Chapter 2-B

POWER COMPUTATIONS(B)

focus: non-sinusoidal

voltages/currents

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Effective Value (RMS (root-mean-square))

- Effective value (of a periodic voltage) is based on the average power delivered to a resistor by a dc voltage

$$P = \frac{V_{dc}^2}{R}$$

- For a periodic voltage, "effective voltage" is defined as the voltage that is as effective as the dc voltage in supplying average power.
  - So the effective voltage can be computed using $P = \frac{V_{\text{eff}}^2}{R}$
  - Calculation of P (average power with periodic voltage)

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T v(t)i(t)\,dt = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt \quad = \frac{1}{R}\left[\frac{1}{T}\int_0^T v^2(t)\,dt\right]$$

- Equating the expressions $P = \frac{V_{\text{eff}}^2}{R}$

$$P = \frac{V_{\text{eff}}^2}{R} = \frac{1}{R}\left[\frac{1}{T}\int_0^T v^2(t)\,dt\right]$$

$$V_{\text{eff}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt$$

$$\boxed{V_{\text{eff}} = V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}}$$

- V(t) is first squared, and mean calculated, and rooted
- Root-mean-square (RMS)
- Why RMS?
  - Computing power absorbed by resistances
  - AC systems are given in rms values

• EXAMPLE 2-4

RMS Value of a Pulse Waveform

Determine the rms value of the periodic pulse waveform that has a duty ratio of D as shown in Fig. 2-6.

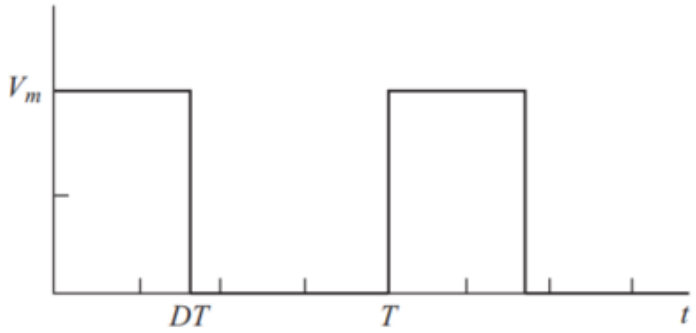


Figure 2-6 Pulse waveform for Example 2-4.

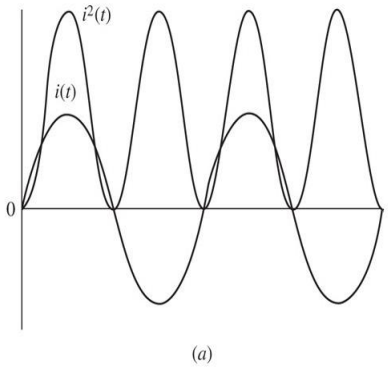
$$v(t) = \begin{cases} V_m & 0 < t < DT \\ 0 & DT < t < T \end{cases}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{T}\left(\int_0^{DT} V_m^2\,dt + \int_{DT}^T 0^2\,dt\right)} = \sqrt{\frac{1}{T}(V_m^2\,DT)}$$

$$V_{\text{rms}} = V_m\sqrt{D}$$

Figure 2.7

EXAMPLE 2-5 RMS values of Sinusoids

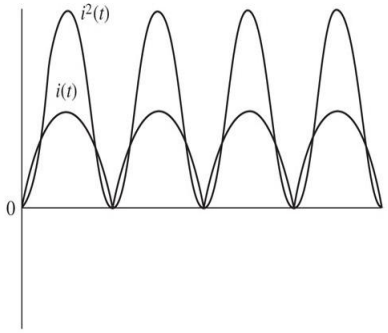


(a)

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \sin^2(\omega t)\,dt} \qquad \text{where } T = \frac{2\pi}{\omega}$$

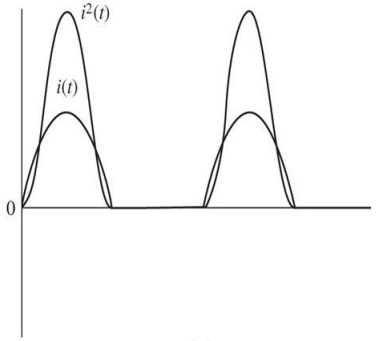
Sinusoidal voltage

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t)\,d(\omega t)} = \frac{V_m}{\sqrt{2}}$$



(b)

Full-wave rectified sinusoid



(c)

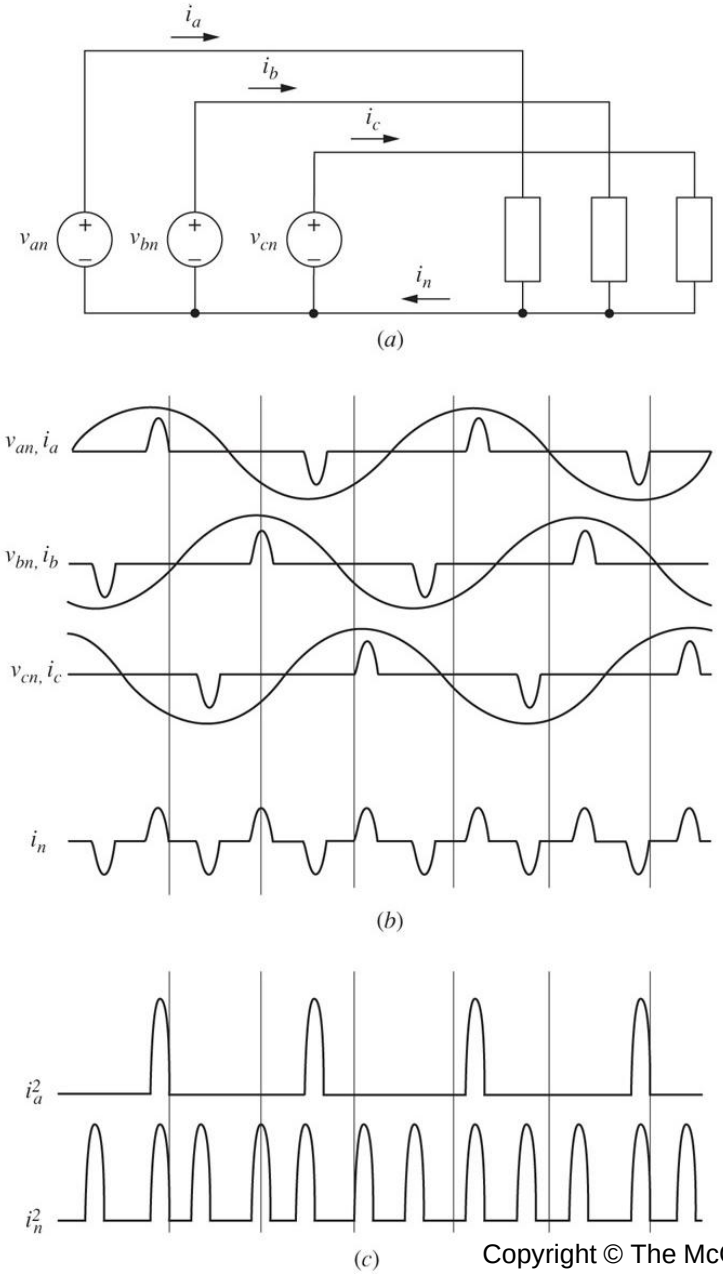
Half-wave rectified sinusoid

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\left(\int_0^{\pi} V_m^2 \sin^2(\omega t)\,d(\omega t) + \int_{\pi}^{2\pi} 0^2\,d(\omega t)\right)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2(\omega t)\,d(\omega t)}$$

