Chapter 2-A

POWER COMPUTATIONS(A)

focus: non-sinusoidal voltages/currents

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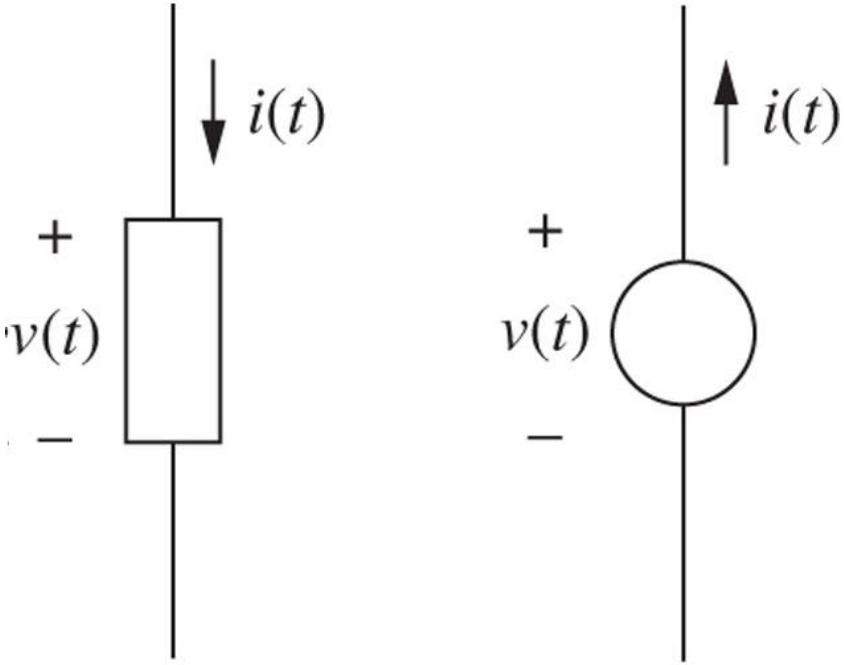
Figure 2.1

THE PASSIVE & (Source following)SIGN CONVENTION



Passive: Absorbing Power (p(t)>0)

Supplying Power (Passive p(t)<0, Source Following p(t)>0))

- Instantaneous Power

$$p(t) = v(t)i(t)$$

- Energy

$$W = \int_{t_1}^{t_2} p(t)\,dt$$

- Average Power

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt$$

$$P = \frac{W}{T}$$

EXAMPLE 2-1

Power and Energy

Voltage and current, consistent with the passive sign convention, for a device are shown in Fig. 2-2*a* and *b*. (*a*) Determine the instantaneous power $p(t)$ absorbed by the device. (*b*) Determine the energy absorbed by the device in one period. (*c*) Determine the average power absorbed by the device.

