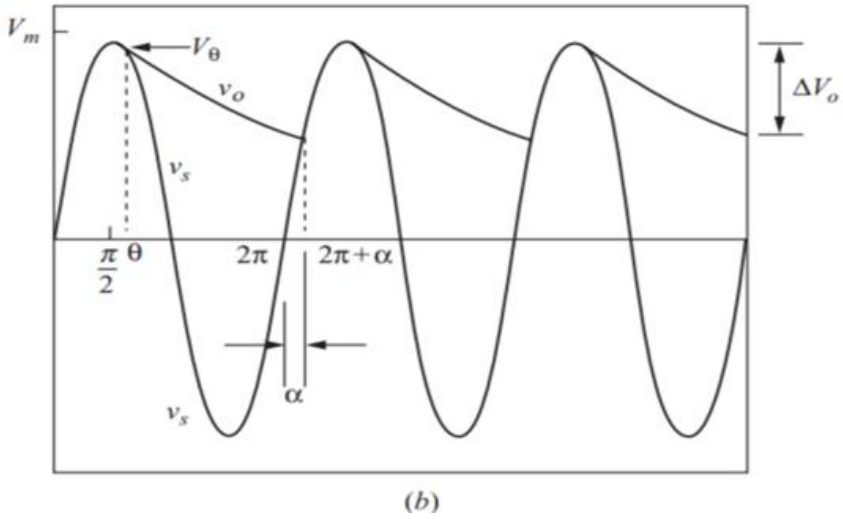
$$\Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m\left(1 - e^{-2\pi/\omega RC}\right)$$

$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

Effectiveness of capacitor filter

- Peak-to-peak ripple voltage

$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

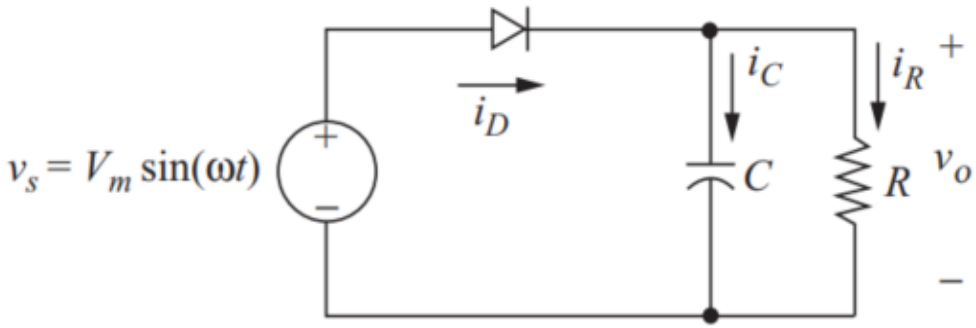


(b)

- **How to reduce the ripple voltage?**
  - Increasing the filter capacitance C
    - The effect?
      - Diode turn-on duration is decreased
      - Diode peak current is increased (why? ID = IR, so keep the average value as IR, during the short period of conduction, the current iD must be high)
  - Increasing the source voltage frequency

- **Example 3-9**

The half-wave rectifier of Fig. 3-11*a* has a 120-V rms source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu F$. Determine (*a*) an expression for output voltage, (*b*) the peak-to-peak voltage variation on the output, (*c*) an expression for capacitor current, (*d*) the peak diode current, and (*e*) the value of C such that ΔV_o is 1 percent of V_m.



$f := 60$ $\quad Vrms := 120$ $\quad R := 500$ $\quad C := 100 \cdot 10^{-6}$

$Vm := Vrms \cdot \sqrt{2} = 169.7056275$

$w := 2 \cdot \pi \cdot f = 376.9911184$

$\tau := R \cdot C = 0.05$

Determination of angle θ

$\theta := -\operatorname{atan}(w \cdot R \cdot C) + \pi = 1.6237983$ rad $\quad \theta \cdot \frac{180}{\pi} = 93.0367887$ deg

Determination of angle α

$$\sin\alpha - (\sin\theta)e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$$

$$g(x) := \sin(x) - (\sin(\theta)) \cdot \exp\left(-\frac{(2 \cdot \pi + x - \theta)}{w \cdot R \cdot C}\right)$$

$$alpha := solve\left(g(x), x, 0, \frac{\pi}{2}\right)$$

$\alpha := alpha = 0.8417867$ rad $\quad \alpha \cdot \frac{180}{\pi} = 48.2308239$ deg