$$V_{\text{rms}} = \left(\sqrt{\frac{1}{2}}\right)\sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2(\omega t)\,d(\omega t)} \qquad V_{\text{rms}} = \sqrt{\frac{1}{2}}\,\frac{V_m}{\sqrt{2}} = \frac{V_m}{2}$$

Figure 2.8

Example 2-6 Neutral current in a 3-phase system



$$I_{n,rms} = \sqrt{\frac{1}{T}\int_0^T i_n^2(t)d(t)} = \sqrt{3\left(\frac{1}{T}\int_0^T i_a^2(t)\,d(t)\right)} = \sqrt{3}\,I_{a,rms}$$

RMS OF THE **SUM** OF VOLTAGE WAVEFORMS

$$v(t) = v_1(t) + v_2(t)$$

For orthogonal functions, this is zero

$$V_{rms}^2 = \frac{1}{T}\int_0^T (v_1 + v_2)^2\, dt = \frac{1}{T}\int_0^T \left(v_1^2 + 2v_1v_2 + v_2^2\right) dt = \frac{1}{T}\int_0^T v_1^2 dt + \frac{1}{T}\int_0^T 2v_1v_2\, dt + \frac{1}{T}\int_0^T v_2^2 dt$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v_1^2(t) dt + \frac{1}{T}\int_0^T v_2^2(t)\, dt = V_{1,rms}^2 + V_{2,rms}^2$$

$$V_{rms} = \sqrt{V_{1,rms}^2 + V_{2,rms}^2}$$

$$V_{rms} = \sqrt{V_{1,rms}^2 + V_{2,rms}^2 + V_{3,rms}^2 + \ldots} = \sqrt{\sum_{n=1}^{N} V_{n,rms}^2}$$

$f := 60 \qquad w := 2 \cdot \pi \cdot f \qquad T := \frac{2 \cdot \pi}{w}$

$T = 0.01666667$

Orthogonal -- zero sum

$$\int_0^T \sin(w \cdot t)^2 \, d(t) = 0.00833333$$

$$\int_0^T \sin(2 \cdot w \cdot t)^2 \, d(t) = 0.00833333$$

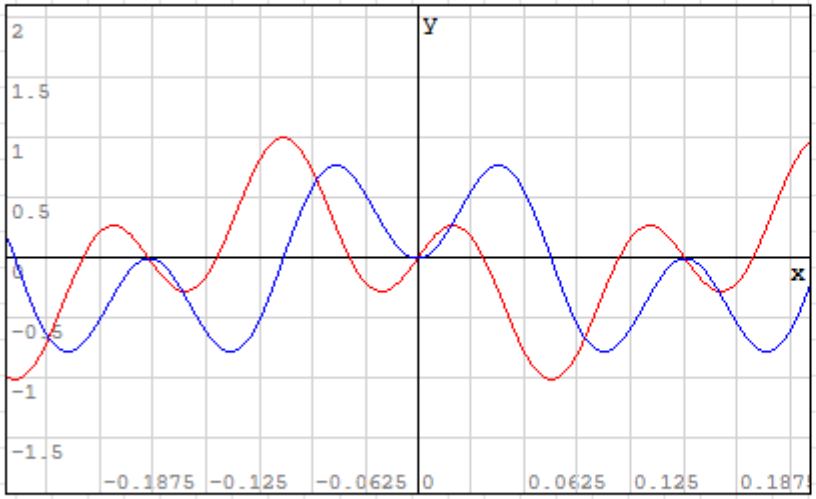
$$\int_0^T \sin(5 \cdot w \cdot t) \cdot \sin(w \cdot t) \, d(t) = 2.18671531 \cdot 10^{-18}$$

$$\int_0^T \sin(5 \cdot w \cdot t) \cdot \cos(5 \cdot w \cdot t) \, d(t) = -5.79484076 \cdot 10^{-18}$$

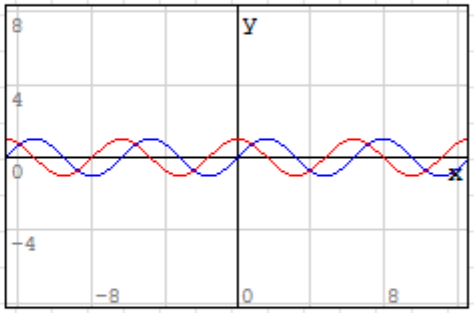
$h(x) := \sin(20 \cdot x) \cdot \sin(2 \cdot 20 \cdot x)$ blue

$g(x) := \sin(20 \cdot x) \cdot \cos(2 \cdot 20 \cdot x)$ red

so the areas are cancelled out +



$h(x)$
$g(x)$



$y := \sin(x)$
$g := \cos(x)$

Example 2-7 RMS value of sum of waveforms

RMS Value of the Sum of Waveforms

Determine the effective (rms) value of $v(t) = 4 + 8\sin(\omega_1 t + 10°) + 5\sin(\omega_2 t + 50°)$ for (*a*) $\omega_2 = 2\omega_1$ and (*b*) $\omega_2 = \omega_1$.