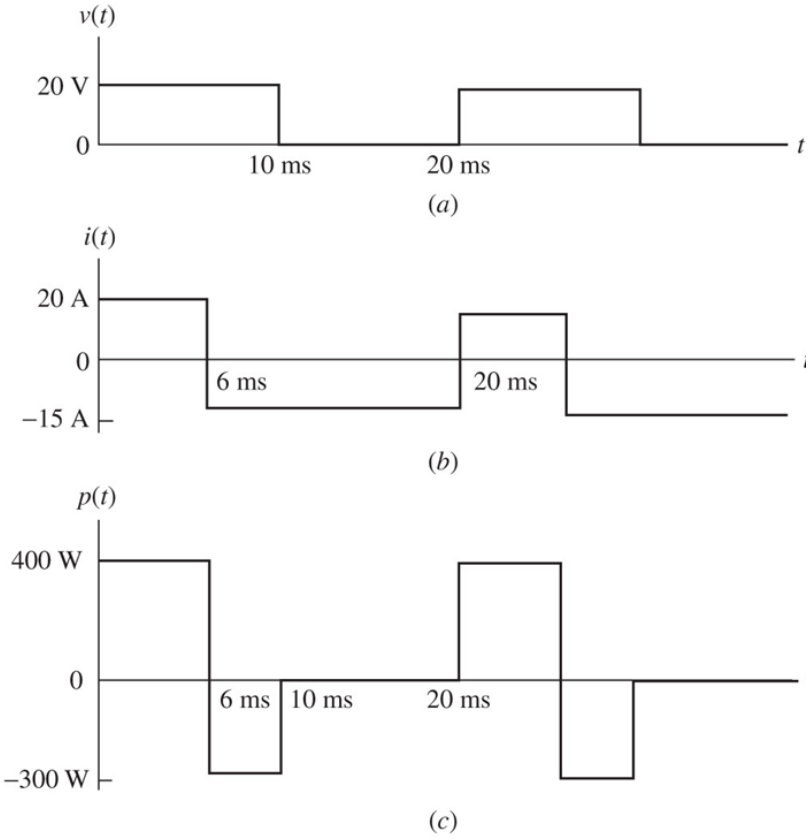


Figure 2.2

Calculation using Smath Studio

$T := 20 \cdot 10^{-3}$

$v1 := 20$ for $0<t<6$ ms

$v2 := 20$ for $10\text{ms} < t < 10\text{ms}$

$v3 := 0$ for $10\text{ms} < t < 20\text{ms}$

$i1 := 20$ for $0<t<6$ ms

$i2 := -15$ for $10\text{ms} < t < 10\text{ms}$

$i3 := -15$ for $10\text{ms} < t < 20\text{ms}$

instantaneous power

$p1 := v1 \cdot i1 = 400$ absorbed

$p2 := v2 \cdot i2 = -300$ supplied

$p3 := v3 \cdot i3 = 0$

Energy

$$W := \int_0^{0.006} p1 \, dt + \int_{0.006}^{0.01} p2 \, dt + \int_{0.01}^{0.02} p3 \, dt = 1.2 \quad J$$

Average Power

$$P := \frac{1}{T} \cdot \left(\int_0^{0.006} p1 \, dt + \int_{0.006}^{0.01} p2 \, dt + \int_{0.01}^{0.02} p3 \, dt \right) = 60 \quad W$$

$$P := \frac{W}{T} = 60 \quad W$$

- Average Power Calculation for a dc source

$$P_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_{dc}\,i(t)\,dt$$

$$P_{dc} = V_{dc}\left[\frac{1}{T}\int_{t_0}^{t_0+T} i(t)\,dt\right]$$

$$\boxed{P_{dc} = V_{dc}\,I_{avg}}$$

- **Inductors and Capacitors**
- Inductor, for periodic current $i(t+T) = i(t)$
- Stored energy in an inductor $w(t) = \frac{1}{2} L i^2(t)$
- [Steady-state periodic operation] Stored energy in an inductor (when the inductor current is periodic) is at the end of one period is the same as at the beginning. → Not net energy transfer → Average power of an inductor is zero $\boxed{P_L = 0}$ → Average voltage of an inductor is zero.

- From V = L di/dt

$$i(t_0 + T) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\,dt + i(t_0)$$

$$i(t_0 + T) - i(t_0) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\,dt = 0$$

$$\text{avg}[v_L(t)] = V_L = \frac{1}{T}\int_{t_0}^{t_0+T} v_L(t)\,dt = 0$$

- [Periodic current] The average voltage across an inductor is zero.

- Capacitor, for periodic voltage $v(t+T) = v(t)$

- stored energy $w(t) = \frac{1}{2} C v^2(t)$

- [Steady-state periodic voltage] stored energy is the same at the end of a periods as at the beginning. → Average power absorbed by a capacitor is zero. $\boxed{P_C = 0}$

- Capacitor, From i = C dv/dt

$$v(t_0 + T) = \frac{1}{C}\int_{t_0}^{t_0+T} i_C(t)\,dt + v(t_0)$$

$$v(t_0 + T) - v(t_0) = \frac{1}{C}\int_{t_0}^{t_0+T} i_C(t)\,dt = 0$$

$$\text{avg}\,[i_C(t)] = I_C = \frac{1}{T}\int_{t_0}^{t_0+T} i_C(t)\,dt = 0$$

- [Periodic voltage] Average current in a capacitor is zero.

Figure 2.3

EXAMPLE 2-2

Power and Voltage for an Inductor

The current in a 5-mH inductor of Fig. in Fig. 2-3*b*. Determine the voltage, insta inductor.