Output voltage

$vo1(wt) := Vm \cdot \sin(wt)$ $\quad 2\pi + \alpha \le \omega t \le 2\pi + \theta$

$vo2(wt) := Vm \cdot \exp\left(\frac{-(wt-\theta)}{w \cdot R \cdot C}\right)$ $\quad \theta \le \omega t \le 2\pi + \alpha$

Ripple voltage

$\Delta V := Vm \cdot (1 - \sin(\alpha)) = 43.1333204$

Capacitor current

$$i_C(\omega t) = \begin{cases} -\left(\frac{V_m \sin\theta}{R}\right) e^{-(\omega t - \theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases}$$

$ic1(wt) := w \cdot C \cdot Vm \cdot \cos(wt)$ $\quad w \cdot C \cdot Vm = 6.3977514$

$ic2(wt) := -\left(\frac{Vm \cdot \sin(\theta)}{R}\right) \cdot \exp\left(\frac{-(wt-\theta)}{w \cdot R \cdot C}\right)$ $\quad \frac{Vm \cdot \sin(\theta)}{R} = 0.3389346$

Peak Diode Current

$$I_{D,\,\text{peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)$$

$IDpeak := Vm \cdot \left(w \cdot C \cdot \cos(\alpha) + \frac{\sin(\alpha)}{R}\right) = 4.5148872$

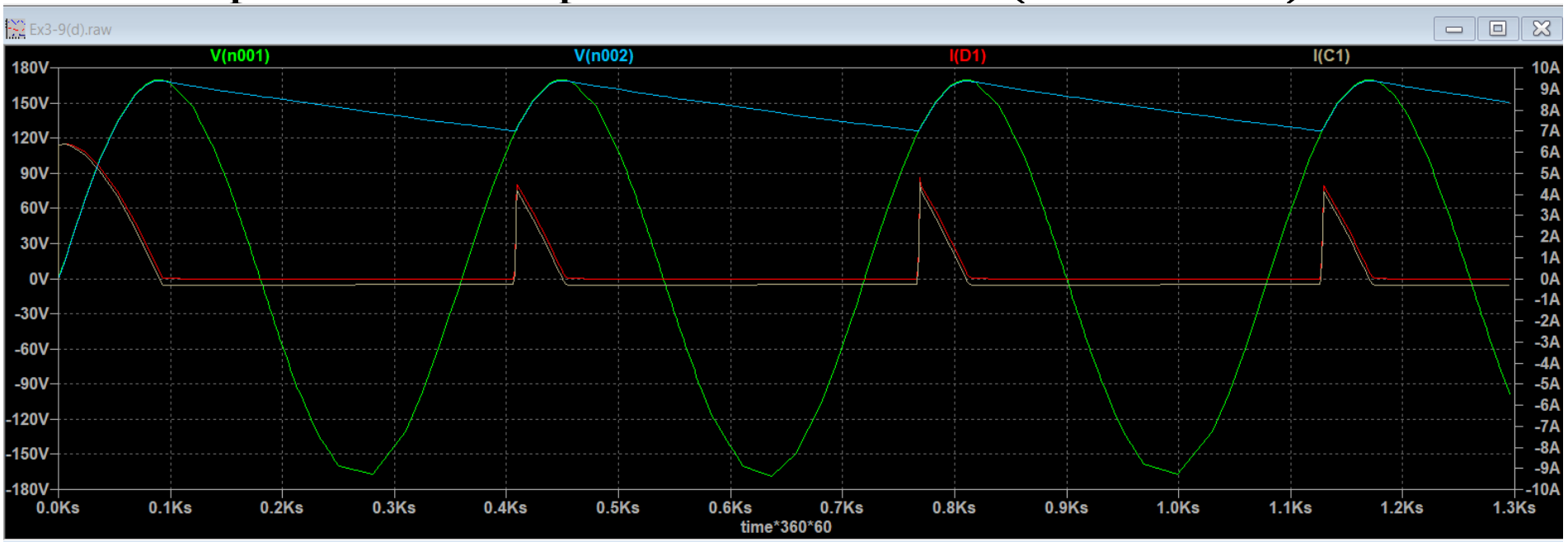
How to make ΔV = 0.01 *Vm ?