(a) Different frequency case

$$V_{\text{rms}} = \sqrt{V_{1,\text{rms}}^2 + V_{2,\text{rms}}^2 + V_{3,\text{rms}}^2} = \sqrt{4^2 + \left(\frac{8}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = 7.78 \text{ V}$$

(b) Same frequency – apply phasor analysis first

Example 2-7 RMS value of sum of waveforms

(a) Different frequency case

$$V_{rms} = \sqrt{V_{1,rms}^2 + V_{2,rms}^2 + V_{3,rms}^2} = \sqrt{4^2 + \left(\frac{8}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = 7.78\ \text{V}$$

(b) Same frequency – apply phasor analysis first

$v(t) := 4 + 8 \cdot \sin(w \cdot t + 10) + 5 \cdot \sin(w \cdot t + 50)$

Phasor representation

$v1 := 4$

$v2 := 8 \cdot e^{i \cdot 10 \cdot \frac{\pi}{180}} = 7.878462 + 1.3891854 \cdot i$

$v3 := 5 \cdot e^{i \cdot 50 \cdot \frac{\pi}{180}} = 3.213938 + 3.8302222 \cdot i$

$\arg(v2) = 0.1745329$

$\arg(v3) = 0.8726646$

$v2 + v3 = 11.0924001 + 5.2194076 \cdot i$

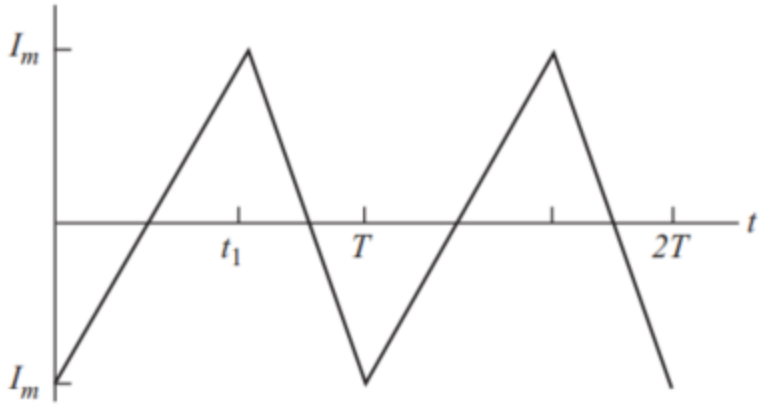
$|v2 + v3| = 12.2590194$ $\quad \arg(v2 + v3) = 0.4398023$ rad

$v(t) := 4 + 12.259 \cdot \sin(w \cdot t + 0.4399)$

$Vrms := \sqrt{4^2 + \left(\frac{12.259}{\sqrt{2}}\right)^2} = 9.5468079$

Figure 2.9

Example 2-8 RMS OF TRIANGULAR WAVEFORMS



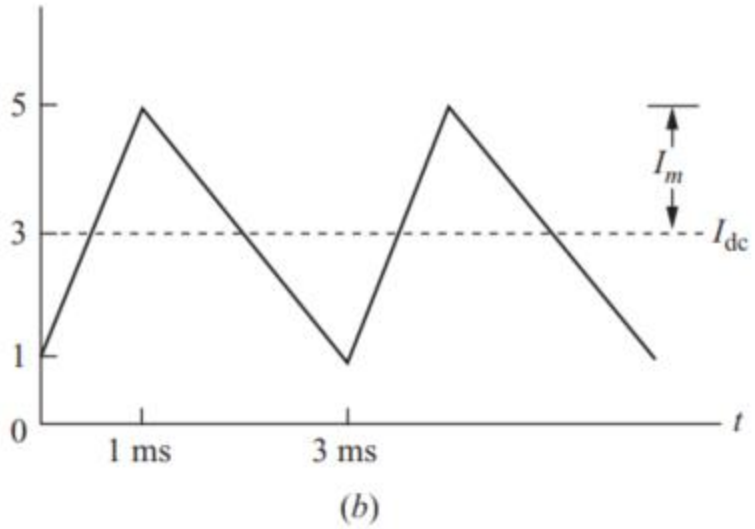
(a)

$$i(t) = \begin{cases} \dfrac{2I_m}{t_1}t - I_m & 0 < t < t_1 \\ \dfrac{-2I_m}{T - t_1}t + \dfrac{I_m(T + t_1)}{T - t_1} & t_1 < t < T \end{cases}$$

$$I_{\text{rms}}^2 = \frac{1}{T}\left[\int_0^{t_1}\left(\frac{2I_m}{t_1}t - I_m\right)^2 dt + \int_{t_1}^{T}\left(\frac{-2I_m}{T - t_1}t + \frac{I_m(T + t_1)}{T - t_1}\right)^2 dt\right]$$

$$I_{\text{rms}} = \frac{I_m}{\sqrt{3}}$$

- **Triangular wave form with offset** (of Idc)



$$I_{rms} = \sqrt{I_{1,rms}^2 + I_{2,rms}^2} = \sqrt{\left(\frac{I_m}{\sqrt{3}}\right)^2 + I_{dc}^2}$$

$$\sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + 3^2} = 3.22 \text{ A}$$

APPARENT POWER (S)

Magnitude of complex power **S**

$$S = V_{rms} I_{rms}$$

POWER FACTOR (pf)

Ratio of average ("real") power to apparent power

$$pf = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}}$$

Power Computation for SINUSOIDAL AC CIRCUITS

- Voltage & currents in Power Electronics are non-sinusoidal
- But they can be represented by a Fourier series sinusoids

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

- Instantaneous power

$$p(t) = v(t)i(t) = [V_m \cos(\omega t + \theta)][I_m \cos(\omega t + \phi)]$$

$$(\cos A)(\cos B) = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$p(t) = \left(\frac{V_m I_m}{2}\right)[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)]$$

- Average Power

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \left(\frac{V_m I_m}{2}\right)\int_0^T [\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)]\,dt$$

Power Computation for SINUSOIDAL AC CIRCUITS

- Average Power:

$$P = \left(\frac{V_m I_m}{2} \right) \cos(\theta - \phi) = V_{rms} I_{rms} \cos(\theta - \phi)$$

- Reactive Power (for inductor and capacitor): no net power absorbed or supplied in steady-state:
  - ½ cycle: energy stored
  - The other ½ cycle: energy retrieved
  - Inductor: absorbs positive reactive power
  - Capacitor: absorbs negative reactive power

$$Q = V_{rms} I_{rms} \sin(\theta - \phi)$$

- Complex Power $\mathbf{S} = P + jQ = (\mathbf{V_{rms}})(\mathbf{I_{rms}})^*$

- Apparent Power

$$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = V_{rms} I_{rms}$$

- Power Factor (pf)

$$pf = \cos(\theta - \phi)$$

Power Computation for Non-Sinusoidal Periodic Signals

- Power electronic circuits have voltage/current periodic, but not sinusoidal
- Common mistake: Application of the formula for sinusoids to non-sinusoidal periodic signals
- Fourier series describes non-sinusoidal periodic signals in terms of a series of sinusoids

FOURIER SERIES

A periodic function f(t): $f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$

where

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, dt$$

Constant, average, dc

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n\omega_0 t)\, dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n\omega_0 t)\, dt$$

FOURIER SERIES

A periodic function f(t):

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

a_0: constant, average, dc

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \cos\left(n\omega_0 t + \theta_n\right)$$

w_0: fundamental frequency

where

$$C_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

n*w0: harmonics

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \sin\left(n\omega_0 t + \theta_n\right)$$

Cn: harmonic coefficient

where

$$C_n = \sqrt{a_n^2 + b_n^2} \quad \text{and} \quad \theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