$T := 2 \cdot 10^{-3}$ $L := 5 \cdot 10^{-3}$

$i1(t) := 4 \cdot 10^{3} \cdot t$ for $0<t<T/2$

$i2(t) := -4 \cdot \left(t - \frac{T}{2}\right) \cdot 10^{3} + 4$ for $T/2 <t<T$

$i1(0) = 0$ $i2(T) = 0$

$i1\left(\frac{T}{2}\right) = 4$ $i2\left(\frac{T}{2}\right) = 4$

$v1 := L \cdot \frac{d}{dt} i1(t) = 20$

$v2 := L \cdot \frac{d}{dt} i2(t) = -20$

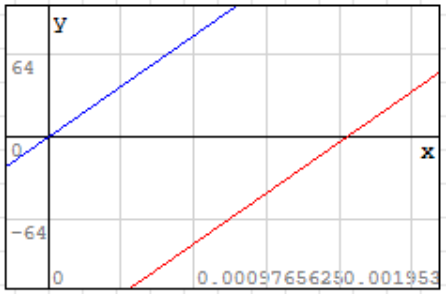
instantaneous power

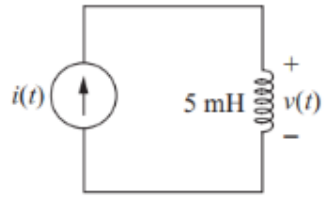
$p1(t) := v1 \cdot i1(t)$

$p2(t) := v2 \cdot i2(t)$

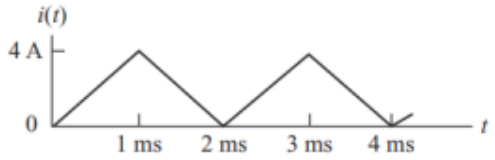
$p1(0) = 0$ $p1\left(\frac{T}{2}\right) = 80$

$p2\left(\frac{T}{2}\right) = -80$ $p2(T) = 0$

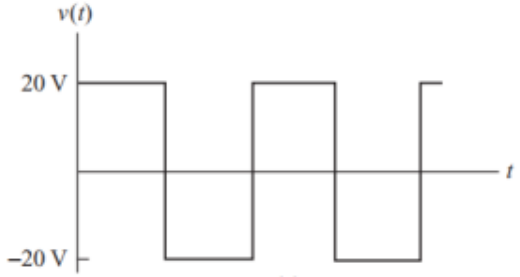




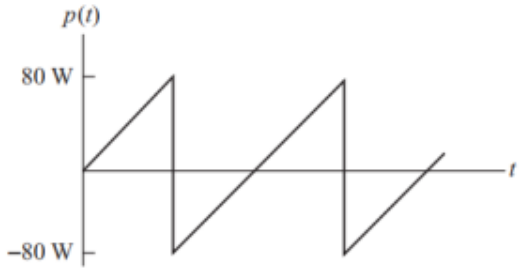
(a)



(b)



(c)



(d)

Figure 2.4

Energy Transfer

Inductors and capacitors are energized and de-energized in Power Electronics

Stored energy must be discharged to avoid escalated increase which leads to damage

Better option is to transfer the stored energy to the load or the source

(EX) ENERGY TRANSFER
FROM THE INDUCTOR
TO THE RESISTOR

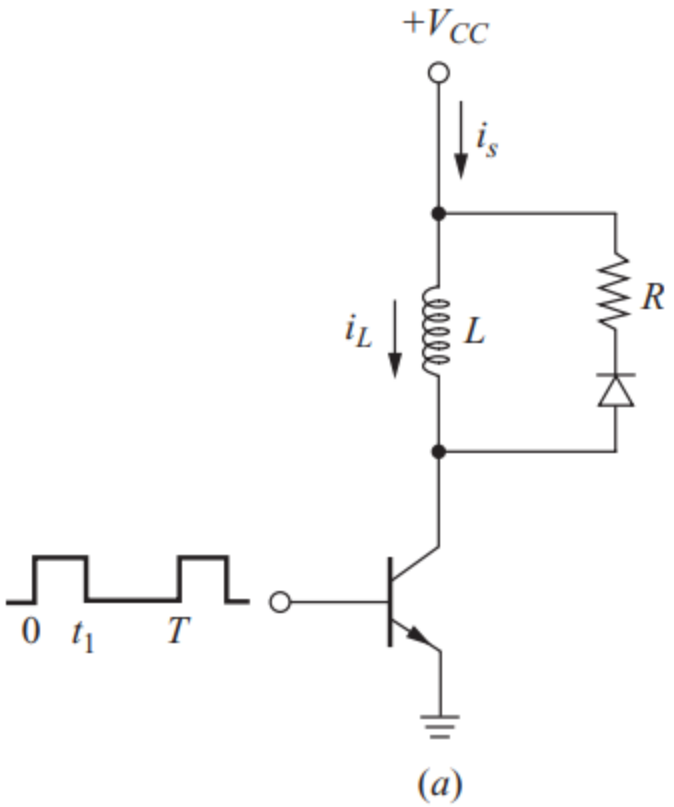
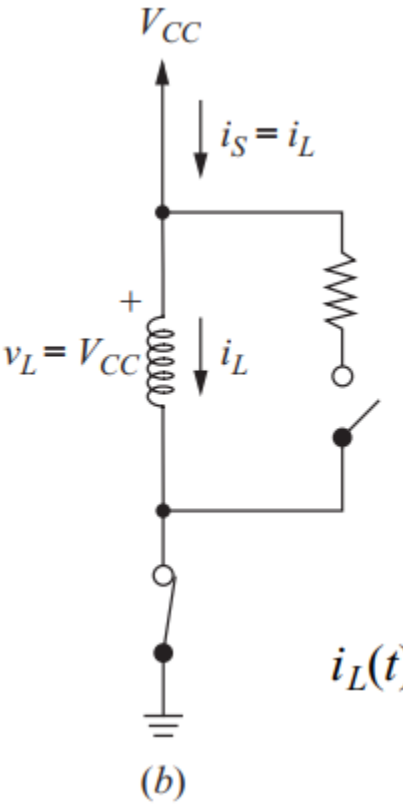