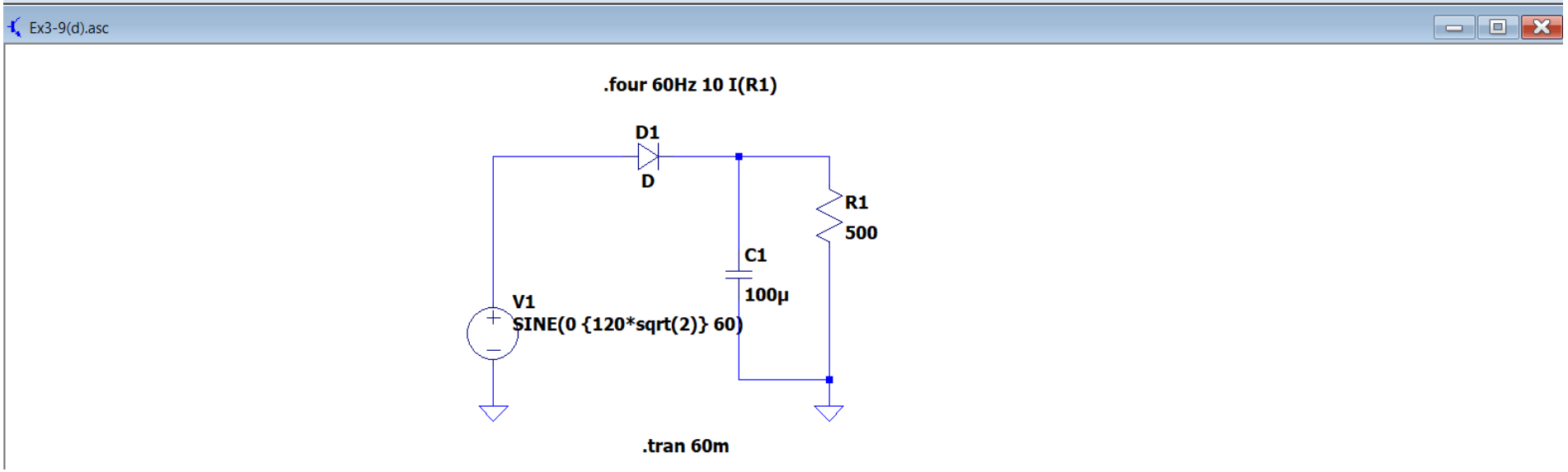
$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

$$C := \frac{Vm}{f \cdot R \cdot (0.01 \cdot Vm)} = 0.0033333$$

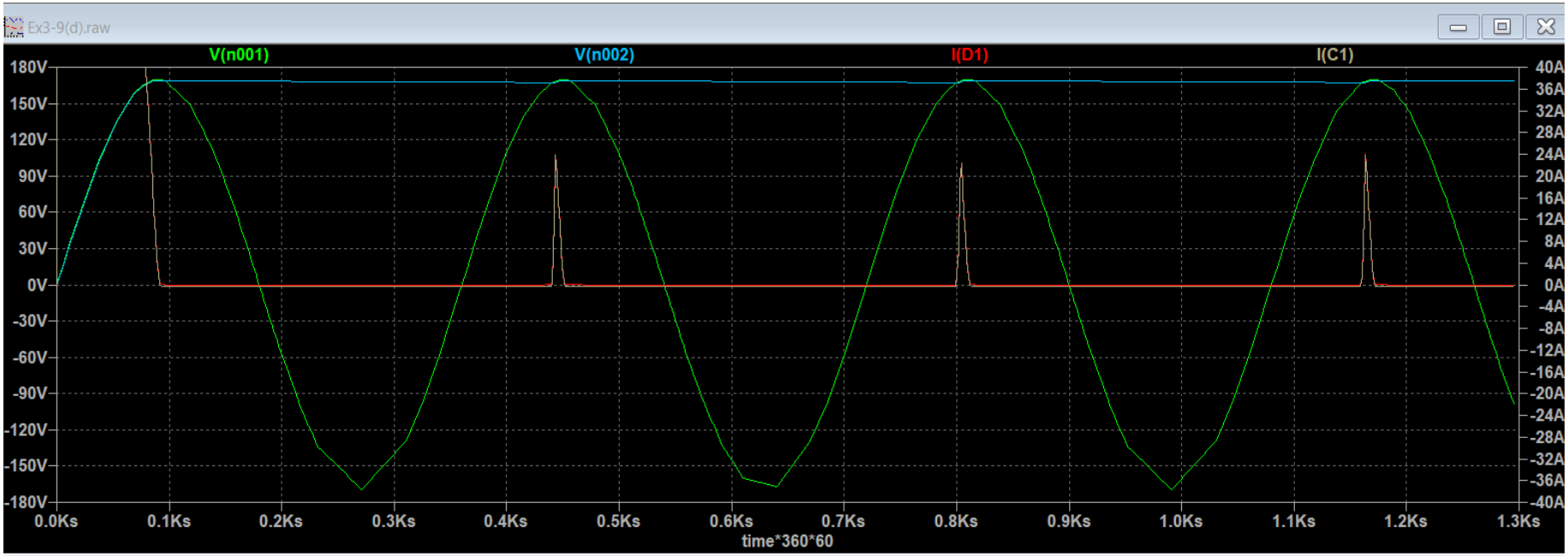
$$C \cdot 10^{6} = 3333.3333333 \quad uF$$