FOURIER SERIES

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n \cos\left(n\omega_0 t + \theta_n\right)$$

RMS value of f(t):

$$F_{rms} = \sqrt{\sum_{n=1}^{\infty} F_{n,rms}^2} = \sqrt{a_0^2 + \sum_{n=1}^{\infty}\left(\frac{C_n}{\sqrt{2}}\right)^2}$$

FOURIER SERIES

Average Power (P):

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t + \phi_n)$$

$$P = \frac{1}{T}\int_0^T v(t)i(t)\,dt$$

$$P = \sum_{n=1}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,rms} I_{n,rms} \cos(\theta - \phi)$$

P(avg power) calculation details

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

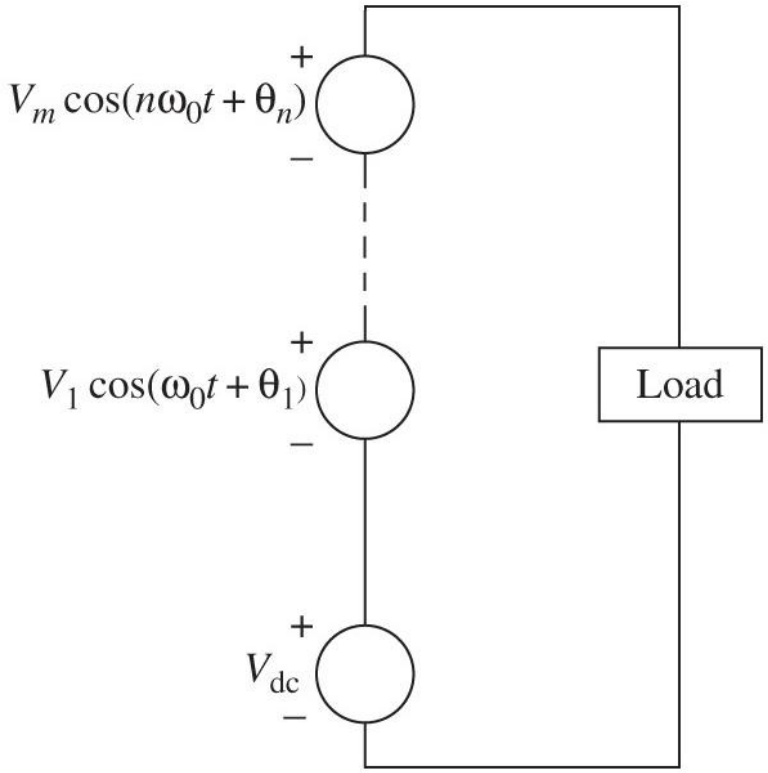
$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t + \phi_n)$$

$$P = \frac{1}{T}\int_0^T v(t)i(t)\,dt$$

$$P = \sum_{n=1}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,rms} I_{n,rms} \cos(\theta - \phi)$$

Figure 2.10

NONSINUSOIDAL SOURCE AND LINEAR LOAD

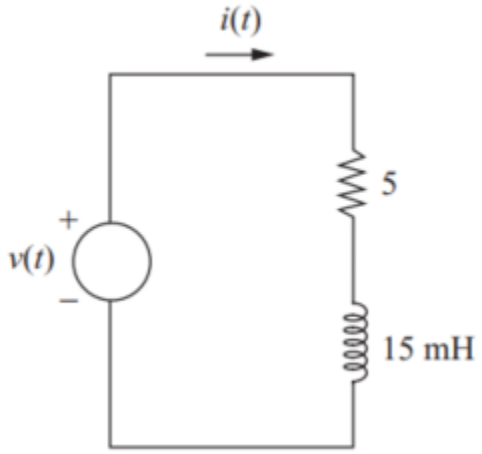


Non-sinusoidal source = **combination** of Fourier series **sinusoids**

Current can be obtained by **superposition**

- Example 2-9 (non-sinusoidal source and linear load)

A nonsinusoidal periodic voltage has a Fourier series of $v(t) = 10 + 20\cos(2\pi 60t - 25°) + 30\cos(4\pi 60t + 20°)$ V. This voltage is connected to a load that is a 5-Ω resistor and a 15-mH inductor connected in series as in Fig. 2-11. Determine the power absorbed by the load.



$$I_0 = \frac{V_0}{R} = \frac{10}{5} = 2 \text{ A}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{R + j\omega_1 L} = \frac{20\angle(-25°)}{5 + j(2\pi 60)(0.015)} = 2.65\angle(-73.5°)$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{V}_2}{R + j\omega_2 L} = \frac{30\angle 20°}{5 + j(4\pi 60)(0.015)} = 2.43\angle(-46.2°)$$

$$i(t) = 2 + 2.65\cos(2\pi 60t - 73.5°) + 2.43\cos(4\pi 60t - 46.2°)$$

dc term: $P_0 = (10 \text{ V})(2 \text{ A}) = 20 \text{ W}$

$\omega = 2\pi 60$: $P_1 = \frac{(20)(2.65)}{2}\cos(-25° + 73.5°) = 17.4 \text{ W}$

$\omega = 4\pi 60$: $P_2 = \frac{(30)(2.43)}{2}\cos(20° + 46°) = 14.8 \text{ W}$

$$P = I_{\text{rms}}^2 R = \left[2^2 + \left(\frac{2.65}{\sqrt{2}}\right)^2 + \left(\frac{2.43}{\sqrt{2}}\right)^2\right]5 = 52.2$$

$$P = 20 + 17.4 + 14.8 = 52.2 \text{ W}$$

SINUSOIDAL SOURCE AND NONLINEAR LOAD

- Current will not be sinusoid
- Current will be represented by Fourier series

$$v(t) = V_1 \sin(\omega_0 t + \theta_1)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \sin\left(n\omega_0 t + \phi_n\right)$$

$$P = V_0 I_0 + \sum_{n=1}^{\infty} \left(\frac{V_{n,\max} I_{n,\max}}{2} \right) \cos\left(\theta_n - \phi_n\right)$$

$$= (0)(I_0) + \left(\frac{V_1 I_1}{2} \right) \cos\left(\theta_1 - \phi_1\right) + \sum_{n=2}^{\infty} \left(\frac{(0) I_{n,\max}}{2} \right) \cos\left(\theta_n - \phi_n\right)$$

$$= \left(\frac{V_1 I_1}{2} \right) \cos\left(\theta_1 - \phi_1\right) = V_{1,\text{rms}} I_{1,\text{rms}} \cos\left(\theta_1 - \phi_1\right)$$