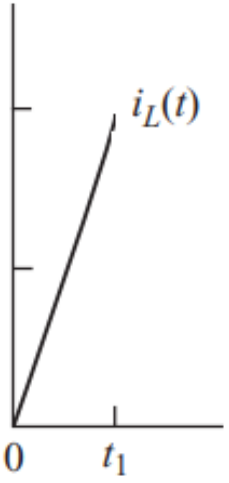
Figure 2.4

SW ON: 0 < t < t1
Diode OFF



$$v_L = V_{CC}$$

$$i_s(t) = i_L(t)$$



$$i_L(t) = \frac{1}{L}\int_0^t v_L(\lambda)\,d\lambda + i_L(0) = \frac{1}{L}\int_0^t V_{CC}\,d\lambda + 0 = \frac{V_{CC}t}{L}$$

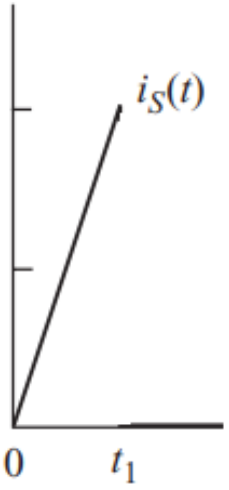
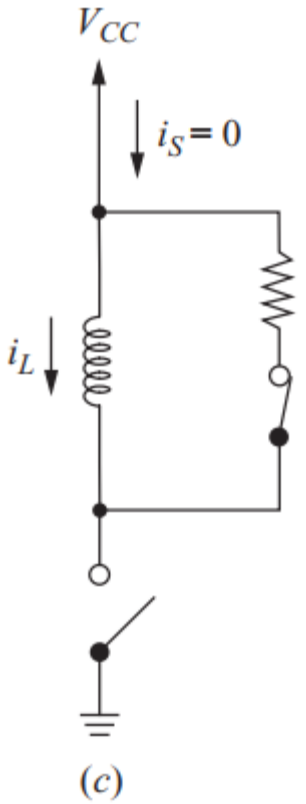


Figure 2.4

SW OFF: t1 <t<T
Diode ON



(c)

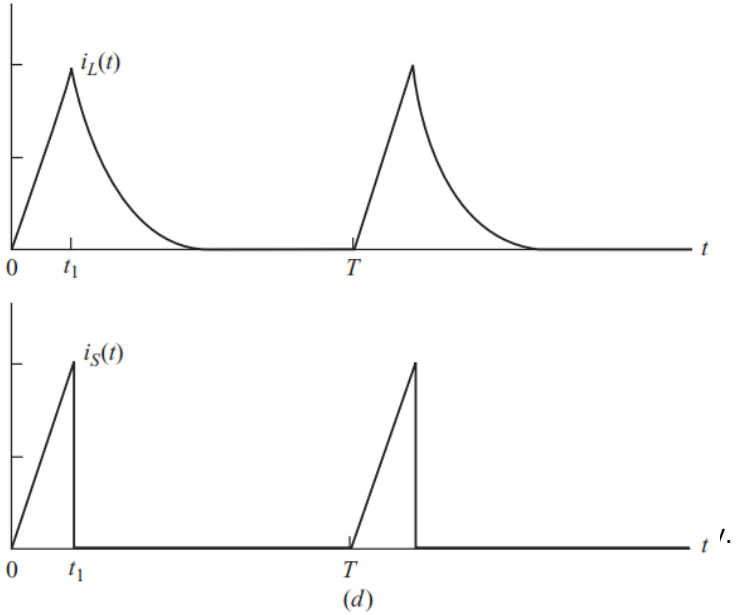
Initial condition for inductor current:

$$i_L(t_1) = \frac{V_{CC} t_1}{L}$$

Inductor current equation:

$$i_L(t) = i_L(t_1) e^{-(t-t_1)/\tau} = \left(\frac{V_{CC} t_1}{L} \right) e^{-(t-t_1)/\tau} \quad t_1 < t < T$$

Time constant: $\tau = L/R$



(d)

- [0 – T] During the switch period
- Average Power by the dc source PS:

$$P_S = V_S I_S = V_{CC}\left[\frac{1}{T}\int_0^T i_s(t)\,dt\right]$$

$$= V_{CC}\left[\frac{1}{T}\int_0^{t_1}\frac{V_{CC}t}{L}\,dt + \frac{1}{T}\int_{t_1}^{T} 0\,dt\right] = \frac{(V_{CC}t_1)^2}{2LT}$$