- Example 3-9 LTSpice Simulation (C=100uF)





- Example 3-9 LTSpice Simulation (new C=3333 uF)



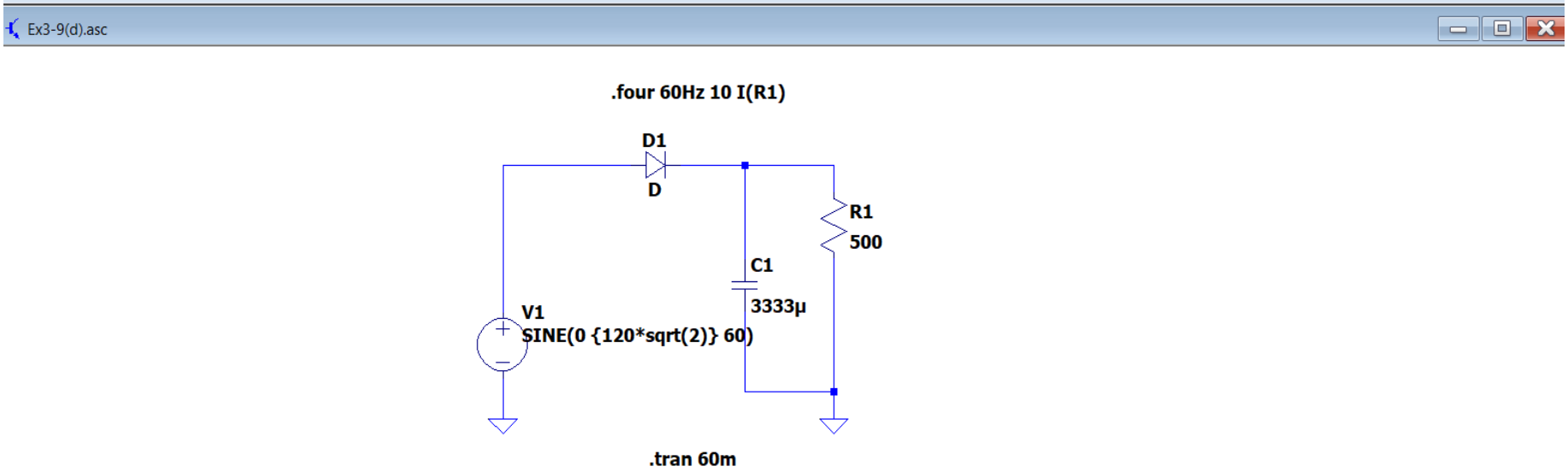
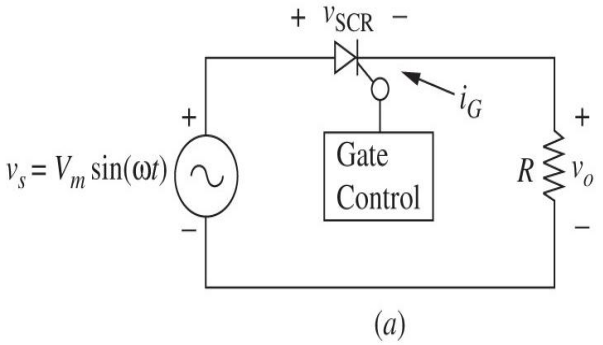
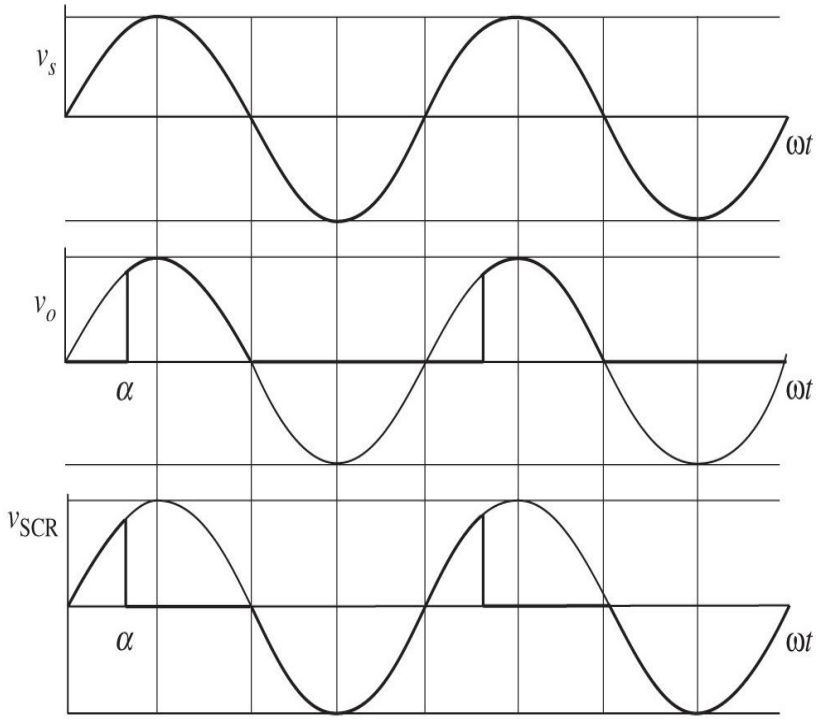