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}}$$

$$\text{pf} = \frac{V_{1,\text{rms}} I_{1,\text{rms}} \cos(\theta_1 - \phi_1)}{V_{1,\text{rms}} I_{\text{rms}}} = \left(\frac{I_{1,\text{rms}}}{I_{\text{rms}}}\right) \cos(\theta_1 - \phi_1)$$

$$I_{\text{rms}} = \sqrt{\sum_{n=0}^{\infty} I_{n,\text{rms}}^2} = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \left(\frac{I_n}{\sqrt{2}}\right)^2}$$

$$\text{DF} = \frac{I_{1,\text{rms}}}{I_{\text{rms}}}$$

$$\text{pf} = [\cos(\theta_1 - \phi_1)]\,\text{DF}$$

$$\text{THD} = \sqrt{\frac{\sum_{n\neq 1} I_{n,\text{rms}}^2}{I_{1,\text{rms}}^2}} = \frac{\sqrt{\sum_{n\neq 1} I_{n,\text{rms}}^2}}{I_{1,\text{rms}}}$$

$$\text{THD} = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1}$$

$$\text{THD} = \sqrt{\frac{I_{\text{rms}}^2 - I_{1,\text{rms}}^2}{I_{1,\text{rms}}^2}}$$

$$\text{DF} = \sqrt{\frac{1}{1 + (\text{THD})^2}}$$

Reactive Power

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta - \phi) \qquad Q = \frac{V_1 I_1}{2} \sin(\theta_1 - \phi_1)$$

Distortion volt-amps

$$S = \sqrt{P^2 + Q^2 + D^2}$$

$$D = V_{1,\,\text{rms}} \sqrt{\sum_{n \neq 1}^{\infty} I_{n,\,\text{rms}}^2} = \frac{V_1}{2} \sqrt{\sum_{n \neq 1}^{\infty} I_n}$$

Form factor and crest factor

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{avg}}}$$

$$\text{Crest factor} = \frac{I_{\text{peak}}}{I_{\text{rms}}}$$

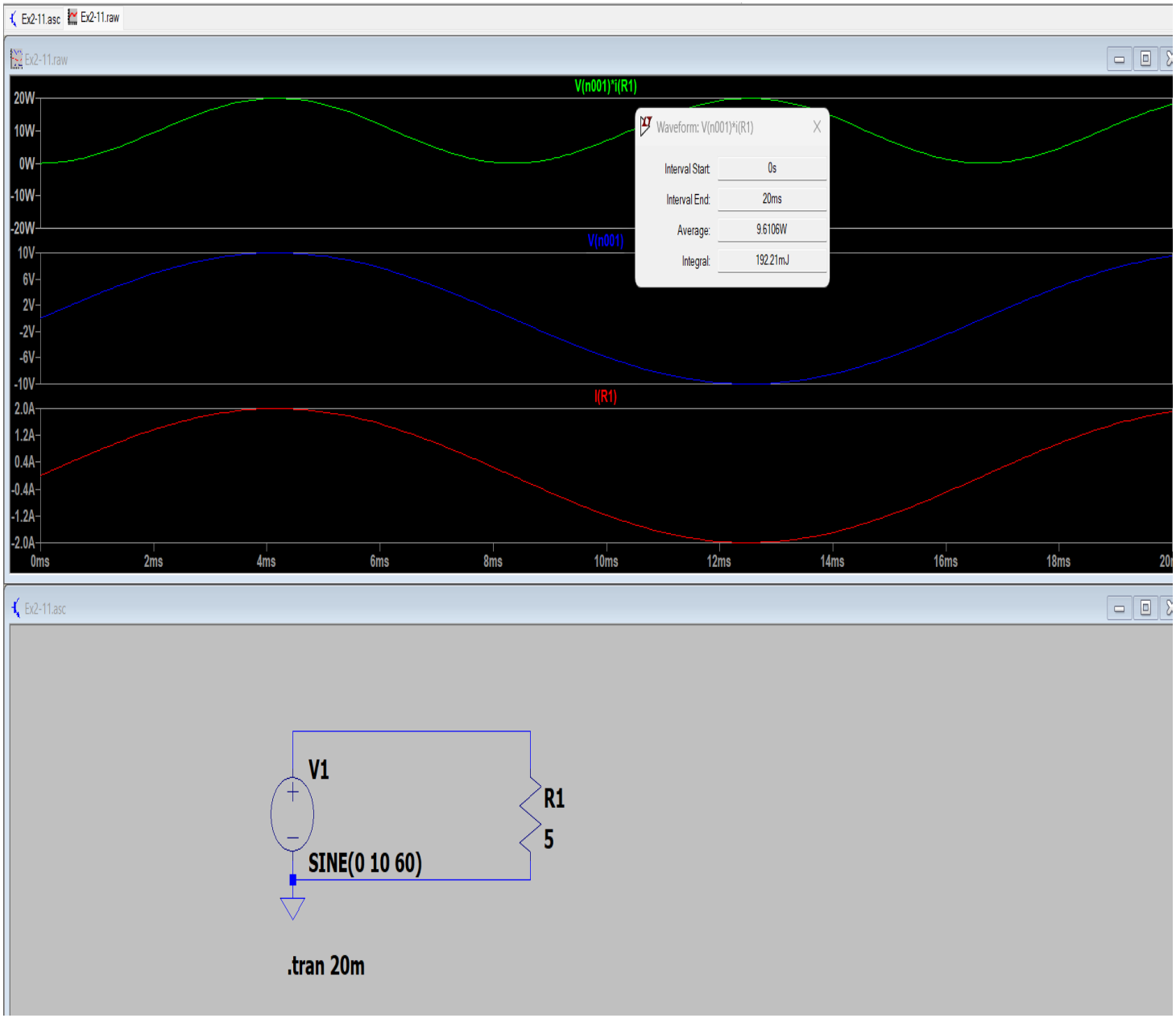
Example 2-10(Sinusoidal source and a nonlinear load)

A sinusoidal voltage source of $v(t) = 100\cos(377t)$ V is applied to a nonlinear load, resulting in a nonsinusoidal current which is expressed in Fourier series form as