- Average power absorbed by the resistor load (no average power absorbed by the inductor):

$$P_R = P_S = \frac{(V_{CC}t_1)^2}{2LT}$$

- [0 – T] During the switch period
- Alternative approach for <u>average power absorbed by the resistor load</u>
- Peak energy stored in the inductor (at t = t1):

$$W = \frac{1}{2}Li^2(t_1) = \frac{1}{2}L\left(\frac{V_{CC}t_1}{L}\right)^2 = \frac{(V_{CC}t_1)^2}{2L} \qquad i_L(t_1) = \frac{V_{CC}t_1}{L}$$

- This energy is transferred to the resistor when SW is OFF.

$$P_R = \frac{W}{T} = \frac{(V_{CC}t_1)^2}{2LT}$$

Figure 2.5

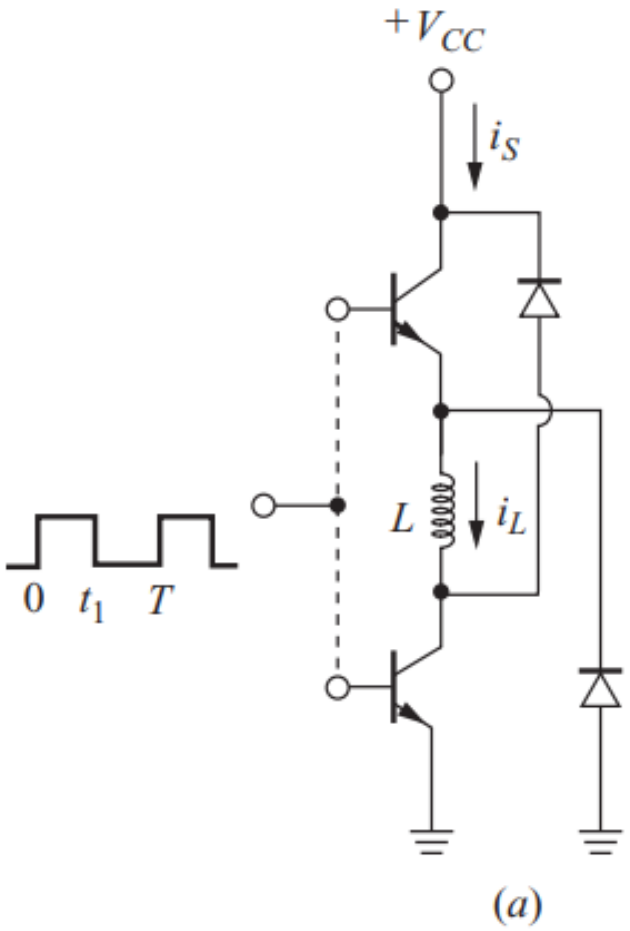
ENERGY RECOVERY

Transfer energy back to the source

SWs are ON and OFF simultaneously

Diodes provide the path for returning the energy stored in the inductor back to the source

SW turns on at t =0 and off at t= t1 in a period T

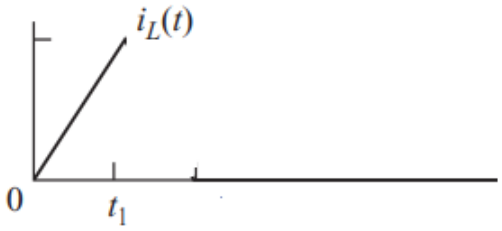


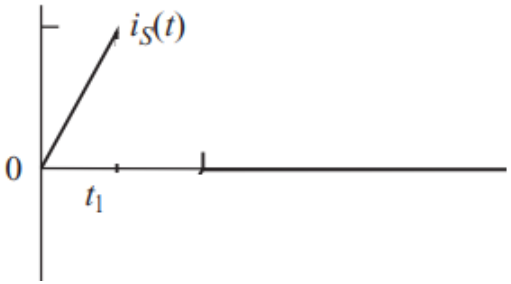
- SW on: o<t<t1
  - Diodes are reverse biased (“off”)
  - Inductor voltage $v_L = V_{CC}$
  - Inductor current