Figure 3.13

CONTROLLED HALF-WAVE RECTIFIER



(a)



(b)

- Control of dc-level of the output
- Use Thyristor (SCR) instead of Diode
- SCR:
  - ON – Gate control
  - OFF – reverse-biased (current=0)
- **R-load case**
- Gate signal applied at wt = α (delay angle)
- Average Output Voltage:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}\ \frac{(1+\cos\alpha)}{2}$$

- RMS Output voltage:

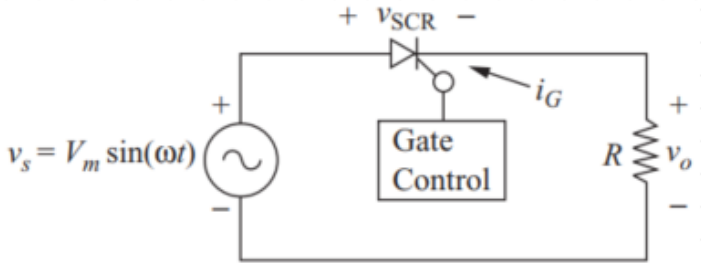
$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v_o^2(\omega t)\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2\, d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

- Example 3-10 (**R-load** case)

Controlled Half-Wave Rectifier with Resistive Load

Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.



$f := 60$ $Vsrms := 120$ $R := 100$

$Vm := Vsrms \cdot \sqrt{2} = 169.7056275$

$w := 2 \cdot \pi \cdot f = 376.9911184$

$Vo := 40$ desired output voltage

From $V_o = \frac{V_m}{\pi} \frac{(1 + \cos\alpha)}{2}$ $g(x) := \frac{Vm}{\pi} \cdot \left| \frac{1 + \cos(x)}{2} \right| - Vo$

$alpha := solve\left(g(x), x, 0, \frac{\pi}{2}\right)$

$\alpha := alpha = 1.0690459$ rad

$\alpha \cdot \frac{180}{\pi} = 61.2518161$ deg

From $V_{rms} = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$ $Vorms := \frac{Vm}{2} \cdot \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2 \cdot \alpha)}{2 \cdot \pi}} = 75.6065059$