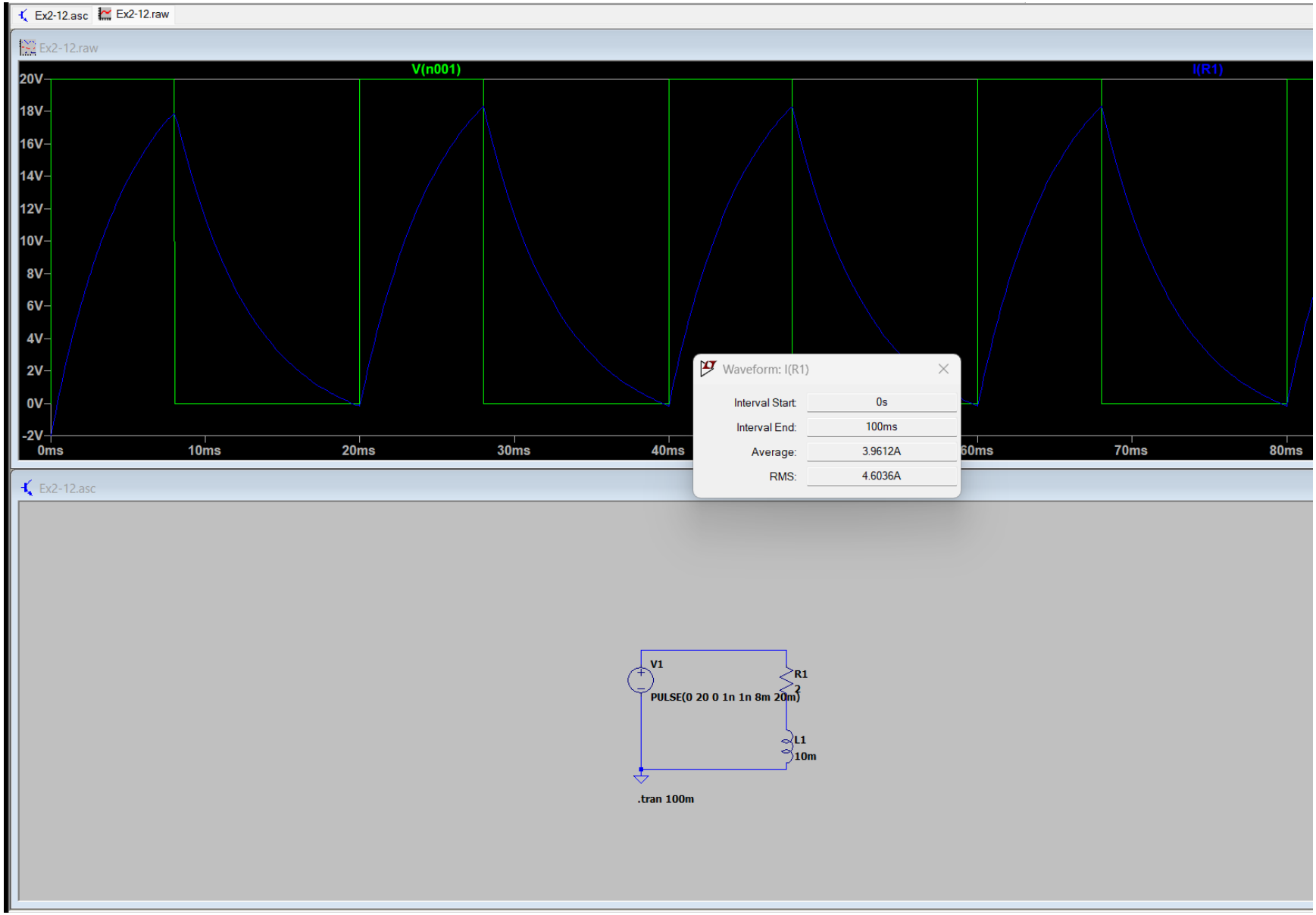
$$i(t) = 8 + 15\cos(377t + 30°) + 6\cos[2(377)t + 45°] + 2\cos[3(377)t + 60°]$$

Determine (*a*) the power absorbed by the load, (*b*) the power factor of the load, (*c*) the distortion factor of the load current, (*d*) the total harmonic distortion of the load current.

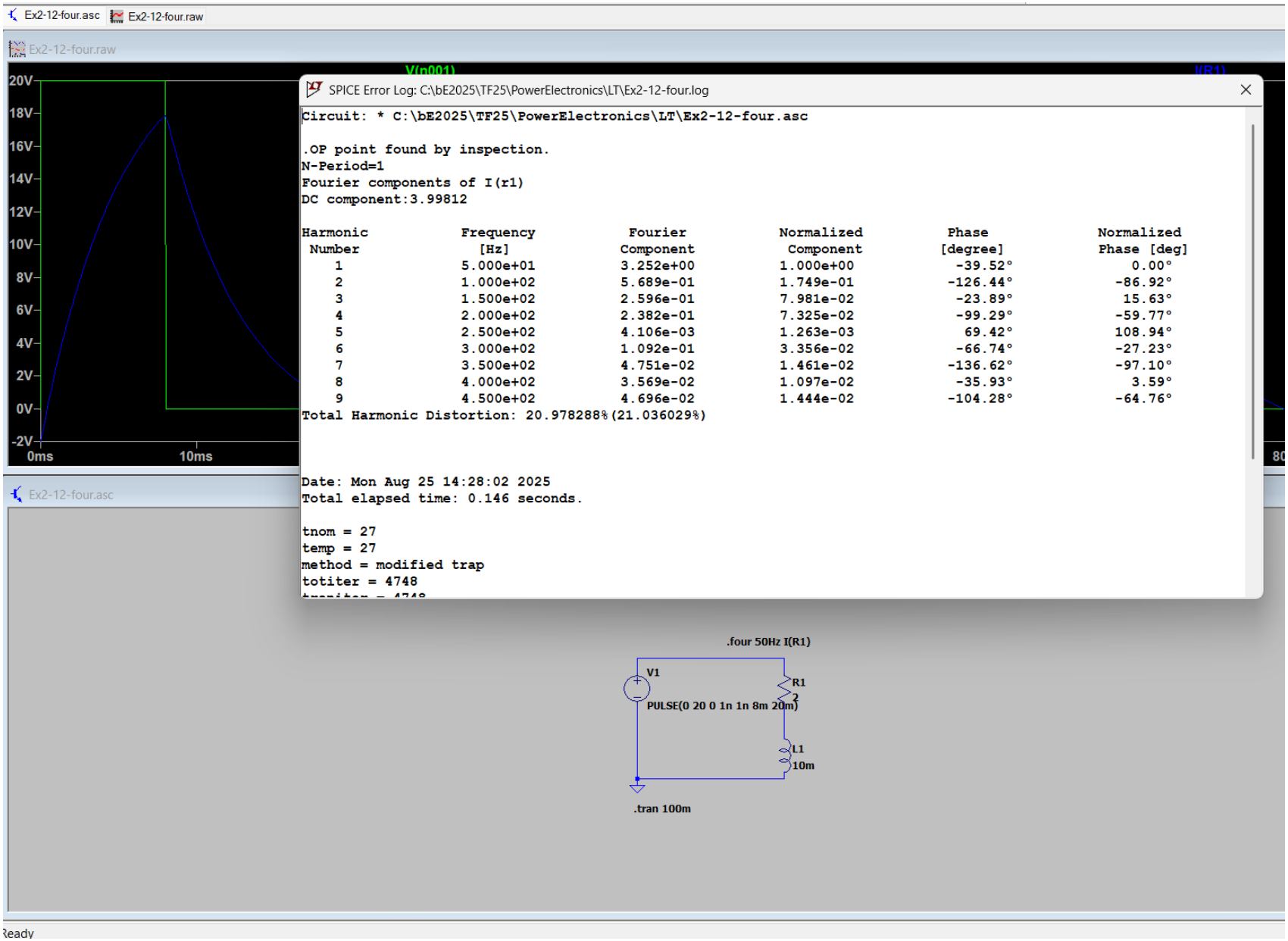
LTSpice (Example 2-11)