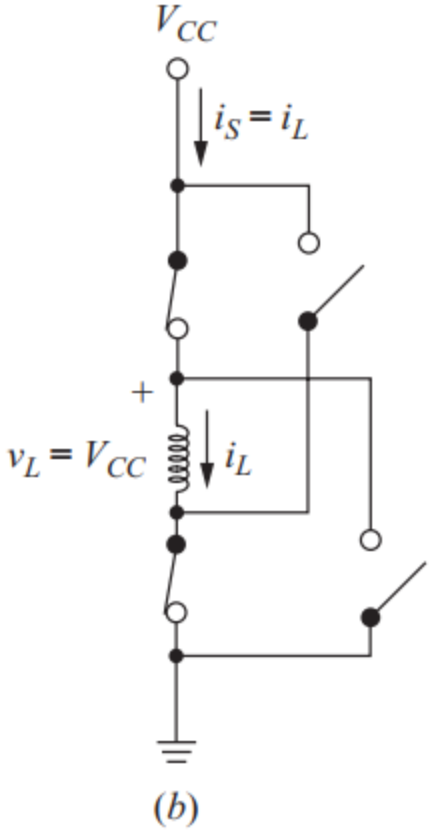
$$i_L(t) = \frac{1}{L}\int_0^t v_L(\lambda)\, d\lambda + i_L(0) = \frac{1}{L}\int_0^t V_{CC}\, d\lambda + 0 = \frac{V_{CC}\, t}{L}$$

  - Source current

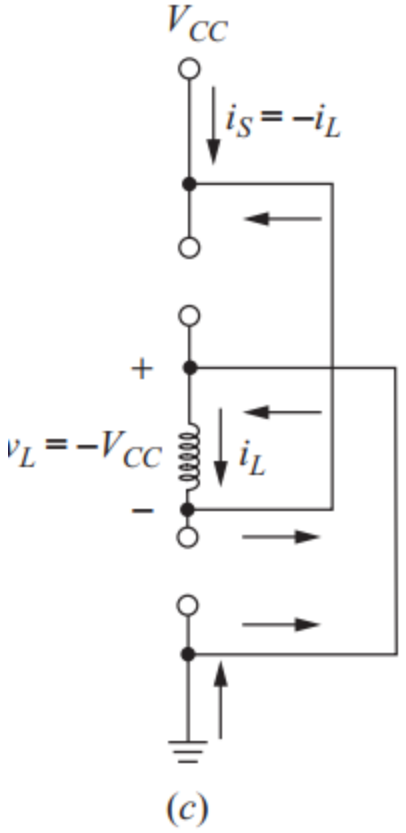
$$i_S(t) = i_L(t)$$







- SW off: t1<t<T
  - Diodes are ON and provides a path for inductor current
  - Inductor voltage is reversed $v_L = -V_{CC}$
  - Inductor current



$$i_L(t) = \frac{1}{L}\int_{t_1}^{t} v_L(\lambda)\,d\lambda + i_L(t_1) = \frac{1}{L}\int_{t_1}^{t} (-V_{CC})\,d\lambda + \frac{V_{CC}t_1}{L}$$

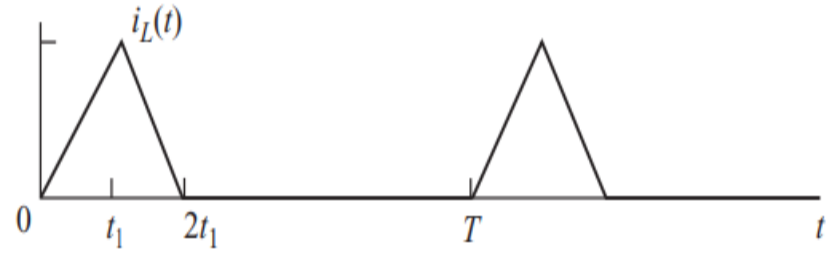
$$= \left(\frac{V_{CC}}{L}\right)[(t_1 - t) + t_1]$$