Load power $P := \frac{Vorms^2}{R} = 57.1634373$ $Irms := \frac{Vorms}{R} = 0.7560651$

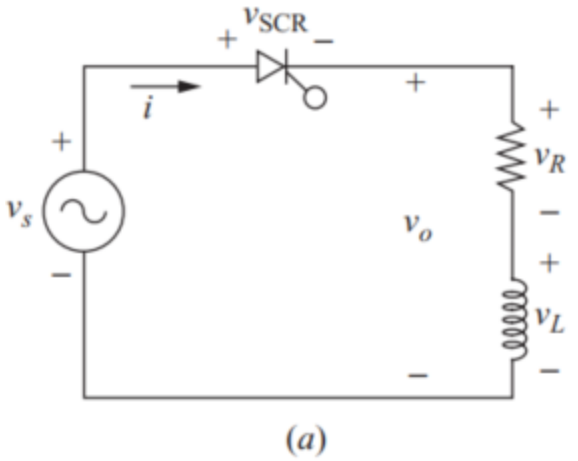
Source apparent Power $S := Vsrms \cdot Irms = 90.727807$

power factor $pf := \frac{P}{S} = 0.6300542$

- Controlled Half-Wave (**R-L load** case)
- SCR ON: current = forced response + natural response

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$



(a)

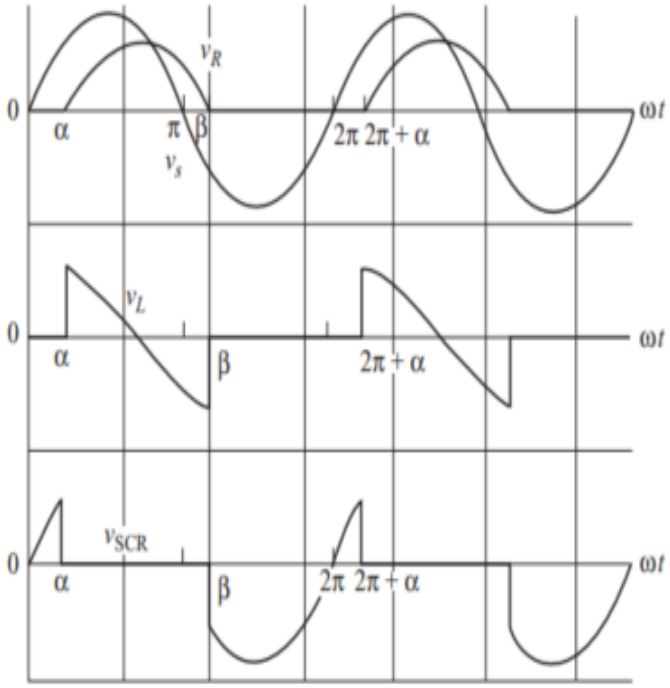
- At wt=α, current = 0

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

- We get A: $A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right]e^{\alpha/\omega\tau}$

- **Final current equation:**

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$



(b)

- β (extinction angle), at which current is zero

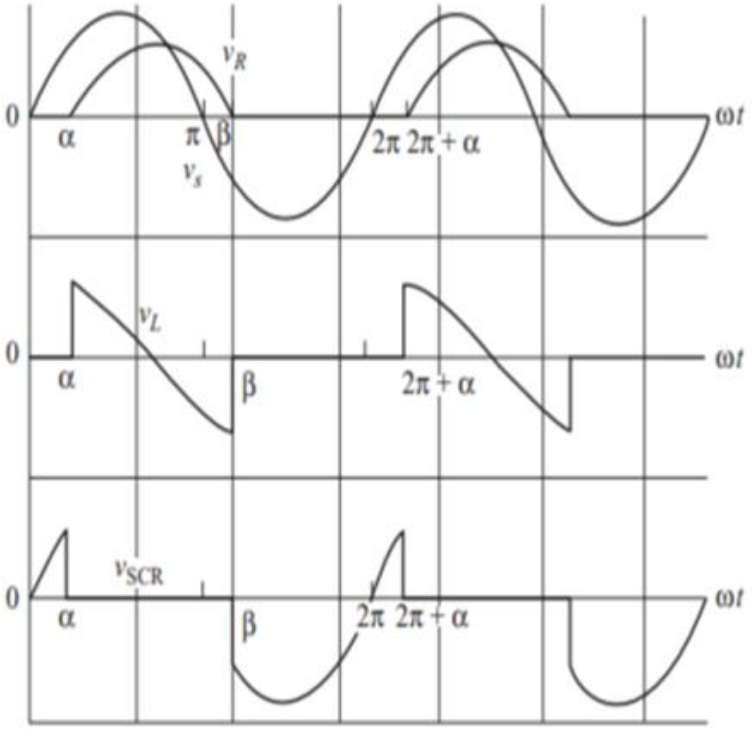
$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha-\beta)/\omega\tau}\right]$$