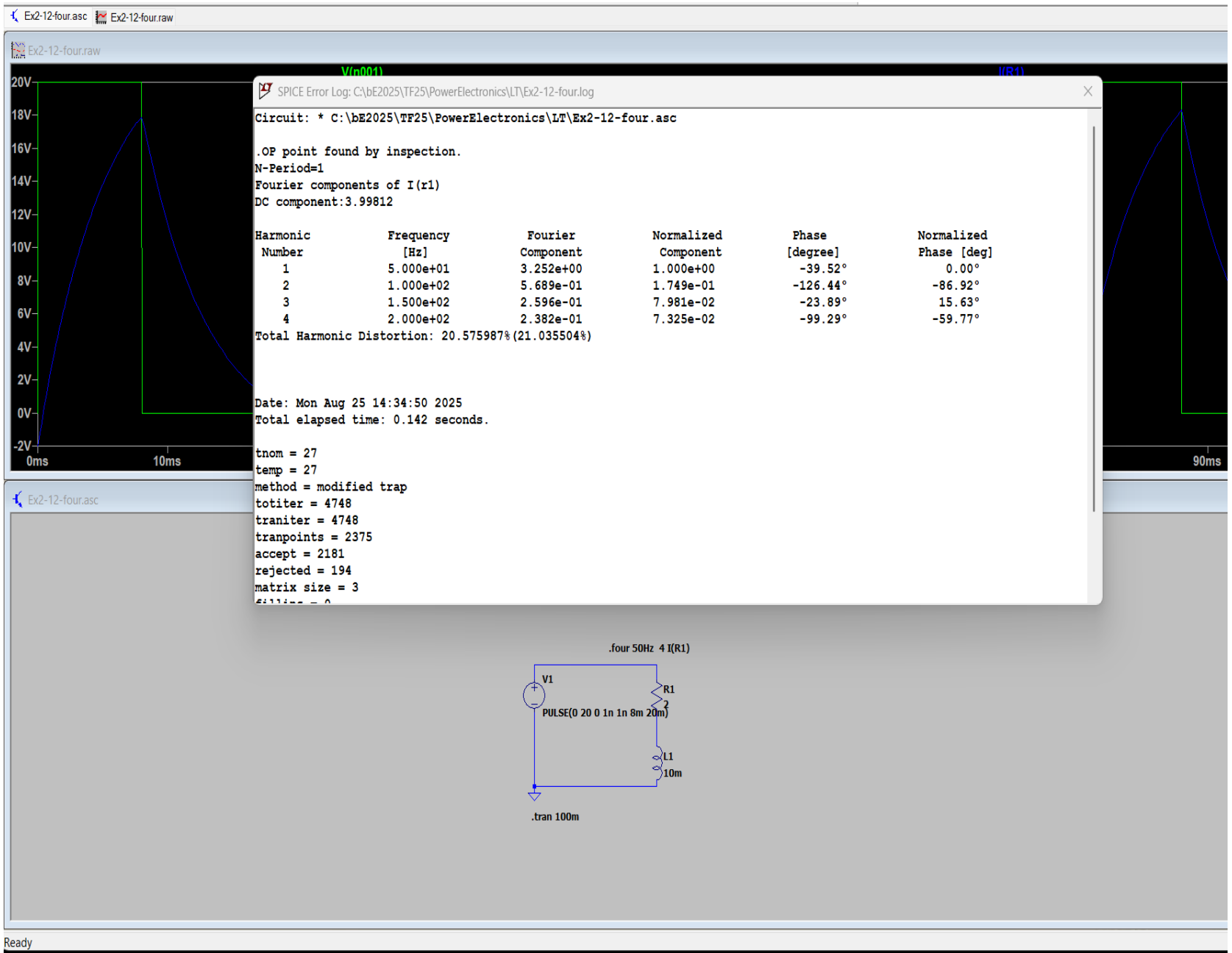
Example 2-12


Ex2-12.asc
Ex2-12.raw
Ex2-12.raw
V(n001)
I(R1)
20V
18V
16V
14V
12V
10V
8V
6V
4V
2V
0V
-2V
0ms
10ms
20ms
30ms
40ms
60ms
70ms
80ms
Waveform: I(R1)
Interval Start: 0s
Interval End: 100ms
Average: 3.9612A
RMS: 4.6036A
Ex2-12.asc
V1
PULSE(0 20 0 1n 1n 8m 20m)
R1
2
L1
10m
.tran 100m