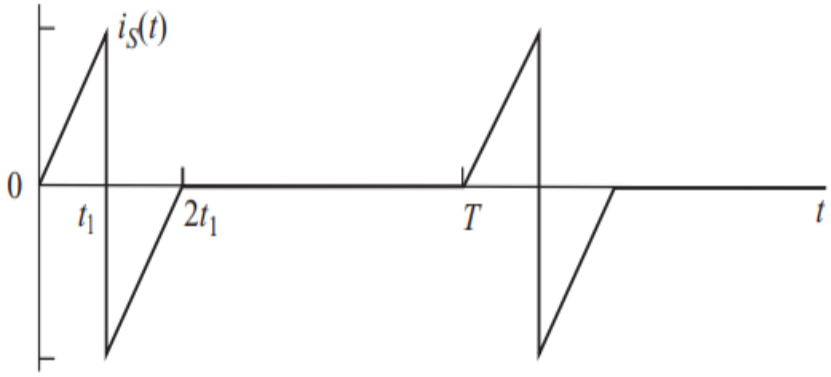
Inductance current=0 when t= 2*t1

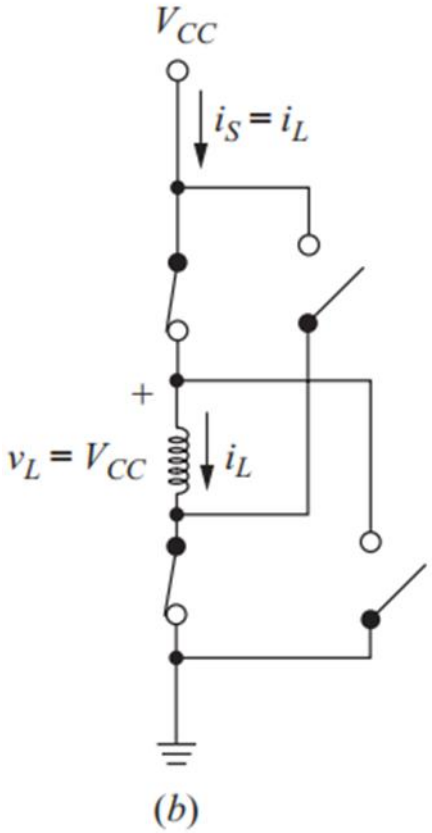
$$i_L(t) = \left(\frac{V_{CC}}{L}\right)(2t_1 - t) \qquad t_1 < t < 2t_1$$

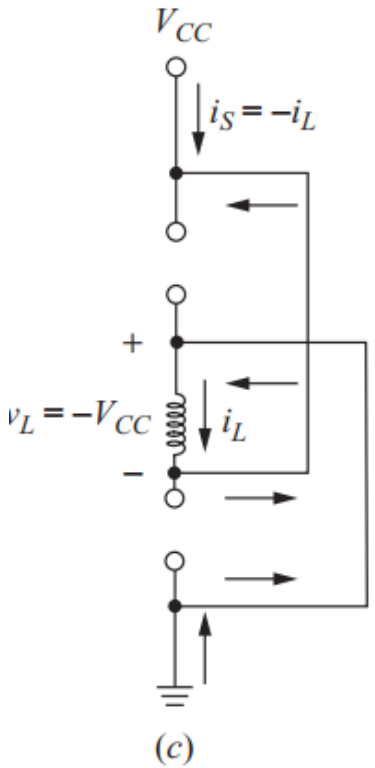
  - Source current $i_S(t) = -i_L(t)$

- Shapes of the currents in the switching period







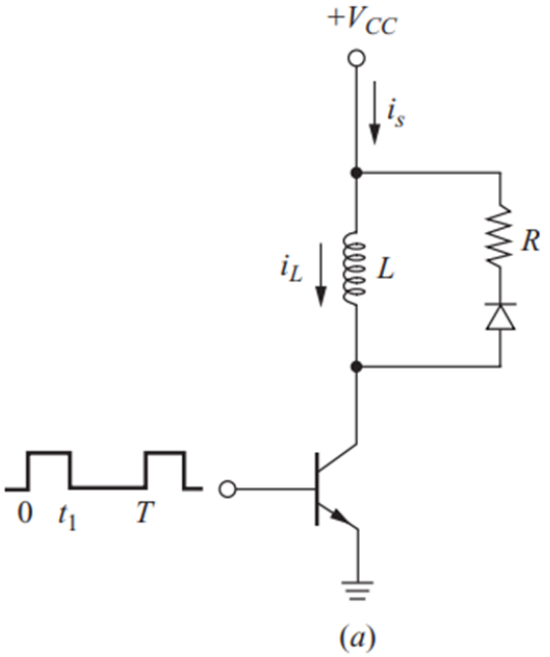


- Source supplies power when the source current ıs positive: SW ON & Diodes OFF
- Source absorbs power when the source current is negative : SW OFF & Diodes ON

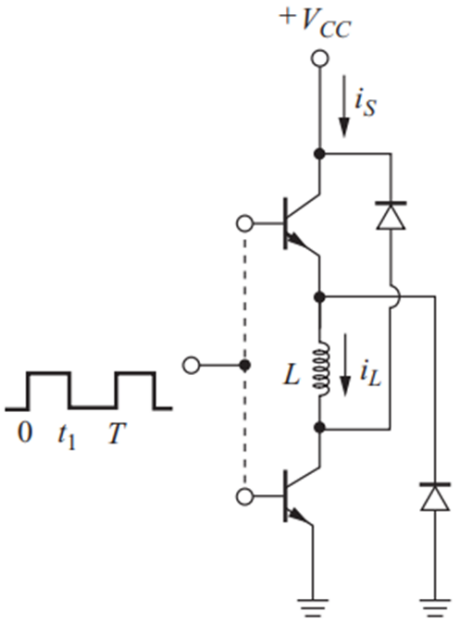
- Example 2-3

Energy Recovery

The circuit of Fig. 2-4*a* has $V_{CC} = 90$ V, $L = 200$ mH, $R = 20\ \Omega$, $t_1 = 10$ ms, and $T = 100$ ms. Determine (*a*) the peak current and peak energy storage in the inductor, (*b*) the average power absorbed by the resistor, and (*c*) the peak and average power supplied by the source. (*d*) Compare the results with what would happen if the inductor were energized using the circuit of Fig. 2-5*a*.