- β is obtained by a transcendental equation → *Solve* in Smath
- α: delay angle, γ=β-α : conduction angle
- Average (dc) Output voltage

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

- Average current

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t)$$



$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

- RMS current

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)}$$

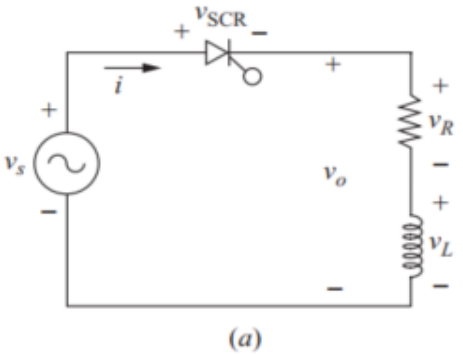
- Power absorbed by the load:

$$P = I_{rms}^2 R$$

- **Example 3-11**

Controlled Half-Wave Rectifier with *RL* Load

For the circuit of Fig. 3-14*a*, the source is 120 V rms at 60 Hz, $R = 20\ \Omega$, $L = 0.04$ H, and the delay angle is 45°. Determine (*a*) an expression for $i(\omega t)$, (*b*) the average current, (*c*) the power absorbed by the load, and (*d*) the power factor.



(*a*)

$f := 60$ $\quad Vsrms := 120$ $\quad R := 20$ $\quad L := 0.04$ $\quad Vm := Vsrms \cdot \sqrt{2} = 169.7056275$

$w := 2 \cdot \pi \cdot f = 376.9911184$

$Z := \sqrt{R^2 + (w \cdot L)^2} = 25.0478679$ $\quad \theta := \operatorname{atan}\left(\frac{w \cdot L}{R}\right) = 0.6460449$ $\quad \tau := \frac{L}{R} = 0.002$

Delay angle $\quad \alpha := 45 \cdot \frac{\pi}{180} = 0.7853982$ rad $\quad w \cdot \tau = 0.7539822$

From

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

$i(wt) := \frac{Vm}{Z} \cdot \sin(wt - \theta) - \left(\frac{Vm}{Z} \cdot \sin(\alpha - \theta)\right) \cdot e^{\frac{(\alpha - wt)}{w \cdot \tau}}$

$\frac{Vm}{Z} = 6.7752524$

$\sin(\alpha - \theta) = 0.1389027$

$\frac{Vm}{Z} \cdot \sin(\alpha - \theta) = 0.9411009$

Extinction angle

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega\tau}\right]$$

$g(x) := \frac{Vm}{Z} \cdot \left(\sin(x - \theta) - \sin(\alpha - \theta) \cdot e^{\frac{(\alpha - x)}{w \cdot \tau}}\right)$

$beta := solve(g(x), x, \pi, 2 \cdot \pi)$

$\beta := beta = 3.7850379$

$\beta \cdot \frac{180}{\pi} = 216.8666953$

Average current

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\,d(\omega t)$$

$$Io := \frac{1}{2\cdot\pi}\cdot\int_{\alpha}^{\beta} i(wt)\,d(wt) = 2.0353545$$

RMS current

$$Irms := \sqrt{\frac{1}{2\cdot\pi}\cdot\int_{\alpha}^{\beta} i(wt)^2\,d(wt)} = 3.2626433$$

Load power

$$P := Irms^2\cdot R = 212.8968289$$

Source apparent Power

$$S := Vsrms\cdot Irms = 391.5171986$$

power factor

$$pf := \frac{P}{S} = 0.5437739$$

- **Example 3-11 in LTSpice**