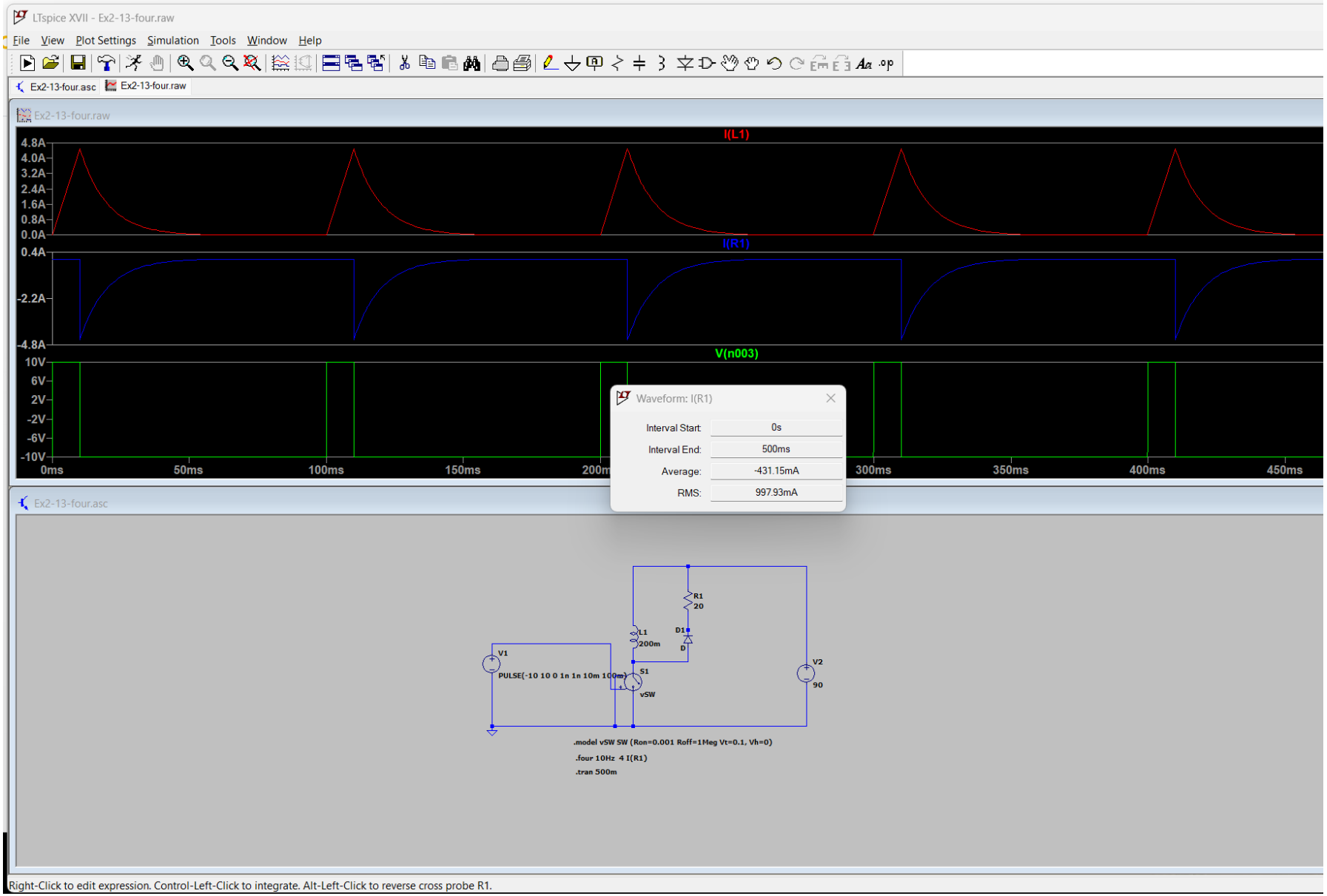
Fourier Analysis

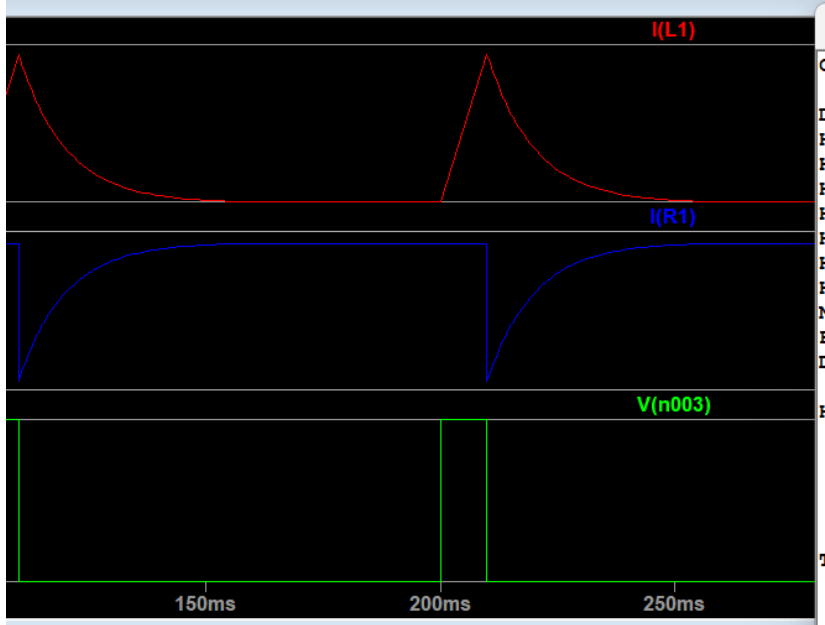


Fourier Analysis – up to 4th harmonic



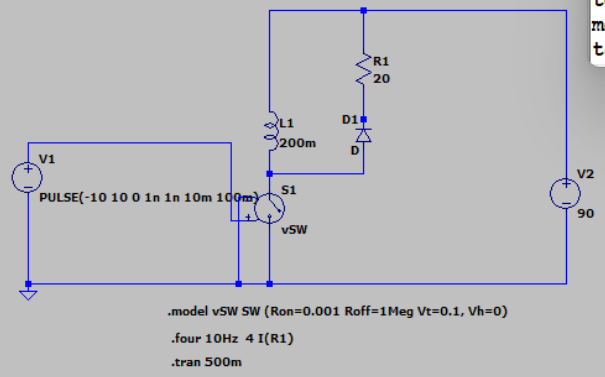
Example 2-13 with Vcc=90 V and Initial pulse voltage = -10V





SPICE Error Log: C:\bE2025\TF25\PowerElectronics\LT\Ex2-13-four.log

```
Circuit: * C:\bE2025\TF25\PowerElectronics\LT\Ex2-13-four.asc

Direct Newton iteration for .op point succeeded.
Heightened Def Con from 0.01 to 0.01
Heightened Def Con from 0.01 ++++++++++++++++++to 0.01
Heightened Def Con from 0.11 to 0.11
Heightened Def Con from 0.11 ++++++++++++++++++to 0.11
Heightened Def Con from 0.21 ++++++++++++++++++to 0.21
Heightened Def Con from 0.31 ++++++++++++++++++to 0.31
Heightened Def Con from 0.41 ++++++++++++++++++to 0.41
N-Period=1
Fourier components of I(r1)
DC component:-0.431161

Harmonic     Frequency     Fourier     Normalized    Phase       Normalized
 Number        [Hz]       Component     Component   [degree]     Phase [deg]
    1       1.000e+01    7.604e-01    1.000e+00    -156.52°       0.00°
    2       2.000e+01    5.620e-01    7.390e-01     146.60°     303.12°
    3       3.000e+01    4.214e-01    5.541e-01     100.44°     256.96°
    4       4.000e+01    3.335e-01    4.385e-01      57.75°     214.27°
Total Harmonic Distortion: 102.249702%(134.228905%)



Date: Mon Aug 25 14:57:54 2025
Total elapsed time: 0.054 seconds.

tnom = 27
temp = 27
method = modified trap
totiter = 14489
```



Summary of Power Computation

- Instantaneous power $p(t) = v(t)i(t)$
- Average Power

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt$$

- RMS values

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

- Apparent Power $S = V_{\text{rms}} I_{\text{rms}}$
- Power factor

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}}$$

- In steady-state periodic condition:
  - L: average voltage = 0
  - C: average current = 0
- Non-sinusoidal periodic signals
  - Use Fourier series of sinusoids
  - Total Power:

$$P = \sum_{n=0}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,\,\text{rms}}\, I_{n,\,\text{rms}} \cos(\theta_n - \phi_n)$$