(*a*)



(*a*)

• Example 2-3

EXample 2-3

Energy Recovery

The circuit of Fig. 2-4*a* has $V_{CC} = 90$ V, $L = 200$ mH, $R = 20\ \Omega$, $t_1 = 10$ ms, and $T = 100$ ms. Determine (*a*) the peak current and peak energy storage in the inductor, (*b*) the average power absorbed by the resistor, and (*c*) the peak and average power supplied by the source. (*d*) Compare the results with what would happen if the inductor were energized using the circuit of Fig. 2-5*a*.

$T := 100 \cdot 10^{-3}$ $L := 200 \cdot 10^{-3}$ $R := 20$ $Vcc := 90$ $t1 := 10 \cdot 10^{-3}$

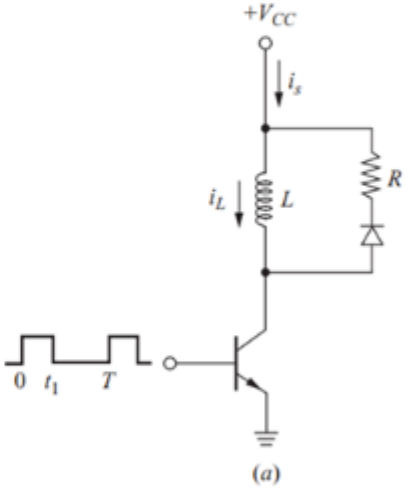
$iL(t) := \frac{Vcc}{L} \cdot t$ $\frac{Vcc}{L} = 450$

$iLpeak := iL(t1) = 4.5$

$WLpeak := \frac{1}{2} \cdot L \cdot iLpeak^2 = 2.025$

$WR := WLpeak = 2.025$

$PR := \frac{WR}{T} = 20.25$



(*a*)

Power by the source

$is(t) := iL(t)$ for SW ON period

$ps(t) := Vcc \cdot is(t)$

Average power by the source

$$Ps := \frac{1}{T} \cdot \left(\int_0^{t1} ps(t)\, dt + \int_{t1}^{T} 0\, dt \right) = 20.25 \quad W$$

Average source current

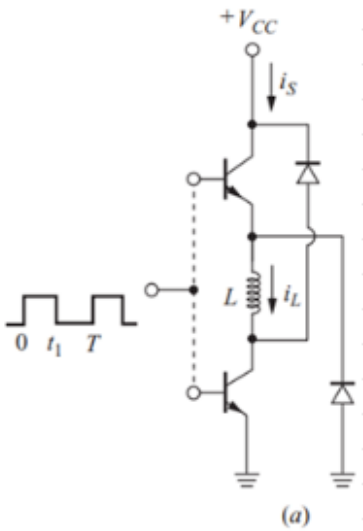
$$Is := \frac{1}{T} \cdot \left(\int_0^{t1} is(t)\, dt + \int_{t1}^{T} 0\, dt \right) = 0.225 \quad A$$

- Example 2-3 (continued)

$$iL1(t) := \frac{Vcc}{L} \cdot t \qquad 0 < t < t1$$

$$iL2(t) := \frac{Vcc}{L} \cdot (2 \cdot t1 - t) \qquad t1 < t < 2*t1$$

$$iL3(t) := 0 \qquad 2*t1 < t < T$$



$$iS1(t) := iL1(t) \qquad 0 < t < t1$$

$$iS2(t) := -iL2(t) \qquad t1 < t < 2*t1$$

$$iS3(t) := 0 \qquad 2*t1 < t < T$$

$$ps1(t) := Vcc \cdot iS1(t) \qquad 0 < t < t1$$

$$ps2(t) := Vcc \cdot iS2(t) \qquad t1 < t < 2*t1$$

$$ps3(t) := 0 \qquad 2*t1 < t < T$$

$$\frac{Vcc}{L} \cdot 2 \cdot t1 = 9$$

Average Power by the source

$$PS := \frac{1}{T} \cdot \left(\int_0^{t1} ps1(t)\, dt + \int_{t1}^{2 \cdot t1} ps2(t)\, dt \right) = 0$$

$$ps1(t1) = 405 \qquad ps2(t1) = -405$$